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Coupling and Alignment of the PHENIX Experiment Solenoid in RHIC

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I. Introduction and Conventions

The PHENIX detector's central magnet (CM) is a split-pole solenoid with a longitudinal magnetic field of $B_s = 0.5$ Tesla along its effective length of $L = 1$ meter. When like hadron species collide in RHIC, the design orbit is parallel to this longitudinal field and the solenoid CM introduces only transverse linear coupling. However, the design orbit through the CM is skewed about the vertical axis when unlike hadron species collide — the angle of this skew is 3.85 milliradians for protons colliding with gold, and could conceivably reach 4.05 milliradians during proton-uranium collisions. This paper examines both linear closed orbit errors and coupling effects caused by the PHENIX central solenoid magnet and the design orbit skew.

The relevant parameters for both injection and storage lattices are shown in Tables 1 and 2. Coupling effects in this paper are quoted at injection where their effects are expected to be worst. Orbital effects scale in a somewhat more complicated fashion — the transverse beam sizes at the interaction point (IP) are

$$\begin{aligned} \sigma^\star &= \sqrt{\frac{\epsilon\,[\mathrm{mm-mrad}]}{6\pi}}\sqrt{\frac{\beta^\star(\mathrm{optics})\,[\mathrm{m}]}{\beta\gamma(\mathrm{dynamics})}} \times 1\mathrm{mm}\,, \\ \sigma'^\star &= \frac{\sigma^\star}{\beta^\star(\mathrm{optics})}\,, \end{aligned} \tag{1}$$

where $\beta\gamma$(dynamics) are relativistic parameters proportional to $B\rho$ (the magnetic rigidity) and the charge-to-mass ratio Z/A of the species being accelerated, and ϵ is the 95% emittance. A convenient dimensionless measure of the solenoid strength is given by [1]

$$\theta = \frac{B_s L}{2|B\rho|}\,, \tag{2}$$

which scales as $(B\rho)^{-1}$; it will be shown in the next section that the transverse coordinate system (x, y) is rotated by an angle θ by a solenoid. Measures of first order solenoid effects

Description	Symbol	Value
CM Solenoid field	B_s	0.5 T
CM Solenoid length	L	1.0 m
Skew angle (proton-gold)	$\psi_{\rm p-Au}$	3.85 mrad
Skew angle (proton-uranium)	$\psi_{\rm p-U}$	4.05 mrad

Table 1: PHENIX solenoid parameters in RHIC

Description	Symbol	Value (injection)	Value (storage)
Magnetic Rigidity	$\|B\rho\|$	97.5 T-m	839.5 T-m
Beta (IP, each plane)	$\beta^\star$	10 m	1.0 m
Emittance (95%, protons)	$\epsilon_{\rm p}$	15π mm-mr	15π mm-mr
Emittance (95%, gold)	$\epsilon_{\rm Au}$	10π mm-mr	40π mm-mr
Beam size (protons at IP)	$\sigma^\star_{\rm p}$	0.90 mm	0.10 mm
Beam size (protons at IP)	$\sigma'^\star_{\rm p}$	0.09 mrad	0.10 mrad
Beam size (gold at IP)	$\sigma^\star_{\rm Au}$	1.15 mm	0.25 mm
Beam size (gold at IP)	$\sigma'^\star_{\rm Au}$	0.12 mrad	0.25 mrad

Table 2: RHIC lattice parameters at PHENIX, injection and storage

in terms of the transverse beam sizes scale as

$$\frac{\theta}{\sigma^\star} \propto \sqrt{\frac{Z/A}{\epsilon\beta^\star B\rho}} \qquad \frac{\theta}{\sigma'^\star} \propto \sqrt{\frac{\beta^\star Z/A}{\epsilon\, B\rho}} \ . \tag{3}$$

These scalings indicate that the worst orbit displacement errors occur with protons at both injection and storage; the closed orbit errors for gold nuclei are smaller than these by factors of 1.5 to 3. At injection with $\beta^\star = 10$ meters, angle errors are more important than displacement errors, while at storage with $\beta^\star = 1$ meter both types of closed orbit errors are of approximately equal importance.

II. Coupling Strength and Solenoid Transfer Matrix

For the PHENIX solenoid at full strength in RHIC at injection, $L = 1$ meter and $B_s = 0.5$ Tesla, which gives

$$\theta = 2.56 \text{ mrad} \ , \quad \text{(RHIC injection)} \tag{4}$$

while at storage $\theta = 0.30$ mrad. In all cases through the ramp this coupling strength is much less than one. The minimal tune separation from this solenoid coupling is given by [2]:

$$\Delta Q \equiv |Q_x - Q_y|_{\min} = \frac{g\theta}{2\pi} \tag{5}$$

with $g = \sqrt{\gamma_x\beta_y + \gamma_y\beta_x + 2(1-\alpha_x\alpha_y)}$ and $\gamma \equiv (1+\alpha^2)/\beta$. At the PHENIX solenoid with design lattice parameters ($\beta_x^\star = \beta_y^\star$ and $\alpha_x^\star = \alpha_y^\star = 0$), $g = 2$ independent of $\beta^\star$. This gives for the worst case at injection,

$$\Delta Q = .00082\ , \quad \text{(RHIC injection)} \tag{6}$$

independent of hadron species and $\beta^\star$. This tune separation is an order of magnitude or more smaller than the RHIC design fractional tune separation of 0.01 and the space between the 0.167 and 0.200 resonances. This solenoid coupling is also equivalent to that introduced by a 3.4 milliradian roll in a *single* typical arc quadrupole.

An unavoidable and significant source of coupling at injection is the approximately 2 magnetic units of systematic skew quadrupole in arc dipoles. Simulation with only these sources of coupling gives an uncorrected minimum fractional tune separation of $\Delta Q = 0.06$, almost two orders of magnitude larger than the coupling from the PHENIX solenoid. The coupling from this solenoid is therefore negligible compared to other sources of coupling that must be corrected by the RHIC global coupling correction system.

For the standard transverse coordinate system used in accelerator physics, (x, x', y, y'), the transfer matrix for a thick solenoid with edge effects is given by Larsen [3] as

$$\mathbf{M} = \frac{1}{2}\begin{pmatrix} 1+\cos(2\theta) & \frac{L\sin(2\theta)}{\theta} & \sin(2\theta) & \frac{L[1-\cos(2\theta)]}{\theta} \\ \frac{-\theta\sin(2\theta)}{L} & 1+\cos(2\theta) & \cos(2\theta)-1 & \sin(2\theta) \\ -\sin(2\theta) & \frac{-L[1-\cos(2\theta)]}{\theta} & 1+\cos(2\theta) & \frac{L\sin(2\theta)}{\theta} \\ 1-\cos(2\theta) & -\sin(2\theta) & \frac{-\theta\sin(2\theta)}{L} & 1+\cos(2\theta) \end{pmatrix} . \tag{7}$$

Because θ is much less than one for all energies of the RHIC ramp, a small-angle approximation can be made keeping up to first order terms in θ:

$$\mathbf{M} \approx \begin{pmatrix} 1 & L & \theta & L\theta \\ 0 & 1 & 0 & \theta \\ -\theta & -L\theta & 1 & L \\ 0 & -\theta & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{D}_2 & \theta\mathbf{D}_2 \\ -\theta\mathbf{D}_2 & \mathbf{D}_2 \end{pmatrix} . \tag{8}$$

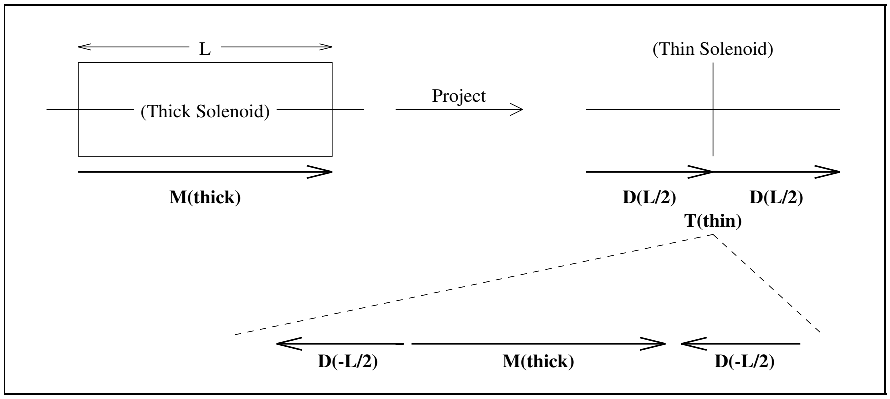

Figure 1: The projection approach to the solenoid transfer matrix.

This matrix represents a first-order rotation in each of the (x, y) and (x', y') planes, as one would expect from a physical rotation of the coordinate system produced by a solenoidal field. Another way this can be viewed is by a 2×2 decomposition with on-diagonal terms $\mathbf{D}_2$ representing drifts of length L and small off-diagonal coupling terms $\theta \mathbf{D}_2$.

Since there is approximately a 90-degree phase advance over the PHENIX solenoid, it is sensible to use a projection formalism [2]. In this approach we make the solenoid thin by projecting out the drift portion of the transfer matrix. If $\mathbf{D}_4(L)$ is the 4×4 length-L drift matrix,

$$\mathbf{M} = \mathbf{D}_4(L/2)\ \mathbf{T}(\theta)\ \mathbf{D}_4(L/2) \tag{9}$$

where

$$\mathbf{T}(\theta) \equiv \mathbf{D}_4^{-1}(L/2)\ \mathbf{M}\mathbf{D}_4^{-1}(L/2) = \begin{pmatrix} \mathbf{I}_2 & \theta \mathbf{I}_2 \\ -\theta \mathbf{I}_2 & \mathbf{I}_2 \end{pmatrix}, \tag{10}$$

and

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{out}} = \mathbf{T}(\theta) \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{in}} . \tag{11}$$

This approach is shown in Figure 1. Note that there are no orbit offsets in this equation — a perfectly aligned solenoid does not distort the closed orbit.

III. Orbit Angle and Misalignments

RHIC is designed to collide a wide variety of hadron species, from protons to bare gold nuclei, and can accommodate different species in the two different rings. The IR optics

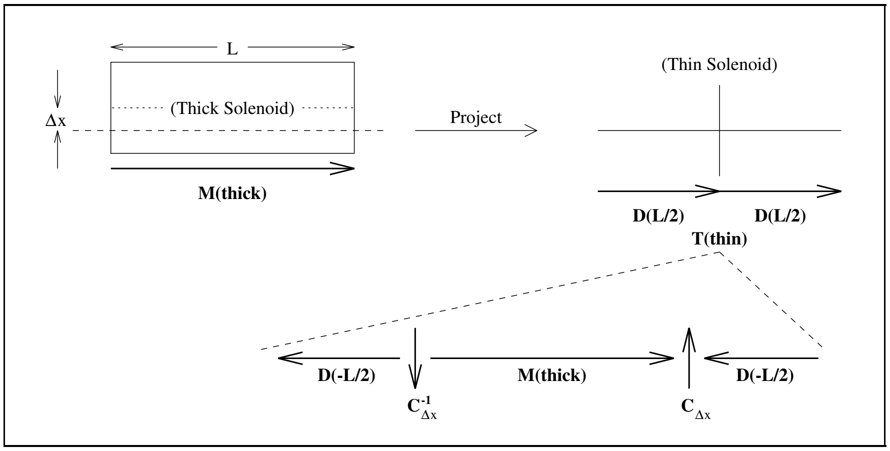

Figure 2: The geometry of a lateral solenoid displacement, with projection.

must therefore handle the maximum asymmetry between rings when protons collide with gold. Since both beams have different Z/A ratios but pass through common DX magnets, the closed orbit through the interaction point solenoids is skewed around the vertical axis. In the case of protons colliding with gold this skew angle $\psi_{\mathrm{p-Au}}$ is 3.85 milliradians. This angle is expected to have a larger effect than PHENIX solenoid misalignments, including transverse misalignments and skew angle errors. However, the effects of a crossing angle through a solenoid and solenoid survey misalignments are of a general importance. For completeness we examine two possible misalignments: a transverse solenoid displacement and a closed orbit skew through the center of the solenoid.

III.1. Lateral solenoid displacement effects.

The geometry of a lateral solenoid displacement is shown in Figure 2. Here the thin solenoid transformation matrix $\mathbf{T}(\theta)$ is modified by adding a coordinate translation before entering the thick solenoid, and this translation is removed after exiting the solenoid. If we denote the translation $(x, x', y, y') \rightarrow (x + \Delta x, x', y, y')$ by $\mathbf{C}_{\Delta x}$, Equation (10) is modified to become

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\mathrm{out}} = \mathbf{T}(\theta) \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\mathrm{in}} + \begin{pmatrix} 0 \\ 0 \\ -\theta\Delta x \\ 0 \end{pmatrix} . \quad (12)$$

Note that, as a mapping, the coordinate translation $\mathbf{C}_{\Delta x}$ commutes with the drift mapping $\mathbf{D}_4$ but does not commute with $\mathbf{M}$.

Outside the solenoid, the net perturbation of the solenoid horizontal displacement in (12) appears to be a second order vertical closed orbit error, as one would expect from the combined effects of coupling and misalignment. There is no additional coupling introduced by a solenoid displacement; instead, a solenoid displacement of $\Delta x = 1$ mm gives a vertical closed orbit distortion of 1.3 microns (0.0015 $\sigma^\star$ at worst case) at the IP. This is much smaller than the orbit error induced by a typical defocusing arc quadrupole (with focal length $f_{\rm quad} \approx 15$ m) misaligned by $\Delta y_{\rm quad} = 1$ mm, which gives a vertical closed orbit error of approximately

$$\Delta y_{\rm co} \approx \sqrt{\beta^\star \beta_{\rm arc}} \frac{\Delta y_{\rm quad}}{f_{\rm quad}} \approx 1.5 \text{ mm} \tag{13}$$

at the IP (1.66 $\sigma^\star$). The orbital effects of a solenoid displacement are therefore small compared to other typical closed orbit errors in RHIC.

III.2. Orbit skew effects.

The first order effect of an orbit skew of angle ψ through the center of a solenoid is a horizontal orbit offset and rotation upon entry into the solenoid. The coordinate transformation

$$\mathbf{C}_{\rm rot\,1} : (x, x', y, y') \to (x + L\psi/2, x' + \psi, y, y') \tag{14}$$

is applied upon entering the solenoid; after exiting the solenoid the rotation is removed and the effect of the drift is subtracted by

$$\mathbf{C}_{\rm rot\,2} : (x, x', y, y') \to (x + L\psi/2, x' - \psi, y, y')\ . \tag{15}$$

This coordinate transformation removes the effect of the skew angle through the drift portion of the solenoid transfer matrix, as shown in Figure 3. Equation (10) for the effect of the solenoid is then modified to become

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\rm out} = \mathbf{T}(\theta) \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\rm in} + \begin{pmatrix} 0 \\ 0 \\ -L\theta\psi \\ -\theta\psi \end{pmatrix} . \tag{16}$$

As mappings, the coordinate translations $\mathbf{C}_{\rm rot\,1}$ and $\mathbf{C}_{\rm rot\,2}$ contain angle displacements which do not commute with the drift mapping $\mathbf{D}_4$ or the thick solenoid transfer mapping $\mathbf{M}$.

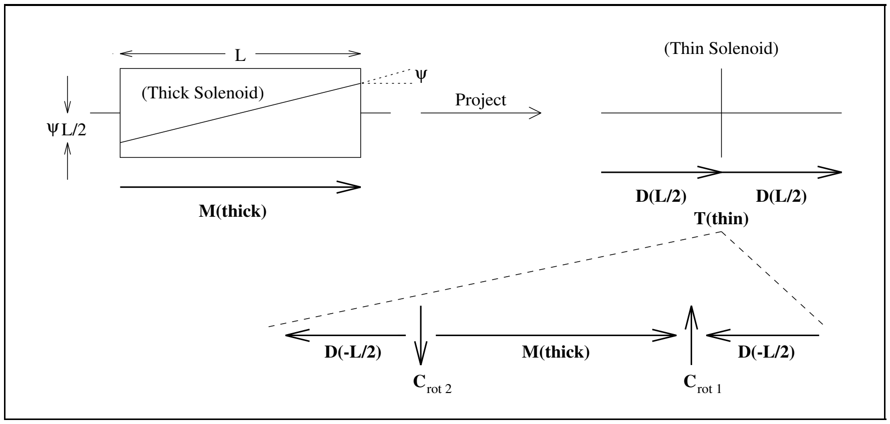

Figure 3: The geometry of a solenoid rotation misalignment, with projection.

Vertical closed orbit errors of $\Delta y = 0.054$ mm and $\Delta y' = 0.004$ mrad are created by the 3.85 mr skew angle through the PHENIX solenoid between gold and proton beams at injection. In terms of beam sizes at the worst case (again for protons at injection) these correspond to

$$\begin{aligned} \Delta y_{\rm co} &= 0.06\,\sigma^\star \ , \\ \Delta y'_{\rm co} &= 0.05\,\sigma'^\star \ . \end{aligned}$$

There is no additional coupling created by the solenoid rotation. The angle error $\Delta y'$ is approximately equivalent to the error introduced by displacing a single typical arc quadrupole by 0.13 mm, and so are approximately the same order of magnitude as errors introduced by quad misalignments in the arcs.

The orbit errors created by the solenoid at the IP propagate downstream in each ring and are correctable by vertical orbit correctors near the ends of the downstream triplets. It is important to note that vertical steering is required at storage to ensure collisions, so the small vertical orbit effects from the collision angle will be naturally corrected by this adjustment. There is a possibility of separating the orbits vertically at injection to avoid low-energy beam-beam effects — however, the impact of such an arrangement on vertical optics (such as vertical dispersion through the IR) has not been fully studied.

IV. Conclusions

The coupling introduced by the PHENIX solenoid is worst at injection energy in RHIC, where the beam rigidity is smallest, and is relatively independent of modest errors in beta function at the IP. At injection this coupling gives a minimum fractional tune separation almost two orders of magnitude smaller than other typical sources of coupling that must be corrected by the global coupling correction system. It is also comparable in effect to other single sources of coupling from magnet rotation misalignments and therefore does not require local correction.

The orbit skew angle required to collide unequal hadron species in RHIC does not affect the coupling introduced by the PHENIX solenoid. This angle instead introduces small vertical orbit errors, on the order of the skew angle times the coupling strength of the solenoid. For the worst case, with protons at injection energy, orbit errors are small compared to the transverse beam sizes and to other typical sources of orbit errors in RHIC. At storage energy and low beta the relative vertical displacement error is roughly the same, while the relative vertical angular error falls by approximately an order of magnitude.

References

[1] K. Brown et al. CERN 73-16 (1973).

[2] S. Peggs, "The Projection Approach to Solenoid Compensation", *Particle Accelerators*, **12**:24, p. 219, 1982. See also S. Peggs, "Coupling and Decoupling in Storage Rings", *IEEE Trans. Nucl. Sci.* , **NS-30**:4, p. 2460, August 1983.

[3] R. Larsen, SPEAR-107 (March 1971).