Requested Systematic Body b_5 Multipoles in RHIC Triplet Quadrupoles

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Abstract

In this report, we summarize the compensation methods of the allowed magnetic multipole b_5 in the interaction region (IR) triplet magnet that consists of body-ends compensation, tuning shimming, and local corrections. From the experience gained so far with the on-site production of Q1 magnets of series number less than 8, the amount of b_5 in the body is requested to be -1.2 unit in order to compensate for the effects caused by the strong b_5 at the lead and return ends. This body-ends compensation allows for the tuning shims to be adequately used only for the correction of random errors in each magnet.

1. Introduction

During the storage of heavy-ion beam in RHIC, the magnetic field quality in the triplet quadrupoles crucially determines the dynamic aperture and the beam lifetime when the β^* is lowered to 1 meter. Define the magnetic multipole in terms of the quadrupole "prime" units,

$$B_y + iB_x = G_0(x+iy)\left[1 + 10^{-4}\sum_{n=1}^{\infty}(b_n + ia_n)\left(\frac{x+iy}{R_0}\right)^{n-1}\right]. \quad (1)$$

and the integrated multipole

$$B_n = \int b_n ds,$$

where the reference radius $R_0 = 40.625$ mm and the nominal quadrupole gradient $G_0 = 47$T/m for the 130 mm diameter triplet quadrupoles. Table 1 shows the expected values of magnetic multipole harmonics in the body, lead end, and return end of the triplet quadrupole at the current 5000 Amps for storage.

Expected values of body harmonics (Unit):

n	$\langle b_n \rangle$	$d(b_n)$	$sig(b_n)$	$\langle a_n \rangle$	$d(a_n)$	$sig(a_n)$
1	0	0	10	0	0	0
2	0.5	0	2.4	0.1	0	1.2
3	0	1.0	0.6	0.3	0	0.7
4	0.3	0	0.6	0.1	0	0.5
5	-1.2	1.5	0.5	-0.4	0	0.6
6	0.12	0	0.11	-0.09	0	0.12
7	-0.1	0.05	0.05	-0.03	0	0.12
8	-0.04	0	0.05	0.05	0	0.09
9	0.0	0.2	0.03	0.03	0	0.03

Expected values of harmonics in lead end (Unit-m):

n	$\langle B_n \rangle$	$d(B_n)$	$sig(B_n)$	$\langle A_n \rangle$	$d(A_n)$	$sig(A_n)$
1	0	0	0	0	0	0
2	-0.1	0	0.7	$-1.$	0	2.
3	-0.3	0	0.3	0.4	0	0.8
4	0.1	0	0.3	0.3	0	0.4
5	4.6	0.5	0.3	-1.5	0.5	0.2
6	0.01	0	0.04	-0.06	0	0.06
7	0.04	0	0.05	-0.02	0	0.01
8	0.04	0	0.05	-0.02	0	0.02
9	-0.5	0.05	0.02	0.2	0.05	0.03

Expected values of harmonics in return end (Unit-m):

n	$\langle B_n \rangle$	$d(B_n)$	$sig(B_n)$	$\langle A_n \rangle$	$d(A_n)$	$sig(A_n)$
1	0	0	0	0	0	0
2	0.3	0	1.8	0.7	0	1.
3	-0.1	0	0.2	-0.1	0	0.3
4	0	0	0.25	0.2	0	0.2
5	1.	0	0.6	-0.1	0	0.1
6	0.06	0	0.03	0.06	0	0.02
7	-0.01	0	0.02	-0.02	0	0.05
8	0.03	0	0.03	-0.01	0	0.02
9	-0.1	0	0.03	0.04	0	0.01

Table 1: Expected values of harmonics in body, lead end, and return end of the 130 mm insertion quadrupoles at 5000 A. Here, $\langle b_n \rangle$ = mean, $d(b_n)$ = uncertainty in mean, $sig(b_n)$ = sigma for b_n.

The allowed magnetic multipoles in the quadrupole configuration are b_1, b_5, b_9, $\cdots$. The effect of b_1 errors can be easily compensated by the tuning quadrupoles during the operation. The correction of b_5 errors consists of body-ends compensation on systematic errors, tuning shimming on random errors, and additional local correction by triplet correctors. Since there is no correction on multipoles of higher order than 5, the tolerable amount of b_9 errors is determined by tune-spread analysis and dynamic aperture studies.

Based on the measurements of the RHIC triplet quadrupole magnet Q1 of serial numbers less than 8, the lead and return ends of the magnets have significant amount of systematic b_5 multipole component. The integrated b_5 is equal to 4.6 unit·m at the lead end and 1.0 unit·m at the return end, respectively. The integrated b_9 is equal to 0.6 unit·m at the lead end and 1.0 unit·m at the return end, respectively. Preliminary studies show that these b_5 errors produce significant tune spreads when the beam is stored in the 1 meter β^* lattice.

This report describes the compensation of b_5 multipole in the triplets. In Section 2, we generally discuss the compensation method based on the minimization of the tune shift and local "kick". In Section 3, we investigated the effectiveness of the body-ends systematic b_5 compensation for the RHIC triplet and compare several alternative compensation schemes. The correction of random b_5 errors is discussed in Section 4. The additional b_5 correction by using two local correctors per triplet is discussed in Section 5. Conclusions and discussion are given in Section 6.

2. Tune Spread and Kick Minimization

The amplitudes of the betatron oscillations reach their maximum values when the particle passes the triplets. On the other hand, the dispersion at the triplet is relatively small. We therefore only consider the amplitude effects on the particle motion from the nth-order magnetic multipole error. In the following, we discuss the compensation schemes based on the minimization of the transverse tune spreads and the local kicks at each triplet.

2.1. Kick minimization

We minimize the multipole kick using the horizontal plane as an example. The analysis for the vertical plane is similar. The change of transverse momentum $p_x \equiv dx/ds$ for the on-momentum particle produced by the magnetic multipole b_n is

$$\frac{dp_x}{ds} = -\left(\frac{10^{-4}G_0}{B_0\rho R_0^{n-1}}\right) b_n x^n \tag{2}$$

where $B_0\rho$ is the momentum of the particle. The change of the momentum results in a change in the action J_x, which is otherwise a constant of motion. The action J_x can be written as

$$J_x = \frac{1}{2\beta_x}\left[x^2 + (\alpha_x x + \beta_x p_x)^2\right] \tag{3}$$

where

$$x = \sqrt{2J_x\beta_x}\cos\chi, \quad p_x = -\sqrt{\frac{2J_x}{\beta_x}}(\sin\chi_x + \alpha_x\cos\chi_x) \tag{4}$$

with χ_x the betatron phase in the absence of perturbation. Here, $2\pi R$ is the circumference, and α_x and β_x are the Courant-Snyder lattice functions. In the presence of the multipole error b_n, the change of the action can be derived from Eq. 2,

$$\frac{\Delta J_x}{J_x} = \left(\frac{10^{-4}G_0}{B_0\rho R_0^{n-1}}\right) b_n L \beta_x^{\frac{n+1}{2}} (2J_x)^{\frac{n-1}{2}} (2\sin\chi_x\cos^n\chi_x) \tag{5}$$

where L is the length of the magnet. Typically, the total length (e.g. about 10 meters for RHIC) of each triplet is much smaller that the value of the amplitude function β (e.g. from 600 to 1400 meters for RHIC 1 m β^* operation). Therefore, there is essentially no betatron phase advance within the triplet. Consequently, the quantity $\Delta J_x/J_x$ can be minimized if the quantities

$$K_{nx} = \int_{\text{trip}} \frac{G}{G_0}\beta_x^{\frac{n+1}{2}} b_n ds, \quad \text{and} \quad K_{ny} = \int_{\text{trip}} \frac{G}{G_0}\beta_y^{\frac{n+1}{2}} b_n ds \tag{6}$$

are both minimized. Here, the integrals extend over all the elements within one triplet. The quadrupole gradients G of the RHIC triplet quadrupoles Q1, Q2, and Q3 at storage are given by Table 2, where DFD and FDF denote polarity of the magnets Q1, Q2, and Q3 of the triplet pair near the interaction point (IP). The nominal gradient G_0 is chosen to be 47 T/m.

2.2. Tune spread minimization

In order to accommodate the independent adjustment of each IP, we minimize the tune spread generated by the pair of triplets located near each IP. In the case that the multipole of interest produces first-order tune shifts, we use the well-known tune-shift formula (e.g. Eq. 7 in Ref. 1)

$$
\begin{aligned}
\Delta\nu_x &= \frac{10^{-4}G_0}{B_0\rho R_0^{n-1}}\oint\frac{\beta_x ds}{2\pi}\left\{C_1+3C_2\beta_xJ_x-6C_2\beta_yJ_y\right.\\
&\quad\left.+\frac{15}{2}C_3\beta_x^2J_x^2-45C_3\beta_x\beta_yJ_xJ_y+\frac{45}{2}C_3\beta_y^2J_y^2\right\}\\
\Delta\nu_y &= \frac{10^{-4}G_0}{B_0\rho R_0^{n-1}}\oint\frac{\beta_y ds}{2\pi}\left\{-C_1+3C_2\beta_yJ_y-6C_2\beta_xJ_x\right.\\
&\quad\left.-\frac{15}{2}C_3\beta_y^2J_y^2+45C_3\beta_x\beta_yJ_xJ_y-\frac{45}{2}C_3\beta_x^2J_x^2\right\}
\end{aligned}
\tag{7}
$$

where the integrals are performed along the ring, and

$$
\begin{aligned}
C_1 &= -\frac{1}{2}b_1\delta+b_2\Delta_x-a_2\Delta_y+\frac{3}{2}(b_3\Delta_x^2-a_3\Delta_y^2)\\
C_2 &= \frac{1}{4}b_3+b_4\Delta_x-a_4\Delta_y\\
C_3 &= \frac{1}{6}b_5+b_6\Delta_x-a_6\Delta_y.
\end{aligned}
\tag{8}
$$

Here, $\Delta_x = D_x\delta + x_c$, $\Delta_y = D_y\delta + y_c$, D_x and D_y are the dispersions, δ is the fractional momentum deviation, and x_c and y_c are the closed-orbit displacement from the magnet center. Because β_x and β_y achieve their maximum values at different location, it can be verified that the major effect of the nth multipole on particles of constant total action $J_x + J_y$ is from the two leading terms proportional to $J_x^{(n-1)/2}$ and $J_y^{(n-1)/2}$, respectively. Therefore, the tune spread due to one pair of triplets can be minimized when the quantities

$$
K_{nx\mathrm{IR}} = K_{nx\mathrm{FDF}} + K_{nx\mathrm{DFD}} \quad \text{and} \quad K_{ny\mathrm{IR}} = K_{ny\mathrm{FDF}} + K_{ny\mathrm{DFD}} \tag{9}
$$

are minimized over each IR.

3. Body-Ends Compensation of b_5 Multipole

The systematic multipole b_5 in the lead and return ends of the triplet quadrupole can be compensated by the systematic b_5 in the quadrupole body. The compensation is based on the following three principles. Firstly, both the tune shift over the triplet pair near each IP and the local kicks in each triplet should be minimized. Secondly, the net amount of body b_5 should be made as small as possible in order not to "introduce" undesired errors when the compensation is made imperfect by the lattice deviation from the ideal design. Furthermore, the choice of b_5 should be technically simple and flexible for the magnet manufacturing.

The analysis on the triplet multipole compensation is based on the simulation study of the so called "ideal" lattice where both misalignment errors and magnetic errors are excluded. β^* is equal to 1 meter at six and eight o'clock interaction points (IP), and is equal to 10 meter at the rest four IP's. The sextupole magnets are adjusted so that both horizontal and vertical chromaticities are equal to 2. Tables 3a and 3b list the lattice functions at positions where the DFD and FDF triplets are located, respectively. As shown in Figure , the lead ends of Q3 are towards IP, while the lead ends of Q1 and Q2 are away from IP. This arrangement of lead-end locations makes the errors from the lead ends relatively less important (as shown by the relative coefficients in Eq. 14).

3.1. Compensation scheme

The best way to compensate for b_5 in the lead and return ends is to adjust the amount of requested body b_5, identically in all triplet quadrupoles, to minimize the quantities $K_{5x\mathrm{IR}}$ and $K_{5y\mathrm{IR}}$ in Eq. 9 of each IR. Notice that among the triplet pair near each IP, both the kicks and tune shifts are mainly produced by the F quadrupoles where the amplitude function $\beta_{x,y}$ reaches its maximum either Q2 in the DFD or Q3 in the FDF triplet. Therefore, the body b_5 of these F quadrupole must be of the opposite sign of that of their ends. For engineering convenience, we assume that the body b_5 is the same for all the magnets Q1, Q2, and Q3. Due to the optical anti-symmetry of the x and y plane over DFD and FDF triplet,

$$K_{5x\mathrm{DFD}} \approx -K_{5y\mathrm{FDF}}, \quad \text{and} \quad K_{5y\mathrm{DFD}} \approx -K_{5x\mathrm{FDF}}, \tag{10}$$

the condition for the quantities of Eq. 9 to be equal to zero is

$$D_{\mathrm{FDF}} + D_{\mathrm{DFD}} + b_5 L_{eff} = 0 \tag{11}$$

	Q1	Q2	Q3
G(FDF) (T/m)	48.4	−47.0	47.3
G(DFD) (T/m)	−48.4	47.0	−47.3

Table 2: Gradients of triplet quadrupoles at storage.

Figure 1: Schematic layout of the RHIC triplet, showing the quadrupoles, the orientation of the quadrupole lead ends, and the local correctors C1, C2, and C3.

where the drive D comes from both the lead (L) and the return (R) ends

$$D = \frac{B_{5L}}{G_0} \sum_{i=1}^{3} G_i \beta_{xi}^3 \Big|_L + + \frac{B_{5R}}{G_0} \sum_{i=1}^{3} G_i \beta_{xi}^3 \Big|_R , \tag{12}$$

the subscripts $i = 1, 2, 3$ represent the quadrupoles Q1, Q2, and Q3, and the effective length is defined as

$$L_{eff} = \frac{1}{G_0} \sum_{i=1}^{3} \int G_i \beta_{xi}^3 ds \Big|_{\mathrm{FDF}} + \frac{1}{G_0} \sum_{i=1}^{3} \int G_i \beta_{xi}^3 ds \Big|_{\mathrm{DFD}} . \tag{13}$$

Using the lattice function from Table 3, the desired b_5 in the body is obtained as

$$b_5 = -0.17\ B_{5L} - 0.35\ B_{5R} = -1.2\ (\mathrm{unit}). \tag{14}$$

On the other hand, if the local kicks are to be minimized to zero, the required b_5 in the body becomes, respectively,

$$b_5 = \begin{cases} -0.31\ B_{5L} - 0.59\ B_{5R} = -2.0\ (\mathrm{unit}) & \text{for } K_{5x} = 0 \text{ in FDF or } K_{5y} = 0 \text{ in DFD} \\ -0.078\ B_{5L} - 0.19\ B_{5R} = -0.54\ (\mathrm{unit}) & \text{for } K_{5y} = 0 \text{ in FDF or } K_{5x} = 0 \text{ in DFD} \end{cases} . \tag{15}$$

The desired b_5 value given by Eq. 14 is, by chance, about the average of the values in Eq. 15.

Figure 2 shows the tune footprint for on-momentum particles produced by the b_5 multipole error at the lead and return ends of one pair of triplet (6 o'clock). The mesh of points represents a spectrum of particles launched with initial amplitudes between 0 to 5σ in each plane individually, or along several contours of constant total action $(J_x + J_y)$ where the ratio (J_x/J_y) of horizontal and vertical action is smoothly varied. The normalized 95% emittance is assumed to be 40πmm·mr. The energy of the particle is 100 GeV/u ($\gamma = 107$). Figure 3 shows the similar diagram when the body b_5 is set to be equal to -1.2 unit. The correction in tune spread is obviously satisfactory.

3.2. Comparison with alternative schemes

Several alternative schemes have been explored to determine the optimum compensation method. Table tab:4 lists the possible 9 schemes denoted A1 through A9. The one discussed in Section 3.1 is denoted A7. In Table 1, the notation Q, T, and IR indicates that the

Table 3: Lattice functions for the triplets.

correction is applied by minimizing the undesired effect in each quadrupole, triplet, and IR, respectively.

In scheme A1 and A2, each quadrupole magnet is compensated independently. The fact that the weight in A1 is equal to 1 indicates that the integrated b_5 over each quadrupole is made to be zero. These methods result in a relatively large b_5 in the body of each magnet, which is against the second principle for b_5 compensation.

Because of the large variation of β_x and β_y over the quadrupole body, the minimization of the kicks and tune spreads is not good when the weight is taken to be 1. The desired values of b_5 body are also different when β_x^3 and β_y^3 are used as weight. The optimum schemes are found when both the triplet in each IR are taken into account (A7, A8, and A9). In scheme A8, the amount of desired b_5 is large. In scheme A9, the sign of the body b_5 is allowed to vary for different magnet. Obviously, the amount of body b_5 can be made smaller if the effective length L_{eff} is made longer. In this case, the amount of b_5 required for tune-spread minimization is

$$|b_5| = |-0.13\ B_{5L} - 0.26\ B_{5R}| = 0.9\ \text{(unit)} \tag{16}$$

which is about 75% of the amount given by the original method (Eq. 16). However, the complication caused by the sign flipping makes this method less attractive (against the third principle for b_5 compensation). Therefore, the scheme one should adopt is A7, as discussed in Section 3.1.

4. Tuning Shimming

Tuning shims have been used to make the integrated multipoles from order A_2 to A_5, and from order B_2 to B_4 to be equal to zero[2] in each triplet quadrupole. On the other hand, due to the large value of b_5 in the triplet quadrupole ends, the available range of shim (about ± 3.2 mm in thickness) is not adequate for compensation.

The body-ends compensation on b_5 multipole allows for the tuning shims to be used only for the correction of random b_5 errors in each magnet. According to the correction scheme A7, the total b_5 in the body after shimming should be equal to -1.2 unit. The available range of shimming is adequate for the correction of the relatively small value of random b_5, as shown in Table 1. Even if the systematic body b_5 of the Q1 quadrupoles first constructed on site is not equal to the desired value of -1.2, the tuning shims will still only respond to the random b_5 values around the new systematic mean. In this case, the requested systematic b_5 body will be slightly modified for Q2 and Q3 production.

5. Local Correction

As shown in Figure , there exists three multipole corrector packages C1, C2, and C3 in each triplet for local correction. Among the correctors (a_1, $b2$, and a_5) in the middle C2 package, only a_1 correctors are currently planned to be powered. Since the betatron phase advance from dipole D0 to triplet quadrupoles Q1, q2, and Q3 is very small (Table 3), the strength of the a_1 corrector at each C2 may be set to compensate for the a_1 on these elements using the relation

$$\sqrt{\beta_x\beta_y}a_1L\Big|_{C2} = \int_{\mathrm{D0,trip}} \sqrt{\beta_x\beta_y}a_1ds, \tag{17}$$

where L is the length of the corrector, and the integral extends over D0, Q1, Q2, and Q3.

The C1 and C3 packages consisting of b_3, b_4, and b_5 correctors are used to correct the residual b_3, b_4, and b_5 after the manufacture (with body-ends compensation and tuning shimming) and installation of the D0 and triplet magnets. The location of C1 and C3 are chosen such that the horizontal and vertical β functions are large at different correctors (i.e. β functions orthogonal at two locations). The strengths of the two correctors are adjusted according to the measurement of the multipole errors of triplet quadrupoles and D0 and the lattice functions such that the total kicks K_{nx} and K_{ny} (n=3,4,5) in both x and y planes from the triplet and D0 are independently adjusted to zero for each triplet and D0. The maximum amount of b_5 achievable in each corrector of 0.5 meter length with 50 Amp power supply corresponds to about 20 units b_5 (normalized to G_0 = 47 T/m). If body-ends compensation and shimming are performed within reasonable accuracy, the corrector strength is adequately for the compensation of residual b_5 errors.

6. Conclusions and Discussion

In this note, we have discussed the compensation of the allowed magnetic multipole b_5 in the IR triplet magnet where the amplitude of the transverse particle oscillation is the largest. From the experience gained so far with the on-site production of Q1 magnets of series number less than 8, the amount of b_5 in the body is requested to be -1.2 unit in order to compensate for the effects caused by the strong b_5 at the lead and return ends. If the actual value of b_5 in Q1 magnets differs significantly from its desired value of -1.2, further fine tuning might be requested in the later production runs of Q3 and Q2. This allows for the tuning shims to be used only for the correction of random errors in each magnet. Consequently, the tuning shim magnetic non-linear effects and the the "feed-up" effects (coupling desired body b_5 changes to undesired body b_9 changes) become less important.

Compared with the b_5 compensation, the systematic body compensation for b_9 from the ends is less effective partly due to the strong dependence of the b_9 kicks on the variation of the oscillation amplitude. Previous studies indicate that b_9 in Q2 and Q3 magnet ends should be made less than 0.3 unit-m for proper beam storage. The requested b_9 in the magnet body is thus zero.

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References

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Method	Correction Unit	Weight	Sign flip?	Polarity	b5(Body) Q1	b5(Body) Q2	b5(Body) Q3
A1	Q	1	-	-	-3.9	-1.7	-2.7
A2	Q	β_x^3	-	F/D	$-3.6/-5.1$	$-1.1/-3.0$	$-2.0/-4.3$
A3	T	1	no	-	-40.0	-40.0	-40.0
A4	T	1	yes	-	-0.8	0.8	-0.8
A5	T	β_x^3	no	FDF/DFD	$-2.0/-0.5$	$-2.0/-0.5$	$-2.0/-0.5$
A6	T	β_x^3	yes	FDF/DFD	$-1.5/0.4$	$1.5/-0.4$	$-1.5/$ 0.4
A7	IR	β_x^3 and β_y^3	no	both	-1.2	-1.2	-1.2
A8	IR	β_x^3 and β_y^3	yes	both	5.2	-5.2	5.2
A9	IR	β_x^3 and β_y^3	yes	FDF/DFD	$-0.9/0.9$	$0.9/-0.9$	$-0.9/0.9$

Table 4: Possible body-ends compensation schemes for the b_5 multipole in the triplet quadrupoles.

Figure 2: Tune shift of on-momentum particles with betatron amplitude from 0 to 5 σ with the 1 meter β^* storage lattice produced by one pair of triplet with lead end error $B_5 = 4.6$ unit·m and return end error $B_5 = 1.0$ unit·m. The horizontal and vertical integer tunes are 28 and 29, respectively.

Figure 3: Similar to Figure 2, with $b_5 = -1.2$ unit in the body for compensation.