Parameterization of Sextupole Correctors

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An estimate is made of sextupole requirements for chromaticity correction. The most severe case occurs at storage where the magnetic rigidity is large and $\beta^* = 1$m insertions make large contributions to the natural chromaticity. Uncertainties in the 8 cm dipoles of +4 prime units of systematic sextupole errors pose no problem to operation with two $\beta^* = 1$m insertions, and operation with up to four $\beta^* = 1$m insertions seems possible.

The 8 cm dipoles are designed to have the following expected systematic sextupole errors: $\langle b2 \rangle = -3.5$ at injection, 0 at transition, and +1 at storage (primed units). These values are expected for dipoles made with zero tolerance. The systematic multipoles are strongly dependent on geometry, so dipoles satisfying the nonzero tolerances specified to Grumman can have systematic multipoles that differ from the expected values. The differences, denoted by d<bn>, have been estimated by the Magnet Division. The uncertainty in the systematic sextupole, d<b2>, of the magnet body is expected to lie in the range $-4 < d\langle b2 \rangle < 4$ primed units. As d<b2> arises from geometry, it should be independent of magnet excitation.

The SF and SD sextupoles for chromaticity correction are more than adequate at injection where the magnetic rigidity is low. However, at storage, the magnetic rigidity is high, the low β insertions make large contributions to the chromaticity, and nonzero d<b2> can further increase requirements on these sextupoles. In the event that requests are made for $\beta^* = 1$m in more than two insertions, the requirements on the sextupoles could exceed their capabilities. The question of the adequacy of these sextupoles has been addressed by parameterizing the integrated sextupole strength required to correct the chromaticity contributions from known configurations and of then using this parameterization to estimate their adequacy to correct other configurations.

The strength of each sextupole family is divided into three parts:

1. a portion necessary to correct chromaticity of error free arcs,
2. a portion necessary to correct chromaticity from systematic b2's in the 8 cm dipoles, and
3. a portion necessary to correct chromaticity from the insertions.

The third component varies as $\sum k\beta$; as the strengths of most insertion quadrupoles are essentially constant, the contribution is dominated by the triplet quadrupoles where β is large. The β's in the triplets vary inversely as β^* resulting in their contribution to the chromaticity being proprotional to $\sum 1/\beta^*$. We define a variable $n^* = \sum 1/\beta^*$ as a measure of the chromaticity contribution from the insertions. The values of n^* for combinations of $\beta^* = 1$m and 10m insertions are: $n^* = 0.6, 2.4$ and 4.2 when there are 0, 2, or 4 $\beta^* = 1$ insertions, respectively.

The integrated sextupole fields are parameterized according to the three contributions listed above, and the coefficients d_0, d_2, d_*, f_0, f_2 and f_* are determined from simulation results.

$$\left(B''L\right)_d = d_0 + d_2 \quad b2 + d_* \quad n*$$
$$\left(B''L\right)_f = f_0 + f_2 \quad b2 + f_* \quad n*$$

Average sextupole strengths, SF and SD, were determined with Teapot for four "machines" when all elements had expected systematic magnet errors and randomly generated displacement, rotation, and field errors. Measurements were made at 100 GeV (Bρ = 839.5 Tm) with <b2>= 1, d<b2> = +4, and the chromaticity = +2 in both x and y planes. The following multipole configurations in the bodies of all 8 cm dipoles were considered:

b2(sys) = <b2> $(6 \times \beta^* = 10\text{m})$ (1)

b2(sys) = <b2> $(2 \times \beta^* = 1\text{m})$ (2)

b2(sys) = <b2>+d<b2> $(2 \times \beta^* = 1\text{m})$ (3)

b2(sys) = <b2>−d<b2> $(2 \times \beta^* = 1\text{m})$ (4)

The coefficient d_2 is found from difference of (3) and (4), d_* is found from the difference of (1) and (2), and d_0 is obtained by replacement of d_2 and d_* in (2). The "f" coefficients are determined in the same manner.

We obtain the following parameterization:

$$(B''L)_d = -167.20 - 11.021\ b2 - 74.661\ n*$$
$$(B''L)_f = 61.01 - 6.103\ b2 + 40.844\ n*$$

The strength of the SD's is larger than that of the SF's with the most stringent requirement being at storage when <b2> = +1 and d<b2> = +4.

The relation $B''L(T/m) = 2B\rho S$ was used to convert the integrated sextupole strength S, determined by Teapot, to the integrated sextupole field $B''L(T/m)$, and a transfer function, the average measured transfer function from six sextupole magnets,[1] was used to obtai the corresponding excitation current.

Integrated sextupole field ($B''L$), the transfer function (TF), and the currents are listed in Table 1 for lattice configurations having zero, two, and four $\beta^* = 1$m insertions. The sextupole current of 85 Amps required for operation with four $\beta^* = 1$m insertions is comfortably within the 100 Amp limit for sextupole leads. The maximum current of 100A corresponds to an integrated sextupole field of 580 T/m. Inserting this into the relation given above for $(B''L)_d$ when b2=5, the maximum value of $n*$ is obtained:

$$-50 = -167.20 - 11.021 \times 5 - 74.661\ n*$$

The result, $n*$(max) = 4.8, indicates there is not adequate sextupole strength to correct five $\beta^* = 1$m insertions, but correction of four $\beta^* = 1$m insertions should be possible and uncertainty of the systematic b2 in the Grumman 8 cm dipoles does not impose a limitation on operation with up to four $\beta^* = 1$m insertions.

Table 1. N is the number of $\beta^* = 1$m insertions, $B''L(T/m)$ indicates integrated sextupole field TF is the transfer function with units (T/m/A), and I(A) is the required current.

N		SD		SF		
$(\beta^* = 1)$	$B''L(T/m)$	TF	I(A)	$B''L(T/m)$	TF	I(A)
0	-267.1	9.000	-29.7	55.0	9.055	6.1
2	-401.5	8.407	-47.8	128.5	9.041	14.2
4	-535.9	6.342	-84.5	202.0	9.017	22.4

References

1. Sextupole Transfer Functions from Animesh Jain, 2/17/94.