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Measurements of Helical Magnetic Fields Using Flat Rotating Coils

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1 Introduction

Rotating coils are commonly used to measure the magnetic field coefficients (a_n, b_n) inside straight magnets [1]. They can also be employed to determine the multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Ref. [2]) in helical magnets.

We use a cylindrical coordinate system (r, θ, s) where s designates the coordinate along the longitudinal magnet axis. The area of a flat rotating coil ranges from r_1 to r_2 and from s_1 to s_2. The magnetic flux through the coils is

$$\Phi(\theta) = N \int_{s_1}^{s_2} \int_{r_1}^{r_2} B_\theta(r, \theta)\ dr\ ds \tag{1}$$

where N is the number of coils windings. For rotating coils one has $\theta = \omega t$ and the induced voltage

$$U = -\frac{d\Phi}{dt} \tag{2}$$

is proportional to the angular velocity ω.

We will present formulae for the magnetic flux through a rotating coil for straight and helical magnets. Assuming the induced voltage is parameterized in terms of ordinary multipole coefficients (a_n, b_n) conversion formulas will be given to obtain the helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$.

2 Straight Magnetic Fields

The azimuthal field in straight magnets can be expressed in multipole coefficients (a_n, b_n) as [2]

$$B_\theta = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{r_0}\right)^n \left[b_n \cos\Big((n+1)\theta\Big) - a_n \sin\Big((n+1)\theta\Big)\right]. \tag{3}$$

B_0 is the main magnetic field and r_0 a reference radius. The magnetic flux (1) becomes

$$\Phi(\theta) = N B_0 (s_2 - s_1) \sum_{n=0}^{\infty} K_n \left[b_n \cos\Big((n+1)\theta\Big) - a_n \sin\Big((n+1)\theta\Big)\right] \tag{4}$$

where the coefficients K_n are defined by

$$K_n = \frac{r_0}{n+1}\left[\left(\frac{r_2}{r_0}\right)^{(n+1)} - \left(\frac{r_1}{r_0}\right)^{(n+1)}\right] \tag{5}$$

and result from the integration over r in (1).

3 Helical Magnetic Fields

The azimuthal helical field can be written in terms of helical multipole coefficients $(\tilde{a}_n, \tilde{b}_n)$ (cf. Ref. [2]) as

$$B_\theta = \frac{B_0}{kr}\sum_{n=0}^{\infty} f_n I_{n+1}\Big((n+1)kr\Big)\Big[\tilde{b}_n \cos\Big((n+1)(\theta - ks)\Big) - \tilde{a}_n \sin\Big((n+1)(\theta - ks)\Big)\Big], \tag{6}$$

with

$$f_n = \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}}\frac{1}{r_0^n k^n}. \tag{7}$$

Here B_0 denotes the transverse component of the main field close to the magnet axis. This field is vertical at the location $s = 0$. The magnetic flux (1) can be expressed as

$$\Phi(\theta) = NB_0\sum_{n=0}^{\infty} R_n\Big[\hat{b}_n \cos\Big((n+1)\theta\Big) - \hat{a}_n \sin\Big((n+1)\theta\Big)\Big] \tag{8}$$

with new coefficients

$$R_n = \frac{f_n}{k}\int_{r_1}^{r_2} \frac{1}{r} I_{n+1}\Big((n+1)kr\Big)\,dr. \tag{9}$$

The integral in (9) can be computed numerically. In (8) new magnetic multipole coefficients

$$\begin{aligned} \hat{a}_n &= +\tilde{a}_n T_n + \tilde{b}_n S_n, \\ \hat{b}_n &= -\tilde{a}_n S_n + \tilde{b}_n T_n. \end{aligned} \tag{10}$$

are used for which

$$\begin{aligned} S_n &= \frac{1}{(n+1)k}\Big[\cos\Big((n+1)ks_2\Big) - \cos\Big((n+1)ks_1\Big)\Big] \\ &= -\frac{2}{(n+1)k}\sin\frac{(n+1)k(s_2 - s_1)}{2}\sin\frac{(n+1)k(s_2 + s_1)}{2} \end{aligned} \tag{11}$$

and

$$
\begin{aligned}
T_n &= \frac{1}{(n+1)k}\left[\sin\Big((n+1)ks_2\Big) - \sin\Big((n+1)ks_1\Big)\right] \\
&= +\frac{2}{(n+1)k}\sin\frac{(n+1)k(s_2-s_1)}{2}\cos\frac{(n+1)k(s_2+s_1)}{2}
\end{aligned}
\tag{12}
$$

have been defined.

4 Conversion

We assume now a device that parameterizes the voltage (2) in terms of multipole coefficients (a_n, b_n) for straight magnets. If the measured magnetic field has helical symmetry, the coefficients $(\tilde{a}_n, \tilde{b}_n)$ in Eq. (8) can be derived as

$$
\begin{aligned}
\tilde{a}_n &= \frac{K_n}{R_n}(s_2 - s_1)\cdot\frac{a_nT_n - b_nS_n}{S_n^2+T_n^2}, \\
\tilde{b}_n &= \frac{K_n}{R_n}(s_2 - s_1)\cdot\frac{a_nS_n + b_nT_n}{S_n^2+T_n^2}.
\end{aligned}
\tag{13}
$$

We consider three special cases.
(a) Measuring coil of one helical wavelength with $s_1 = s$, $s_2 = s + \lambda$.
From equations (11) and (12) we obtain

$$
S_n = T_n = 0 \tag{14}
$$

and with (10)

$$
\hat{a} = \hat{b} = 0. \tag{15}
$$

The magnetic flux (8) is therefore zero and the coefficients can not be obtained.

(b) Measuring coil of half helical wave length with $s_1 = 0$, $s_2 = \lambda/2$.
In this case one has

$$
S_n = \begin{cases} -\frac{2}{(n+1)k} & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{cases} \qquad \text{and} \qquad T_n = 0.
$$

Only coefficients with n even (i.e. helical dipole, sextupole etc. coefficients) can be measured. For those we have

$$
\begin{aligned}
\tilde{a}_n &= +\frac{K_n}{R_n}\frac{(n+1)\pi}{2}b_n, \\
\tilde{b}_n &= -\frac{K_n}{R_n}\frac{(n+1)\pi}{2}a_n.
\end{aligned}
\tag{16}
$$

(c) Infinitely short measuring coil with $s_1 = s$, $s_2 = s + ds$.
Expanding (11) and (12) to first order in ds we obtain

$$\begin{aligned} S_n &= -\sin\Big((n+1)ks\Big)ds, \\ T_n &= +\cos\Big((n+1)ks\Big)ds \end{aligned} \tag{17}$$

and

$$\begin{aligned} \tilde{a} &= \frac{K_n}{R_n}\Big[+a_n\cos\Big((n+1)ks\Big)+b_n\sin\Big((n+1)ks\Big)\Big], \\ \tilde{b} &= \frac{K_n}{R_n}\Big[-a_n\sin\Big((n+1)ks\Big)+b_n\cos\Big((n+1)ks\Big)\Big]. \end{aligned} \tag{18}$$

If in addition $s = 0$, the $(\tilde{a}_n, \tilde{b}_n)$ can by obtained from the (a_n, b_n) by multiplication with K_n/R_n.

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References

[1] P. Schmüser, "Magnetic measurements of the superconducting HERA magnets and analysis of systematic errors", DESY HERA-p 92-1 (1992).

[2] W. Fischer, "Magnetic field error coefficients for helical dipoles", RHIC/AP/83 and AGS/RHIC/SN/17 (1996).