Effect of Longitudinal Variation of Multipoles in QRJ Magnets

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The recent warm measurements of RHIC QRJ (Q3) quadrupoles indicate that the magnetic multipoles a_2 and b_3 vary significantly lengthwise along the magnet body. In the case of QRJ105, the difference in a_2 between the lead-end half and non-lead-end half is about 10 units, and the difference in b_3 is about 3 units. In the case of QRJ106, the average a_2 is about 6 units. Here, we first estimate the effect of such a longitudinal variation of the multipoles harmonics, and then study the effect of the large average a_2.

We express the variation of the multipole b_n along a magnet of length L by the expression

$$b_n(s) = b_{n0} + b_{n1}(s), \qquad \int_0^L b_{n1}(s)ds = 0 \tag{1}$$

where b_{n0} is the average value of the multipole harmonics, and s is the distance along the magnet axis. The relative change (kick) in particle action caused by b_n can be derived to be[1]

$$\frac{\Delta J_x}{J_x} = \left(\frac{10^{-4}G_0}{B_0\rho R_0^{n-1}}\right)(2J_x)^{\frac{n-1}{2}} \int_0^L 2\beta_x^{\frac{n+1}{2}} \sin\chi_x \cos^n\chi_x b_n(s)\; ds, \tag{2}$$

where $R_0 = 0.04$ m is the reference radius, G_0 ($G_0 = 48$ T/m at storage) is the reference gradient, $B_0\rho$ ($B_0\rho = 840$ Tm at storage) is the magnetic rigidity, and χ_x is the betatron phase. Since the amplitude functions $\beta_{x,y}$ vary significantly inside the magnet, the effect of variation b_{n1} is not negligible. From Eq. 2, we define the effective multipole $\Delta b_{n,eff}$ as

$$\Delta b_{n,eff} = \frac{1}{2}\left\{\left|\frac{\int_0^L \beta_x^{\frac{n+1}{2}}(s)b_{n1}(s)ds}{\int_0^L \beta_x^{\frac{n+1}{2}}(s)ds}\right| + \left|\frac{\int_0^L \beta_y^{\frac{n+1}{2}}(s)b_{n1}(s)ds}{\int_0^L \beta_y^{\frac{n+1}{2}}(s)ds}\right|\right\}, \tag{3}$$

and similarly the effective $\Delta a_{n,eff}$. The effect (e.g. action kick and tune shifts) of the harmonic variation b_{n1} can be approximated by the effective harmonic $\Delta b_{n,eff}$ multiplied by the average amplitude function to the appropriate power (Eq. 2).

Figure 1 shows the variation of the amplitude functions $\beta_{x,y}$ in the Q3 magnet in a typical IR region operating at $\beta^* = 1$ m. Using a simple model to approximate the variation of

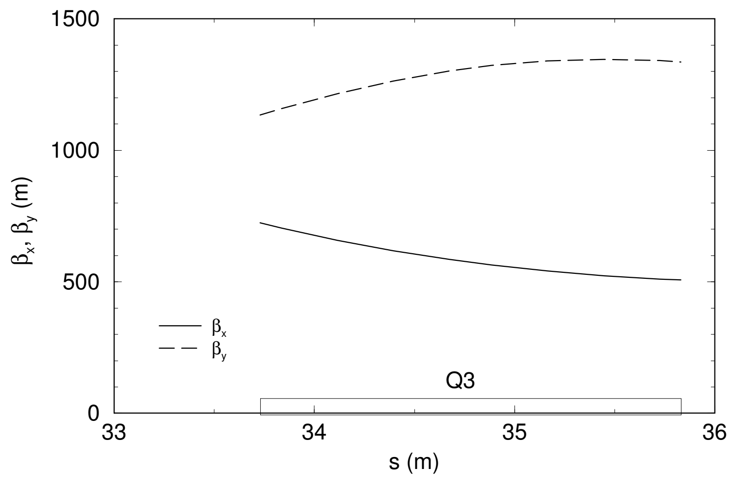

Figure 1: Amplitude function variation in the RHIC Q3 magnet at 6 o'clock outer insertion region during $\beta^* = 1$ m operation.

the harmonics,

$$b_{n1} = \frac{\Delta b_n}{2} \, \mathrm{sign}\left(\cos\frac{2\pi\lambda s}{L}\right), \tag{4}$$

where sign(x) is equal to 1 if x is positive, and is equal to -1 if x is negative, and λ is the frequency of the variation. Using Eq. 3, Table 1 shows the effective values of $a_{n,eff}$ or

n	$\lambda = 1$	$\lambda = 2$	$\lambda = 4$
1	0.055	0.037	0.027
2	0.10	0.075	0.050
3	0.16	0.11	0.080
4	0.26	0.18	0.13
5	0.42	0.28	0.21

Table 1: Effective strengths of the longitudinal multipole variations (i.e., the effective $a_{n,eff}$ or $b_{n,eff}$ that correspond to one units of Δa_n or Δb_n, respectively) as functions of the variation cycle λ for various multipole harmonics n.

$b_{n,eff}$ that correspond to one units of $a_{n,eff}$ or $b_{n,eff}$, respectively, i.e., the relative strength of the multipole variation. Table 1 implies that the effect of the 10 unit difference in a_2 between the lead and the non-lead halves in QRJ magnets is about the same as that of an average a_2 of about 1 unit. Similarly, the 3 unit difference in b_3 can be approximated by an average b_3 of 0.5 units.

If installed in the maximum-β location of the IR region where the operating β^* is 1 meter, the magnet with average $a_2 = 6$ units (similar to QRJ106) produces a significant kick of $\Delta J_x/J_x \approx 3 \times 10^{-2}$ for a 5-σ particle at emittance 40πmm·mr, and is likely to cause problem. Therefore, measures should be taken to investigate and to correct the large a_2 multipole harmonics. Compared with the a_2, the effect of longitudinal b_3 variation is less alarming, and is likely be solved by sorting and similar methods.

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References

1. J. Wei and S. Peggs, RHIC/AP/19 (1993).