RHIC/AP/47

ORBIT MATRICES FOR HELICAL SNAKES

E.D. Courant

November 8, 1994

Helical snakes may be useful for preventing spin resonances in RHIC and the Tevatron. We must evaluate the impact they may have on orbit stability.

Blewett and Chasman[1] show that in a helical snake the fields are, up to terms quadratic in the displacements from the axis,

$$\begin{aligned} B_x &= -B_0 \left\{ \left[1 + \frac{k^2}{8}(3x^2 + y^2) \right] \sin kz - \frac{k^2}{4} xy \cos ks \right\} \\ B_y &= B_0 \left\{ \left[1 + \frac{k^2}{8}(x^2 + 3y^2) \right] \cos kz - \frac{k^2}{4} xy \sin kz \right\} \\ B_z &= -kB_0 (x \cos kz + y \sin kz) \left[1 + \frac{k^2}{8}(x^2 + y^2) \right] \end{aligned} \tag{1}$$

where $k = 2\pi/\lambda$ is the wave number of the helical field, B_0 its value on the axis, and x and y the displacements from the axis, z being the distance along the longitudinal axis. This is in agreement with the field expressions obtained by Ptitsin[2].

The equations of motion for x and y are (to lowest order in x and y and their derivatives)

$$\begin{aligned} x'' &= (y' B_z - B_y)/B\rho \\ y'' &= (B_x - x' B_z)/B\rho \end{aligned} \tag{2}$$

A solution of these equations is the helical trajectory

$$\begin{aligned} x_0 &= r_0 \cos kz \\ y_0 &= r_0 \sin kz \end{aligned} \tag{3}$$

where

$$r_0 = \frac{1}{k^2 \rho} \tag{4}$$

[1] J.P. Blewett and R. Chasman, J. App. Phys. **48**, 2692-2698 (1977).
[2] V. Ptitsin, Note RHIC/AP/41 (Oct. 10, 1994).

is the radius of the helical orbit centered on the axis, and $\rho \equiv B\rho/B_0$ is the radius of curvature of the particle in a field B_0.

We may describe the actual motion of the particle as an oscillation about (3). Note that, if the helix is centered on the central orbit $x = y = 0$, the actual orbit will, in fact, not be the helical orbit (3) but an oscillation about it, with an amplitude of the order of r_0.