RHIC/AP/66
August 1995

Parasitic beam-beam collisions and crossing angles in RHIC

S. Peggs

Introduction

It has recently been suggested that the RHIC acceleration and storage frequencies become 28.62 MHz ($h = 366, \lambda_{acc} = 10.475$ m) and 200.3 MHz ($h = 2,562, \lambda_{store} = 1.496$ m). If so, the ratio of harmonic numbers is changed from 22:3 to 7:1, and the number of bunches in the nominal (initial upgrade) scenario is changed from 57 (114) to 61 (122). This change in RF system frequencies also opens the door to further upgrade scenarios, in which 183 or 366 bunches circulate. If 122/183/366 bunches are stored, the azimuthal distance between beam-beam crossings - half the bunch spacing - is 3/2/1 times 5.238 meters - half the acceleration wavelength.

In the nominal vacuum chamber layout, the two beams enter separate beam pipes at a crotch 17.73 meters from the interaction point (IP). In this case there would be 1/1/3 parasitic beam-beam collisions on each side of the IP when 122/183/366 bunches are present. It is natural to label the potential locations of head-on parasitic collisions "1", "2", and "3", according to the distance from the IP, measured in units of $\lambda_{coll} = 5.238$ meters. The location of the crotch is still somewhat flexible. Its distance from the IP might be decreased to be less than 15.71 meters, suppressing parasitic beam-beam collision "3". In this case there would be 0/1/2 parasitic collisions with 122/183/366 bunches.

This paper evaluates the potential strength of the parasitic beam-beam collisions when the crossing angle is zero. A non-zero crossing angle is required in some operational scenarios. The RHIC lattice is shown to easily accommodate even conservatively large crossing angles. A modest loss in luminosity is incurred when gold ions collide at an angle after 10 hours of storage.

Beam-beam tune shift parameters

When two identical Gaussian beams collide, the horizontal and vertical beam-beam tune shift parameters are given by

$$\xi_{H,V} \;=\; \frac{r}{2\pi\gamma}\,\frac{N\,\beta_{H,V}}{\sigma_{H,V}(\sigma_H+\sigma_V)} \tag{1}$$

where N is the single bunch population, the classical radius r is $r_p = 1.5347 \times 10^{-18}$ meters for protons and $r_{Au} = 48.992 \times 10^{-18}$ meters for gold, $\beta_{H,V}$ is the beta function in the appropriate plane, and γ is the Lorentz factor. Assuming from here on that the beam is round ($\beta_H = \beta_V\,, \sigma_H = \sigma_V$), the transverse beam size is given (in the relativistic limit) by

$$\sigma \;=\; \sqrt{\frac{\epsilon_N \beta}{6\pi\gamma}} \tag{2}$$

where ϵ_N is the "6π" normalized emittance used at RHIC. Equation 1 is succinctly rewritten as

$$\xi \;=\; \frac{3Nr}{2\,\epsilon_N} \tag{3}$$

Note that the tune shift parameter is independent of energy (γ), and independent of β. The tune shift of small amplitude particles is equal to the parameter, ξ, no matter what the azimuthal location of the collision, if the beams are round and if they collide head-on.

Nominally there are $N = 10^{11}$ protons per bunch, with a 95% normalized transverse emittance of $\epsilon_N = 20\pi$ microns. When gold ions are stored, there are $N = 10^9$ ions per bunch, with an emittance that rises from $\epsilon_N = 10\pi$ microns at injection to $\epsilon_N = 40\pi$ microns at the end of a 10 hour store. Centered on these nominal parameters, it is convenient to numerically parameterize the proton and gold tune shift parameters as

$$\xi_p \;=\; 0.00366\,\frac{N}{10^{11}}\,\frac{20\pi\mu\mathrm{m}}{\epsilon_N} \tag{4}$$

and

$$\xi_{Au} \;=\; 0.00117\,\frac{N}{10^{9}}\,\frac{20\pi\mu\mathrm{m}}{\epsilon_N} \tag{5}$$

For comparison purposes, strong beam-beam effects are noticed in proton colliders when $\xi = 0.004$, with 6 head-on collisions per turn [1].

Beam separation geometry

Between the IP and the first quadrupole of the interaction region triplet, a trajectory encounters two dipole magnets, DX and D0. The large bore magnet DX is immediately adjacent to the experiment and is common to both beams. A drift follows, allowing the two trajectories to diverge far enough that they can enter one or the other of side-by-side D0 dipoles. The D0 magnets remove most - but not all - of the divergence angle applied by DX. Each D0 shares a cryostat with three triplet quadrupoles, the first of which is immediately adjacent. Table 1 lists the nominal geometrical parameters of this region.

Quantity	Units	Value
DX magnetic length	[m]	3.70
DX bending radius	[m]	196.17
DX bend angle	[mrad]	18.86
DX bend center (from IP)	[m]	11.65
D0 magnetic length	[m]	3.60
D0 bending radius	[m]	237.06
D0 bend angle	[mrad]	−15.19
D0 bend center (from IP)	[m]	22.30

Table 1: DX and DO dipole parameters, when the crossing angle is zero. Bend center locations are measured in meters from the IP.

If a TOTAL central collision crossing angle of α is required, then the DX and D0 bend angles, $\theta_X(\alpha)$ and $\theta_0(\alpha)$, need to be adjusted so the beam has the correct angle and displacement when entering the first quadrupole. These constraints are met, to a very good approximation, if

$$\theta_X(\alpha) = \theta_X(0) - \frac{\alpha}{2}\frac{s_0}{s_0 - s_X} = 18.86 - 1.047\,\alpha \quad [\mathrm{mrad}] \tag{6}$$

$$\theta_0(\alpha) = \theta_0(0) + \frac{\alpha}{2}\frac{s_X}{s_0 - s_X} = -15.19 + 0.547\,\alpha \quad [\mathrm{mrad}] \tag{7}$$

where s_X and s_0 are the bending center locations listed in Table 1. Note that the absolute value of the bend angles, and the fields of DX and D0, both decrease with increasing crossing angle. A typical value of α in the various scenarios discussed below is 1 milliradian.

Parasitic collision parameters

The total beam separation d at each of the collision points, when the crossing angle is zero, is recorded in Table 2. Using equation 2, the beam size σ_{20} for a standard normalized emittance of 20π microns is also given for protons and gold, at injection and storage. It is the relative size of the total beam separation, measured in units of the beam size σ, that is the relevant parameter when evaluating the potency of parasitic beam-beam collisions. Hence, Table 2 also records d/σ_{20} in all cases.

Quantity	Units	"0"	"1"	"2"	"3"
Location s	[m]	0.00	5.24	10.48	15.71
Total separation d	[m]	.0000	.0000	.0024	.1533
Injection β	[m]	10.0	12.74	20.97	34.69
Storage β	[m]	1.00	28.43	110.8	247.9
Proton inject σ_{20}	[mm]	1.03	1.17	1.50	1.93
Proton store σ_{20}	[mm]	0.11	0.59	1.17	1.76
Gold inject σ_{20}	[mm]	1.63	1.84	2.36	3.03
Gold store σ_{20}	[mm]	0.18	0.94	1.85	2.76
Proton inject d/σ_{20}	[m]	0.00	0.00	1.60	79.6
Proton store d/σ_{20}	[m]	0.00	0.00	2.04	87.3
Gold inject d/σ_{20}	[m]	0.00	0.00	1.02	50.6
Gold store d/σ_{20}	[m]	0.00	0.00	1.30	55.5

Table 2: Parasitic collision parameters, with no crossing angle. The central and parasitic collisions are labeled "0" through "3", according to their distance from the IP. A standard emittance of 20π microns is assumed throughout. The Lorentz factor at injection(storage) for protons is $\gamma = 31.17(268.2)$, while for gold ions at injection(storage) it is $\gamma = 12.6(108.4)$.

The horizontal and vertical beta functions in the region of interest - before the triplet - behave as if they are in a pure drift, except for negligible edge focusing at the ends of the DX magnet. They are therefore equal, and given by

$$\beta(s) \quad = \quad \beta^* + \frac{s^2}{\beta^*} \tag{8}$$

where s is the azimuthal distance from the IP, and β^* is the beta function at the IP. The relative luminosity of a head-on parasitic collision is just

$$\frac{L}{L^*} = \frac{\beta^*}{\beta} \tag{9}$$

since the luminosity scales as the inverse square of the beam size. Assuming that $\beta^* = 1.0$ meter at storage, collision "1" only generates about 3.5% as many background events as the central collision at the IP, even with no crossing angle. However, the background contamination increases rapidly - initially quadratically - as β^* increases to 10.0 meters, the value assumed for injection.

When is a crossing angle required?

When 122 bunches are stored, only collision "3" is active. Table 2 shows that the beams are so well separated there - at least 50 σ_{20} in all cases - that further beam-beam suppression is not required. No crossing angle is needed, and it is not necessary to move the crotch closer to the IP.

When 183 bunches are stored, only collision "2" is active. The total separation - between 1 and 2 σ_{20} - is maximally bad, not only generating a significant contribution to the total tune shift, but also driving both odd and even order resonances. The nominal proton tune shift parameter $\xi_p \approx 0.0037$ is large enough to disallow this - or any head-on - parasitic collision. While protons must have a crossing angle when 183 bunches collide, the nominal gold tune shift parameter $\xi_{Au} \approx 0.0011$ is POSSIBLY low enough to allow gold bunches to collide at "2" without a crossing angle.

Parasitic collisions occur at "1", "2", and "3", when 366 bunches are stored. Protons definitely need a crossing angle. If gold ions collide without a crossing angle, they see three head-on collisions of equal strength, plus two ugly "2" collisions. Gold ions probably need a crossing angle.

Crossing angles, and their side effects

The small amplitude tune shift due to a single long range collision scales like

$$|\Delta Q_{LR}| \simeq \xi \left(\frac{\sigma'^*}{\alpha} \right)^2 \tag{10}$$

where

$$\sigma'^* = \frac{\sigma^*}{\beta^*} \tag{11}$$

is the root mean square angular size at the central collision point [2]. For this reason it is conservative to make the total crossing angle

$$\alpha \;=\; 7\,\sigma'^{*}_{20} \tag{12}$$

where, as before, a standard normalized emittance of 20 π microns is assumed. Table 3 lists the crossing angle that this leads to, for protons and gold at injection and at collision. It also lists the modified bend angles of the DX and D0 dipoles, according to equations 6 and 7.

Quantity	Units	Proton inject	Proton store	Gold inject	Gold store
RMS angle σ'^{*}_{20}	[mrad]	0.10	0.11	0.16	0.18
Crossing angle α	[mrad]	0.70	0.77	1.14	1.26
DX angle θ_X	[mrad]	18.13	17.81	17.67	17.54
D0 angle θ_X	[mrad]	-14.45	-14.38	-13.99	-13.87
RMS bunch length σ_z	[m]	0.353	0.072	0.467	0.206
Luminosity $L(\alpha)/L(0)$		0.993	0.970	0.987	0.811

Table 3: Various angles, and luminosity performance, when RHIC beams collide at an angle. The worst case bunch length has been used for gold ions in storage, after 10 hours of intra beam scattering.

A potentially serious side effect is the loss of luminosity that a crossing angle incurs. This is given by

$$\frac{L(\alpha)}{L(0)} \;=\; \frac{1}{\sqrt{1+(\alpha\sigma_z/2\sigma^{*}_{20})^2}} \tag{13}$$

where σ_z is the root mean square length of a bunch [3]. Table 3 indicates that negligible luminosity is lost when protons cross at an angle, but that as much as 19% of the nominal luminosity is lost at the end of a 10 hour gold ion store.

Conclusions

It is not necessary to move the vacuum pipe crotch closer to the IP, in order to ameliorate parasitic collision number "3".

A crossing angle is not required when 122 or fewer bunches are stored. Although a crossing angle is required when 183 nominal proton bunches collide, it might be possible for 183 gold bunches to collide head-on. A crossing angle is required when 366 bunches of protons, or gold ions, collide.

The largest crossing angle is required for gold ions. A conservative estimate is $\alpha \simeq 1.26$ milliradians. This is easily achieved by reducing the magnetic field of DX and D0 dipoles by 7.0% and 8.7%, respectively.

Gold ions might see a luminosity decrease of about 19% at the end of a 10 hour store. This loss can be minimized by using a less conservative crossing angle - or by decreasing the storage time.

All the estimates above assume a fixed standard normalized emittance of 20π microns.

References

[1] "Beam-Beam Interaction Effects in the Fermilab Collider", Donna Siergiej, PhD Thesis, University of New Mexico, March 1995.

[2] "Conceptual Design" of the Superconducting Super Collider, p. 198 et seq., SSC-SR-202, March 1986.

[3] "Conceptual Design" of the Superconducting Super Collider, p. 104, SSC-SR-202, March 1986.