RHIC/AP/42

Measurement of β^* and α^* in RHIC Insertions

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Summary

A procedure to measure β^* and α^* in RHIC insertions is proposed. It requires a measurement of tune change resulting from a change in the quadrupole gradient in a pair of triplets. The viability of this procedure depends on the accuracy of tune measurements and on the accuracy of integrated quadrupole gradients. A few examples are given to illustrate the accuracy of β^* and α^* values expected from this procedure.

1. Introduction

In section 3.1 of RHIC/AP/38, a procedure to find the values of β^* (and possibly α^* as well) in an insertion has been outlined. It involves measurements of tune change when a quadrupole in a pair of triplets undergoes a small change in its strength, one at a time. The relevant formula is given in Courant–Snyder, Eq. (4.37),

$$\cos(2\pi\nu) - \cos(2\pi\nu_0) = -\frac{1}{2}\sin(2\pi\nu_0)\, k \int \beta(s)\, ds$$

where only the term linear in $k = \Delta(B'/B\rho)$ and in β is retained. The integral

$$\int \beta(s)\, ds$$

for each quadrupole can be expressed as a linear function of β^*, α^*, and γ^* but with the constraint

$$\gamma^*\beta^* - (\alpha^*) = 1.$$

Quantities to be measured are tuen ν_0 and ν, before and after the change, respectively, in B'.

Because of the constraint, the least–square analysis is not straightforward. It is simpler to resort to a brute–force method of finding the minimum in the two–parameter space of (β^*, α^*). The likely range of these two parameters is known rather well, at least for β^*, but a precise specification is not necessary to find the minimum point.

If there is no uncertainties in the measured tune values or in the integrated quadrupole gradients, one can in principle find β^* and α^* using only two quadrupoles, probably the ones adjacent to the IP. In reality, this will not give reliable results and the use of all six quadrupoles is proposed here. One might also use several different values of k, the change in $(B'/B\rho)$, for each quadrupole to improve the reliability.

A short computer code has been used to simulate the analysis of measured data assuming the following:

1. The ring may not be as designed but it is linear.

   The effect of nonlinearity depends mostly on how one measures the tune. If the tune is measured by pinging the beam and generating a coherent oscillation, the amplitude dependence may invalidate the algorithm. The effect of linear coupling is of course most important since it is independent of the oscillation amplitude. One either must eliminate the effect entirely or at least take into account the effect in the analysis.
2. The term nonlinear in $(k\beta)$ for $\cos(2\pi\nu) - \cos(2\pi\nu_0)$ is negligible.

   This may not be true when β^* is as small as 1m so that beta is very large in quadrupole. Examples studies so far indicates that β^* should be reliable but not α^* when β^* is small.
3. The uncertainties in the measured tune values and in the integrated gradients relative, are known to be within $\pm\Delta_\nu$ and $\pm\Delta_G$ respectively.

   It is assumed in this work that Δ_ν should be between 0.001 and 0.0001, and

Δ_G around 0.0025 (that is, one–quarter percent relative). These must be established before the measurements are undertaken.

2. Examples

In each case, ten random samples have been analyzed to see the range of (β^*, α^*) resulting from this procedure. The fractional part of ν_0 is taken to be 0.19 for all cases.

A. $\beta^* = 10\text{m}$, $\alpha^* = 0.40$

parameter range used in the search: β^* from 2m to 20m,

α^* from -1.0 to 1.0

$\Delta_G = 0.$

A.1	$\Delta\nu$	$= 0.002$:	8.74 to	11.29,	0.406 to	0.573
A.2		$= 0.001$:	9.33 to	10.61,	0.453 to	0.536
A.3		$= 0.0005$:	9.65 to	10.29,	0.476 to	0.517
A.4		$= 0.0001$:	9.92 to	10.05,	0.494 to	0.503

$\Delta_G = 0.0025$

A.5	Δ_ν	$= 0.001$:	9.48 to	10.67,	0.434 to	0.574
A.6		$= 0.0001$:	9.92 to	10.07,	0.493 to	0.507

B. $\beta^* = 1.0\text{m}$, $\alpha^* = -0.25$

parameter range used in the search: β^* from 0.5m to 5m,

α^* from -1.0 to 1.0

$\Delta_G = 0$

B.1	Δ_ν	$= 0.002$:	0.989 to	1.072,	-0.217 to	-0.380
B.2		$= 0.001$:	0.993 to	1.027,	-0.231 to	-0.300
B.3		$= 0.0005$:	0.999 to	1.006,	-0.245 to	-0.260
B.4		$= 0.0001$:	1.003 to	1.005,	-0.256 to	-0.260

$\Delta_G = 0.0025$

B.5	Δ_ν	$= 0.001$:	0.985 to	1.038,	-0.220 to	-0.324
B.6		$= 0.0001$:	0.998 to	1.005,	-0.244 to	-0.260

C. $\beta^* = 2.73,\ \alpha^* = -0.18,\ \Delta_G = 0.0025$

Only one case out of ten random sets has been used but with different values of k.

C. $\beta^* = 2.73,\ \alpha^* = -0.18,\ \Delta_G = 0.0025$

C.1 $\Delta_\nu = 0.001$

k	β	α
0.005	2.89	-0.295
0.01	2.80	-0.239
0.015	2.78	-0.221
0.02	2.77	-0.211
0.025	2.76	-0.205
0.03	2.76	-0.200
0.035	(unstable)	

C.2 $\Delta_\nu = 0.0005$

k	β	α
0.005	2.80	-0.238
0.01	2.77	-0.211
0.015	2.76	-0.203
0.02	2.75	-0.199

D. The following examples illustrate the dependence on how many pairs of quadrupoles are listed in the analysis.

D.1 $\beta^* = 10\text{m},\quad \alpha^* = 0.5,\quad \Delta_\nu = 0.001,\quad \Delta_G = 0.0025$

number of pairs	β	α
1	11.43 – 12.32	0.486 – 0.0714
2	9.44 – 11.49	0.460 – 0.620
3	9.48 – 10.67	0.434 – 0.574

D.2 $\beta^* = 1\text{m},\quad \alpha^* = -0.25,\quad \Delta_\nu = 0.001,\quad \Delta_G = 0.0025$

1	0.945 – 1.007	$-(0.060 - -0.237)$
2	0.987 – 1.028	$-(0.220 - -0.300)$
3	0.985 – 1.038	$-(0.220 - -0.324)$

3. Findings

In addition to examples presented in 2, many more have been tried with varying degree of "success" in predicting β^* and α^*. The fact that the formula used is not exact even in a complete absence of nonlinearity, that is, the linear approximation in $(k\beta)$, becomes important when β^* is as small as 1m and α^* is near zero. The procedure cannot predict α^*. At the same time, the predicted values of β^* are always very good.

The proper choice of k, the change in B' to find the change in tune, depends on the accuracy of tune measurements and on β^*. The optimum choice must be made by trials in the real operation.

Finally, although it is generally true that the use of more quadrupoles give a better result, this again may depend on β^*. When β^* is 1m, increasing the number of quadrupoles from 4 to 6 does not seem to improve the result.