RHIC Magnetic Measurements: Definitions and Conventions

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1. Introduction:

This note describes the formalism and conventions used by the RHIC magnetic measurements group in defining and measuring the multipoles in magnets of various multipolarities, such as dipoles, quadrupoles, etc. Procedures are described for obtaining multipoles for tracking studies when the actual configuration of a magnet in the accelerator is different from that used in the measurements. Much of the information in this note can be found in earlier publications[1] and numerous Magnet Test Group notes by J. Herrera[2]. This note is an attempt to combine all the relevant information at one place, in a manner consistent with measurement and tracking conventions used at RHIC.

2. Magnetic Field Components in Cylindrical Coordinates:

The region of interest within the magnet aperture does not include electric currents. The magnetic field in this region is most conveniently described in terms of a scalar magnetic potential, Φ_m. We have,

$$\nabla \times \mathbf{H} = 0 \;\Rightarrow\; \mathbf{H} = -\nabla\Phi_m \tag{1}$$

$$\nabla.\mathbf{B} = 0 \;\Rightarrow\; \nabla^2\Phi_m = 0; \quad \text{Laplace's Eqn for Scalar Potential} \tag{2}$$

The magnets in RHIC have a cylindrical geometry. The same applies to the rotating coil measuring system used for measurements of harmonics in these magnets. This makes cylindrical coordinates a natural choice for expressing the components of the magnetic induction, **B**.

Solving the Laplace's equation for the scalar magnetic potential, it can be shown that the components of **B** in the region of the magnet aperture (containing the origin) can be expressed in the form

$$B_r(r,\theta) = \sum_{n=1}^{\infty} C(n)(r/R_{ref})^{n-1}\sin[n(\theta-\alpha_n)] \tag{3}$$

$$B_\theta(r,\theta) = \sum_{n=1}^{\infty} C(n)(r/R_{ref})^{n-1}\cos[n(\theta-\alpha_n)] \tag{4}$$

In Eqs.(3) and (4), $C(n)$ are constants (assumed to be positive) having the dimension of magnetic induction (Tesla) and are referred to as the ***strength*** of the $2n$-pole component of the field. The constant α_n essentially controls the orientation of the $2n$-pole field with respect to a chosen coordinate frame and is called the ***phase angle*** of the $2n$-pole component of the field. R_{ref} is an arbitrary ***reference radius***, typically chosen as approximately 5/8 of the magnet coil inner radius for the RHIC magnets. Thus, R_{ref} is chosen as 2.5cm for all the 8cm aperture magnets, 3.1cm for the 10cm aperture D0 dipoles, 4.0cm for the 13cm aperture quadrupoles/correctors and 6.0cm for the 18cm aperture DX magnets.

Since the actual values of $B_r(r,\theta)$ and $B_q(r,\theta)$ must be independent of the choice of a reference radius, it is clear from Eqs.(3) and (4) that the values of $C(n)$ depend on the choice of reference radius, except for the dipole term, n=1. The values of α_n depend on the choice of coordinate axes and rotation of the magnet about its longitudinal axis. Since only the quantity $(n\alpha_n)$ appears in the expansion, a phase angle of α_n is the same as a phase angle of $(\alpha_n + 2\pi/n)$. Thus, without any loss of generality, we may write,

$$0 \leq \alpha_n < 2\pi / n \tag{5}$$

The rotating coil measuring system is designed to obtain the constants $C(n)$ and α_n that characterize the field. Since the values of α_n depend on the choice of coordinate system, a uniform convention is followed for all magnets, as follows:

> *"When looking into the magnet from the LEAD END of the magnet, the positive X-axis points towards the right hand side, while the positive Y-axis points upwards, towards the top of the magnet yoke, as shown in Fig.1. The origin is chosen to be at the center of the magnet's field."*
>
> CONVENTION 1

It should be noted that this measurement coordinate convention is not necessarily the same as the commonly used "MAD" accelerator convention. The implications of this are discussed in detail in Sec. 6.

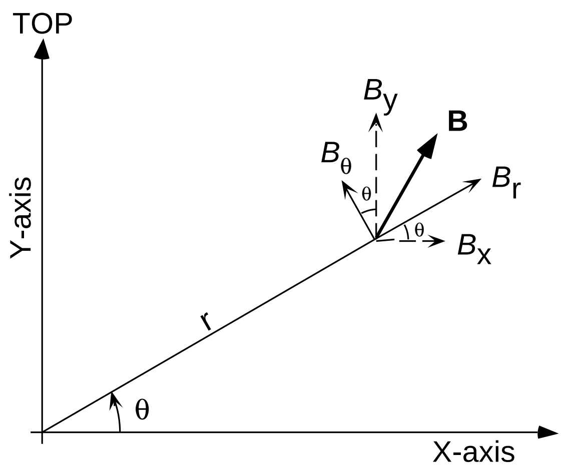

Fig.1 *The coordinate system used in the measurement of magnets, as viewed from the lead end of the magnet. The "Top" denotes top of the magnet yoke.*

3. The Cartesian Components of the Magnetic Field:

Various accelerator codes used for tracking studies prefer to work with the Cartesian, rather than the cylindrical components of the magnetic induction, **B**. To obtain the Cartesian components (see Fig.1), we use,

$$B_x(r,\theta) \;=\; B_r \cos\theta \;-\; B_\theta \sin\theta \tag{6}$$

$$B_y(r,\theta) \;=\; B_r \sin\theta \;+\; B_\theta \cos\theta \tag{7}$$

Substituting for B_r and B_θ from Eqs.(3) and (4), we get,

$$B_x(r,\theta) = \sum_{n=1}^{\infty} C(n)(r / R_{ref})^{n-1} \sin[(n-1)\theta - n\alpha_n] \tag{8}$$

$$B_y(r,\theta) = \sum_{n=1}^{\infty} C(n)(r / R_{ref})^{n-1} \cos[(n-1)\theta - n\alpha_n] \tag{9}$$

In accordance with the prevailing practice at US laboratories, the limits on the summations are changed by re-writing the above equations as

$$B_x(r,\theta) = \sum_{n=0}^{\infty} C(n+1)(r / R_{ref})^{n} \sin[n\theta - (n+1)\alpha_{n+1}] \tag{10}$$

$$B_y(r,\theta) = \sum_{n=0}^{\infty} C(n+1)(r / R_{ref})^{n} \cos[n\theta - (n+1)\alpha_{n+1}] \tag{11}$$

We now define the **Normal** and **Skew** components of the 2(n+1)-pole field as:

$$B_n = \;\; C(n+1)\cos[(n+1)\alpha_{n+1}] \qquad \text{NORMAL COMPONENT: 2(}n\text{+1)-POLE FIELD} \tag{12}$$

$$A_n = -C(n+1)\sin[(n+1)\alpha_{n+1}] \qquad \text{SKEW COMPONENT: 2(}n\text{+1)-POLE FIELD} \tag{13}$$

In terms of the normal and skew components, the Cartesian components of **B** are:

$$B_x(r,\theta) = \sum_{n=0}^{\infty} (r / R_{ref})^{n} [B_n \sin(n\theta) + A_n \cos(n\theta)] \tag{14}$$

$$B_y(r,\theta) = \sum_{n=0}^{\infty} (r / R_{ref})^{n} [B_n \cos(n\theta) - A_n \sin(n\theta)] \tag{15}$$

A 2m-pole magnet is "normal" if the corresponding skew component is zero. Similarly, a magnet is "skew" if the corresponding normal component is zero. From Eqs.(12) and (13), we get,

$$\alpha_m = 0 \quad \text{or} \quad \pi / m \qquad \text{NORMAL } 2m\text{-POLE MAGNET} \qquad (16)$$

$$\alpha_m = \pi / (2m) \quad \text{or} \quad 3\pi / (2m) \qquad \text{SKEW } 2m\text{-POLE MAGNET} \qquad (17)$$

Out of the two possible values in Eqs.(16) and (17), $\alpha_m = 0$ gives $B_{m-1} > 0$, whereas $\alpha_m = \pi/m$ gives $B_{m-1} < 0$. This merely corresponds to a reversal in the direction of the field. Similarly, for a skew magnet, $\alpha_m = 3\pi/2m$ gives $A_{m-1} > 0$ whereas $\alpha_m = \pi/2m$ gives $A_{m-1} < 0$. This leads us to another convention followed in the measurements of field components:

> *"All normal magnets are powered such that the fundamental (2m-pole) term has a phase angle of* a_m *» 0. All skew magnets are powered such that* a_m *» 3p/(2m). The measurement reference frame is then rotated by a small angle such that* a_m *is exactly zero for normal magnets and is 3p/(2m) for skew magnets"*

CONVENTION 2

The locations of magnetic "poles" for any multipolar component are defined by maxima or minima of the radial component, B_r. Using phase angles given by Eqs.(16) and (17), and Eq.(3) for B_r, we get, under the conditions of convention 2,

$$\theta = \pi / (2m), \quad 5\pi / (2m), \ldots. \qquad \text{SOUTH POLES: } 2m\text{-POLE NORMAL MAGNET} \qquad (18)$$

$$\theta = 3\pi / (2m), \quad 7\pi / (2m), \ldots. \qquad \text{NORTH POLES: } 2m\text{-POLE NORMAL MAGNET} \qquad (19)$$

$$\theta = 0, \quad 2\pi / m, \ldots. \qquad \text{SOUTH POLES: } 2m\text{-POLE SKEW MAGNET} \qquad (20)$$

$$\theta = \pi / m, \quad 3\pi / m, \ldots. \qquad \text{NORTH POLES: } 2m\text{-POLE SKEW MAGNET} \qquad (21)$$

The configurations for "positive" normal and skew dipole and quadrupole fields are shown in Fig.2. It is also clear that a "positive" skew magnet ($A_{m-1} > 0$) is obtained by rotating a "positive" normal magnet ($B_{m-1} > 0$) clockwise by an angle of $\pi/(2m)$. It should be recalled that the magnet is being viewed from the lead end.

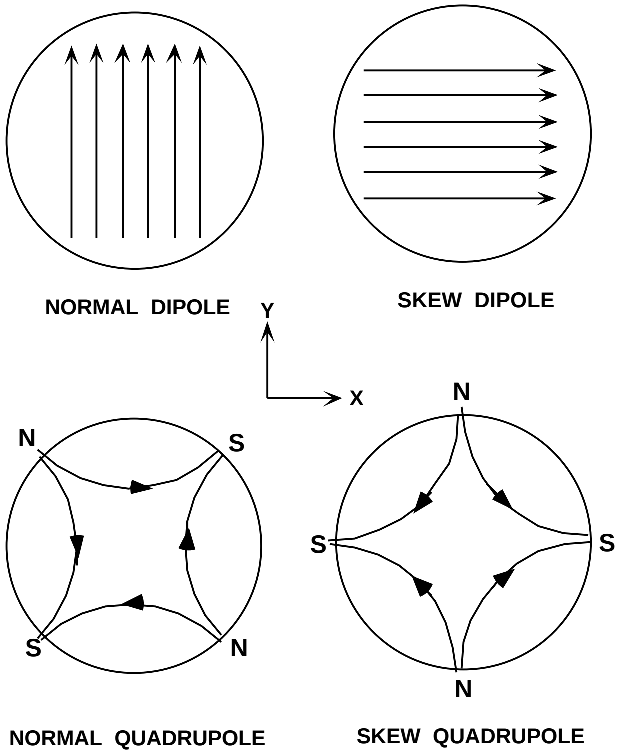

Fig.2 *Views from the lead end for normal and skew dipole and quadrupole magnets, illustrating the power supply polarity convention followed in the measurements of magnets. All normal magnets with* 2*m poles have a south pole located at q=p/*(2*m*)*. For a normal quadrupole, this corresponds to a focusing quadrupole for a positive ion beam traveling from the non-lead end towards the lead end. All skew magnets have a south pole located at q*=0.

4. Field Angles:

When the measurements are made with the magnets on precisely leveled stands employing a measuring coil equipped with gravity sensors, it is possible to choose the x-axis of the measurement reference frame to be precisely in the horizontal plane. With this choice of coordinate frame, the phase angle α_m in a 2m-pole magnet is nearly equal to zero (normal magnet) or $3\pi/(2m)$ (skew magnet), but not exactly equal due to construction features of the magnet. This difference from the ideal value is referred to as **Field Angle**. Typical values of field angle are ~ 1 mrad for almost all the RHIC magnets, except for the 8cm aperture corrector magnets where the field angles are typically ~5mrad.

When a magnet has a **positive field angle**, it means that the measured phase angle $\alpha_m > \alpha_{\text{ideal}}$ [α_{ideal} = 0 for normal magnets, $3\pi/(2m)$ for skew]. The poles (locations of extrema of B_r) of such a magnet are shifted counterclockwise from the ideal positions [see Eq.(3)]. In order to properly orient the field, such a magnet must be rotated clockwise around its axis, as viewed from the lead end. This is illustrated for a normal quadrupole magnet in Fig. 3. Similarly, a magnet with a negative field angle must be rotated counterclockwise, as viewed from the lead end, in order to properly orient the field. All the arc dipoles and arc quadrupoles in RHIC, for example, are installed in this way based on the measured field angles. *For magnets that are not corrected for field angle in installation, the measured field harmonics should be rotated appropriately before using in the beam tracking programs (see Sec. 6.2).*

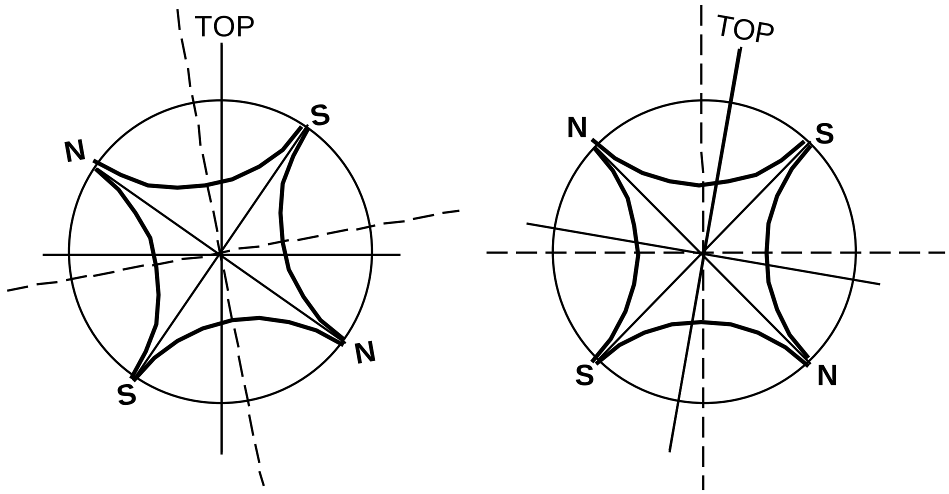

Fig. 3 *Alignment of a normal quadrupole having a positive field angle. The left hand figure shows the magnet with the yoke placed level. The yoke is rotated clockwise in the right hand figure to correct for the positive field angle.*

5. The "Fractional Field Coefficients" or the "Multipoles":

So far, we have described the field components in terms of [$C(n)$, α_n] or [B_n, A_n]. These expansions involve the actual magnitude of the field in Tesla. These coefficients are thus current dependent, even if the field *shape* remains the same. In practice, one is interested in the magnitude of various multipolar terms in relation to the most dominant term. Thus, for a 2*m*-pole magnet, we define the normal and skew multipoles as

$$b_n = 10^4 \times \frac{B_n}{C(m)} = 10^4 \times \frac{C(n+1)}{C(m)} \cos\{(n+1)\alpha_{n+1}\} \quad \text{NORMAL MULTIPOLES (IN UNITS)} \quad (22)$$

$$a_n = 10^4 \times \frac{A_n}{C(m)} = -10^4 \times \frac{C(n+1)}{C(m)} \sin\{(n+1)\alpha_{n+1}\} \quad \text{SKEW MULTIPOLES (IN UNITS)} \quad (23)$$

Since all the terms other than the most dominant term are generally much smaller than $C(m)$, a multiplying factor of 10^4 is used in defining the multipoles in the above equations. The fractional field coefficients b_n and a_n are dimensionless numbers. With the multiplying factor of 10^4, the values are said to be in **"Units"**. Thus "one Unit" represents a field harmonic component whose strength is 10^{-4} of the fundamental harmonic of the field at the reference radius. It should be noted that the numerical values of the multipoles depend on the choice of reference radius. The multipoles reported by the RHIC magnet measurements group are calculated using Eqs.(22)-(23) for all magnet types, while following conventions 1 and 2 for the measurement setup. *In situations where it is physically not possible to power the magnet in accordance with convention 2 due to* *presence of diodes across the magnet, the measured phase angles are transformed at the time of data analysis to conform to convention 2.*

In terms of the normal and skew multipoles, the Cartesian components of field are [see Eqs.(14)-(15)]

$$B_x(r,\theta) = 10^{-4} \times C(m) \sum_{n=0}^{\infty} (r / R_{ref})^n [b_n \sin(n\theta) + a_n \cos(n\theta)] \tag{24}$$

$$B_y(r,\theta) = 10^{-4} \times C(m) \sum_{n=0}^{\infty} (r / R_{ref})^n [b_n \cos(n\theta) - a_n \sin(n\theta)] \tag{25}$$

It is common (and for many purposes, very convenient) to combine the above two equations into a single complex equation as follows:

$$B_y + \mathrm{i}B_x = 10^{-4} \times C(m) \sum_{n=0}^{\infty} (b_n + \mathrm{i}a_n)(\mathbf{z} / R_{ref})^n \tag{26}$$

where $\mathbf{z} = x + \mathrm{i}y = re^{\mathrm{i}\theta}$ is the point of interest in the complex plane.

6. Multipoles for Magnets in the Accelerator:

The purpose of measuring multipoles is to be able to describe the components of the magnetic field at any point in the accelerator. The field components in turn are used for beam tracking studies. The convention of power supply polarity followed during measurements may not apply to the magnets as used in the accelerator. For example, the dipole fields in the arc dipoles of one ring point upwards (α_1=0), thus following convention 2 (see Fig.2), but point downwards ($\alpha_1=\pi$) in the other ring, thus violating convention 2. Similarly, the choice of coordinate system for tracking studies may not coincide with the coordinate system used during the measurements. Such deviations from the measurement conventions require transformation of the reported multipoles before these could be used. This section describes such transformation in various cases of interest.

6.1 Transformation due to choice of a different coordinate system:

Let us first consider a situation where a 2m-pole magnet is powered in the machine in accordance with the measurement convention 2. This means that when viewed from the lead end, the magnet is either a positive normal magnet (α_m~0), or a positive skew magnet (α_m~$3\pi/2m$). In RHIC, following the "MAD" coordinate convention used in most tracking codes, such as TEAPOT, the positive X-axis is always defined to be radially outwards from the center of the ring. We shall denote the machine X-axis by X′, as shown in Figs.4 and 5. Similarly, the positive Y-axis in the machine is always defined to be pointing vertically upwards, and is denoted by Y′ in Figs.4 and 5.

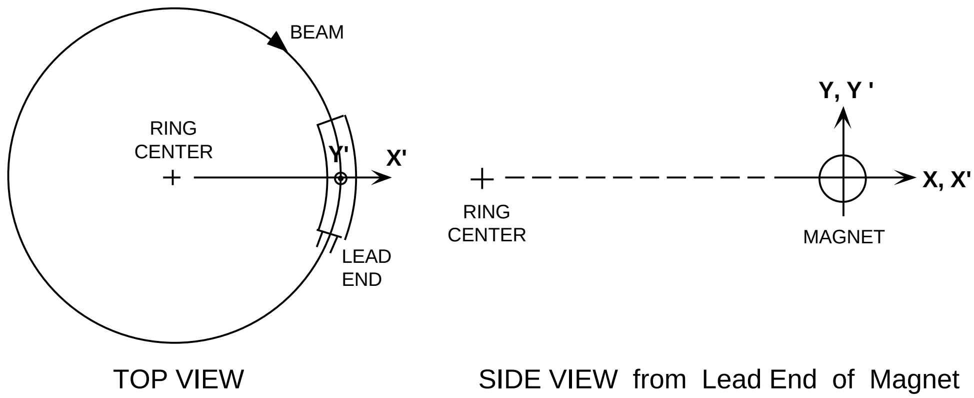

Fig. 4 *The coordinate system X'-Y' used in the machine. The X' axis always points radially outwards from the center of the ring. The Y' axis always points vertically upwards. For a magnet installed with the lead end located in the clockwise direction, the X'-Y' coordinate system is identical to the X-Y coordinate system used in the measurements of multipoles. At the present time, all tracking studies assume that the beams travel in the clockwise direction in both the rings.*

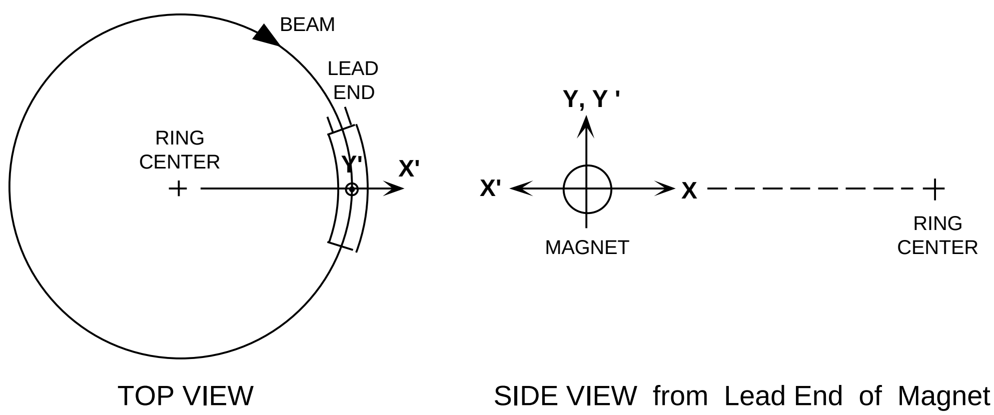

Fig. 5 *The coordinate systems for a magnet installed with the lead end located in the counterclockwise direction. The machine x-axis, denoted by X' is opposite to the X-axis used in the measurements of multipoles. The y-axes in both the coordinate systems are the same, and point vertically upwards.*

A magnet in the ring can have its leads located either in the clockwise direction, as shown in Fig.4, or in the counter-clockwise direction, as shown in Fig.5. It should be recalled that based on convention 1, the X-axis for measurements is always defined as pointing towards the right hand side when viewed from the lead end. Thus, in the case of magnet leads located in the clockwise direction, the X$'$-Y$'$ coordinate system of the machine is identical to the X-Y coordinate system used for measurements. No transformation of multipoles is therefore necessary in this case.

When the magnet leads are located in the counterclockwise direction in the ring, the X′ axis points in a direction opposite to the X-axis of measurements, as shown in Fig.5. The components of field used by the machine are $B_{x'} = -B_x$ and $B_{y'} = B_y$. In analogy to Eqs.(24) and (25), the multipoles b'_n and a'_n in the machine's reference frame are defined by

$$B_{x'}(r,\theta') = 10^{-4} \times C(m) \sum_{n=0}^{\infty} (r / R_{ref})^n [b'_n \sin(n\theta') + a'_n \cos(n\theta')] = -B_x(r,\theta) \qquad (27)$$

$$B_{y'}(r,\theta') = 10^{-4} \times C(m) \sum_{n=0}^{\infty} (r / R_{ref})^n [b'_n \cos(n\theta') - a'_n \sin(n\theta')] = B_y(r,\theta) \qquad (28)$$

To obtain a relationship between (b'_n, a'_n) and (b_n, a_n), we make use of the above two equations along with the following relations:

$$\theta' = \pi - \theta; \quad \cos(n\theta') = (-1)^n \cos(n\theta); \quad \sin(n\theta') = (-1)^{n+1} \sin(n\theta) \qquad (29)$$

Using Eqs.(24)-(25) and (27)-(29), it is easy to show that

$$b'_n = (-1)^n b_n; \quad a'_n = (-1)^{n+1} a_n \qquad \text{REVERSAL OF X-AXIS} \qquad (30)$$

Eq.(30) defines the multipoles in the machine's coordinate frame in terms of the measured multipoles for the case of Fig.5, assuming that the magnet is still powered in accordance with measurement convention 2.

At the time of measurements, it is not known as to how a given magnet is going to be installed in the ring. The magnets database (in the form of FoxPro tables maintained by the magnet measurements group and the MAGBASE tables maintained by the accelerator physics group) therefore contains the multipoles b_n and a_n measured under similar conditions following conventions 1 and 2 for all the magnets.

6.2 Magnets not corrected for field angle during installation:

Most magnets in the RHIC accelerator will be installed after correcting for the field angle. However, for multi-element assemblies, it is only possible to correct for just one of the several elements. Since the reported multipoles are always in a reference frame where the field angle is zero, we must correct for a slight rotation of the magnets in the accelerator. If ε is the field angle of the magnet as installed in the accelerator, then all phase angles are increased by an angle ε. Using the definitions in Eqs.(12)-(13) and (22)-(23) for the normal and skew components, we arrive at the transformation

$$\begin{aligned} b_n &\to \ \ b_n \cos\{(n+1)\varepsilon\} + a_n \sin\{(n+1)\varepsilon\} \\ a_n &\to -b_n \sin\{(n+1)\varepsilon\} + a_n \cos\{(n+1)\varepsilon\} \end{aligned} \qquad \text{FIELD ANGLE } \varepsilon \qquad (31)$$

6.3 Magnets powered in the machine opposite to convention 2:

The power supply convention for measurements is adopted for the sake of uniformity. In actual practice, the magnets in the accelerator could be powered in any of the two possible polarities. For example, the arc dipoles in one ring are powered opposite to the arc dipoles in the other ring. Similarly, the power supply polarity used for a quadrupole depends on the direction of particle motion and on whether it is a focusing or a defocusing quadrupole. Once again, the multipoles used for tracking studies must be suitably modified in order to correctly obtain the components of the magnetic field.

The transformation of multipoles for a change in the direction of current flow are really quite simple to arrive at. Since a change in the direction of current flow changes the direction of all field lines, we have,

$$B_x \to -B_x; \qquad B_y \to -B_y \tag{32}$$

The above transformation can be achieved by having

$$b_n \to -b_n; \qquad a_n \to -a_n \qquad \text{CHANGE IN DIRECTION OF CURRENT KEEPING } C(m) > 0 \tag{33}$$

So far, we have assumed that the quantity $C(m)$ in the expansion of field components in Eq.(24)-(28) is a positive quantity. However, in practice, it is more convenient to implement the transformation in Eq.(33) by simply changing the sign of $C(m)$, instead of changing the sign of every harmonic. As an example, all arc dipoles are installed with their lead ends in the clockwise direction, thus requiring no change in coordinate axes. Thus, one could directly use the measured multipoles for all the arc dipoles in the accelerator with positive $C(1)$ for one ring and a negative $C(1)$ for another. Using a negative value for $C(1)$ has the same effect as changing the sign of all multipoles.

The actual sign of $C(m)$ to be used in a $2m$-pole magnet is determined by convention 2. If this convention is followed, $C(m)$ is positive; otherwise it is negative. Specific examples for dipole and quadrupole magnets in the RHIC arc regions are given in Table I. With this convention for the sign of $C(m)$, we can rewrite the transformation given by Eq.(33) as

$$b_n \to b_n; \qquad a_n \to a_n; \qquad C(m) < 0 \qquad \text{CHANGE IN DIRECTION OF CURRENT} \tag{34}$$

The transformation discussed in this section is summarized in Fig.6. As shown in the figure, there are only two questions to be answered for arriving at the correct expressions for $B_{x'}$ and $B_{y'}$ used by tracking programs. The first question is the location of magnet leads in the ring. For leads located in the clockwise direction, no transformation of the measured multipoles are required. When the leads are located in the counter-clockwise direction, the measured multipoles are transformed according to Eq.(30). The second question addresses the power supply polarity. If the polarity is consistent with the measurements (convention 2), then the sign of $C(m)$ is chosen to be positive, otherwise it is negative. The magnitude of $C(m)$ is given by the product of the measured transfer

TABLE I. Signs of $C(m)$ for Various Magnets in RHIC

Magnet Type	**Beam Enters**	**Sign of $C(m)$**
Arc Dipole (Blue Ring)	Non-Lead End*	$C(1) > 0$
Arc Dipole (Yellow Ring)	Lead End*	$C(1) < 0$
Arc Quad (Focusing)	Non-Lead End	$C(2) > 0$
Arc Quad (Defocusing)	Non-Lead End	$C(2) < 0$
Arc Quad (Focusing)	Lead End	C(2) < 0
Arc Quad (Defocusing)	Lead End	C(2) > 0

* *The direction of beam is not relevant for the dipole magnets.*

function and the magnitude of the current. The Cartesian components of the field can then be obtained using Eqs.(27) and (28). It should be noted that this procedure is uniformly applicable to all types of magnets in RHIC, irrespective of the multipolarity of the most dominant term. In actual practice, the RHIC accelerator physics group implements the transformation somewhat differently, as discussed in the next section.

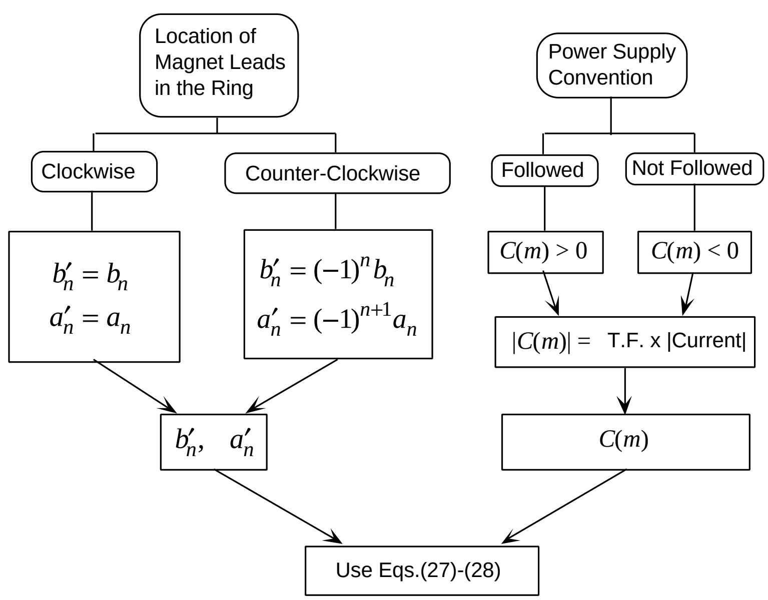

Fig. 6 *Flow chart describing the transformation and sign conventions required to arrive at multipoles for use in tracking studies from the multipoles reported by the RHIC magnetic measurements group. It is assumed that the reported multipoles b_n and a_n are already corrected for field angle using Eq.(31), if required. The RHIC accelerator physics group implements the transformation somewhat differently, as discussed in Sec.7.*

7. Implementation in RHIC Tracking Studies:

The algorithm for transformation discussed in Sec.6, while quite general in approach, can be somewhat confusing in implementation. In particular, it may be difficult to keep track of the sign convention for the quantity $C(m)$. Another "disadvantage" of the approach in Sec.6 is that for some magnets, the most dominant term (for example, b_1 for a normal quadrupole) may become negative when the magnet is installed with the lead end in the counterclockwise direction (see Eq. 30). Furthermore, in RHIC tracking studies, the sign of the quantity $C(m)$ is assigned based on bending/focusing properties of a magnet. This makes it necessary to implement the transformation of multipoles somewhat differently.

A close look at Eq.(30) suggests that we may classify all magnets into two categories– one, where the most dominant term does not change its sign upon a reversal of the X-axis, and another where the most dominant term changes sign. Examples of the first category are normal dipoles, skew quadrupoles, normal sextupoles, skew octupoles, and so on. The primary field in these magnets has a reflection symmetry about the Y-Z plane, as can be seen from Fig.2. Similarly, skew dipoles, normal quadrupoles, skew sextupoles, normal octupoles, and so on fall into the second category. The primary field in these magnets has a reflection anti-symmetry about the Y-Z plane, also seen in Fig.2. We rewrite the transformation in Eq.(30) separately for the two categories of magnets in such a way that the most dominant term of a magnet remains positive:

$$b'_n = (-1)^n b_n; \quad a'_n = (-1)^{n+1} a_n \qquad \text{Magnets with Reflection Symmetry about Y-Z Plane} \tag{35}$$

$$b'_n = (-1)^{n+1} b_n; \quad a'_n = (-1)^n a_n \qquad \text{Magnets with Reflection Anti-Symmetry about Y-Z Plane} \tag{36}$$

With this choice of sign convention (the most dominant term always positive), it is possible to relate the quantity $C(m)$ to the behavior of the field at the origin, helping in the determination of its appropriate sign, without referring to the measurement sign convention 2. Let us first consider a $2(m+1)$-pole normal magnet. The most dominant term in this case will be $b'_m \sim 10^4$. From Eq.(28), the vertical component of the magnetic field at any point along the X′-axis is given by

$$B_{y'}(x',0) = 10^{-4} \times C(m+1) \sum_{n=0}^{\infty} b'_n (x'/R_{ref})^n \tag{37}$$

Taking partial derivatives m times, and evaluating at the origin, we get,

$$\left(\frac{\partial^m B_{y'}}{\partial x'^m} \right)_{x'=0,\, y'=0} = 10^{-4} \times \frac{m!\, C(m+1) b'_m}{R_{ref}^m} \tag{38}$$

Noting that $b'_m \sim 10^4$ and always positive for a normal $2(m+1)$-pole magnet, we can write,

$$C(m+1)=\left(\frac{\partial^m B_{y'}}{\partial x'^m}\right)_{x'=0,\,y'=0}\times\frac{R_{ref}^m}{m!}\qquad \text{NORMAL } 2(m+1)\text{-POLE MAGNET} \quad (39)$$

This relates the quantity $C(m+1)$ to the derivatives of the field at the origin. For example, for a normal dipole, $C(1)$ is positive if $B_{y'}$ is positive. Similarly, for a normal quadrupole, $C(2)$ is positive if $(\partial B_{y'} / \partial x') > 0$, and so on. It should be noted that the power supply polarity sign convention used by the RHIC magnet measurements group is also consistent with this analysis and gives a positive m-th derivative of the field at the origin with a positive $C(m+1)$.

For a skew magnet with a dominant $2(m+1)$-pole term, a similar analysis shows that

$$C(m+1)=\left(\frac{\partial^m B_{x'}}{\partial x'^m}\right)_{x'=0,\,y'=0}\times\frac{R_{ref}^m}{m!}\qquad \text{SKEW } 2(m+1)\text{-POLE MAGNET} \quad (40)$$

7.1 Equivalence with the approach in Section 6:

The purpose of the transformation discussed in this note is to obtain the coefficients to use in order to arrive at the correct components of the magnetic field. The general approach in Sec.6 must yield the same result as the one followed by RHIC accelerator physics group and described in this section. Let us examine this equivalence for a specific case of normal quadrupoles.

At present, the RHIC tracking studies assume that the beams in both the rings travel in the clockwise direction, as shown in Fig.4. Also, in the tracking studies all focusing quadrupoles are assigned a positive strength and all defocusing quadrupoles are assigned a negative strength. Under the assumption of a clockwise beam entering a quadrupole from the non-lead end, as shown in Fig.4, a focusing magnet must be powered in accordance with the measurement sign convention 2. The conformance with sign convention 2 for other configurations can be similarly determined. The various possibilities for a normal quadrupole are summarized in Table II. As can be seen from the table, the product of b_1' and $C(2)$ has the same sign, irrespective of the approach taken for the transformation. Based on Sec.6, b_1' is positive for the lead end placed clockwise, while it is negative for the lead end placed counter clockwise. The sign of $C(2)$ depends on whether the magnet is focusing or defocusing *and* which end the beam enters. On the other hand, based on the transformation in Sec.7, b_1' is always positive and the sign of $C(2)$ simply depends on whether the magnet is focusing or defocusing (since the beam always travels in the clockwise direction). A similar equivalence between the two procedures can be verified for other magnet types.

TABLE II. **Comparison of Transformations based on Sec.6 and Sec. 7 for a Normal Quadrupole**

Lead End	Quadrupole Type	From Section 6		From Section 7	
		b_1'	$C(2)$	b_1'	$C(2)$
CW	Focusing	> 0	> 0	> 0	> 0
CW	Defocusing	> 0	< 0	> 0	< 0
CCW	Focusing	< 0	< 0	> 0	> 0
CCW	Defocusing	< 0	> 0	> 0	< 0

References:

[1] E. Willen, P. Dahl and J. Herrera, SLAC Summer School on *Physics of Particle Accelerators,* Stanford Linear Accelerator Center, Stanford, CA, 1985, in AIP Conference Proceedings 153.

[2] See for example:

J. Herrera, *Magnet Test Group Note* **30**, September 9, 1980.

J. Herrera, *Magnet Test Group Note* **181**, January 27, 1982.

J. Herrera, *Magnet Test Group* Note **244**, October 15, 1982.

J. Herrera, *Magnet Test Group* Note **250**, November 16, 1982 (This note describes the transformation of field parameters due to a shift in the origin – a topic not covered in the present note).