Comparison of Perturbation Method and TEAPOT Tracking on Tune Shift Calculations

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1. Introduction

Correction schemes for the magnetic multipole errors of the RHIC insertion triplets are based on the minimization of the tune spreads and local kicks. Because the tune spreads produced by multipoles are evaluated using the analytical formulae derived from perturbation theory, it is necessary to determine the range of validity of the perturbative approach.

In this note, we compare the tune spreads evaluated by the perturbation method, HARMON subroutine, and TEAPOT numerical tracking using the storage and injection RHIC lattices. Section 2 describes the methods and procedures used for the comparison. Section 3 presents the results. The conclusion is given in section 4.

2. Methods and Procedures

With given momentum deviation and transverse action, the transverse tune shifts of particle motion can be evaluated analytically by the well known formulae derived using perturbation theory.[1] Based on these formulae, a computer program (temporarily named TUNE_TPOT) has been developed.[1,2] This program reads as input the standard TEAPOT output[3] file 7 for normal and skew multipoles on each element from order 0 to 9. Tune shifts, including feed-downs contributed from multipoles of order 3 (octupole) and higher, are evaluated using the analytical formulae to first-order in multipole strength. The tune shifts contributed from the bare machine lattice (which consists of dipole, quadrupole, and sextupole magnets for tune and chromaticity adjustments) are evaluated according to the coefficients calculated by the HARMON subroutine[4] of the MAD program.

Alternatively, the transverse tune shifts are evaluated by tracking the particle motion using the TEAPOT program. First, the same input file (TEAPOT file 7) that includes magnetic multipoles is read in. Tuning, de-coupling, and chromaticity fitting are performed to obtain the desired tunes and chromaticities. With the same initial betatron amplitudes and momentum deviations as those used for analytical calculation, particles are then tracked for 1024 revolutions without synchrotron oscillations. The turn-by-turn data are finally analysed by the TEALEAF program to evaluate the tune shifts.

Tune shifts are evaluated for $^{197}\mathrm{Au}^{79+}$ ions both at injection ($\gamma = 12.6$) and at the end of storage ($\gamma = 107$), with momentum deviations of $-2.5\ \sigma_p$, 0, and $2.5\ \sigma_p$. Here, the r.m.s. relative momentum spread σ_p is equal to 0.44×10^{-3} at injection and 0.89×10^{-3} at the end of storage. The initial normalized actions of the particles are chosen to satisfy

$$J_x + J_y = n^2\left(\frac{\epsilon_N}{6\pi}\right), \quad \text{and} \quad J_x = \frac{m}{4}\, n^2\left(\frac{\epsilon_N}{6\pi}\right) \qquad m = 0, 1, 2, 3, 4, \tag{1}$$

where the 95% normalized emittance ϵ_N of the beam is 10π mm·mr at injection and 40π mm·mr at the end of storage. In the case of storage, n is chosen to be 0, 1, 2, 3, 4, and 5, respectively, while in the case of injection, n is chosen to be 0, 1, 2, 3, 4, 5, 6, and 7, respectively.

To compare the two different methods, we calculate the root mean square deviation of the differences in the tune shift based on every five data points ($m = 0, 1, 2, 3$, and 4 in Eq. 1) for particles of different total action $n^2\epsilon_N/6\pi$ and momentum deviation $\Delta p/p$. In the following section, we summarize the results for various cases.

3. Comparison

First, we compare the tune shifts from the bare storage lattice. The β^*'s at the two interaction regions (six and eight o'clock) are set to 1 meter, while the rest are set to 10 meters. Both horizontal and vertical chromaticities are adjusted to 2. Magnet misalignments are not included.

Fig. 1a shows the tune spreads calculated by using the 0th, 1st, 2nd, and 3rd order chromaticity and 1st order action dependence evaluated by the HARMON subroutine of the MAD program. The origin of the tune diagram corresponds to the split integer tunes of $\nu_x = 28$ and $\nu_y = 29$. The working point (ν_x, ν_y) is at (28.19, 29.18). The three tune triangles from lower left to upper right in Fig. 1a correspond to the momentum deviations of $-2.5\,\sigma_p$, 0, and $2.5\,\sigma_p$, respectively. Fig. 1b shows the corresponding tune spreads calculated by the program TEALEAF using 1024–turn TEAPOT tracking data. The maximum r.m.s. tune difference is about 4×10^{-4}. The relative error is about 20%.

Similar to Fig. 1, Fig. 2 compares the tune shifts from the storage lattice when the only multipole errors present are those listed in Table 1 for the IR triplets. Although the local triplet correctors are not activated, the tuning shims[2] have been inserted into the triplet quadrupoles to compensate for the systematic and random multipoles. The random multipole errors are generated by random seed #6 from the Machine Advisory Committee '93 (MAC93) tracking run. Misalignments are set to zero. Fig. 2a shows the perturbation result combined with the HARMON output for the bare lattice. Fig. 2b shows the TEAPOT tracking result. Table 2 lists the r.m.s. tune difference for various momentum deviations $\Delta p/p$ and betatron amplitudes n. The relative error is typically less than 30% for particles up to 5 σ betatron amplitude. The point with the largest error sits on the $\nu_x = \nu_y$ coupling line in Fig. 2b, corresponding to the particle with $\Delta p/p = -2.5\,\sigma_p$ and initial 5 σ horizontal amplitude ($n = 5$ and $m = 4$). This fact is consistant with the MAC93 tracking results showing that particle loss starts to occur when the betatron amplitude is larger than 5 σ. It is thus indicated that the perturbative approach fails when the betatron amplitude is larger than 5 σ in the case of $\beta^* = 1$ m storage lattice.

Fig. 3 corresponds to the storage lattice when the triplet magnets are not shimmed. Because of the large multipole errors, the perturbative approach (Fig. 3a) fails to describe the actual particle motion (Fig. 3b).

Fig. 4 corresponds to the case of the injection lattice when β^*'s at the six IRs are all equal to 10 meters. The multipole errors are provided by the MAC93 tracking run injection seed #0. Although insignificant in this case, tuning shims have been inserted into the triplet magnets. Neither octupole nor decapole corrections are activated. The chromaticities are set to -3. For better comparison, tune shifts are evaluated up to 7 σ betatron amplitude. Table 3 lists the r.m.s. tune difference between the two methods. The discrepancy for the two small-amplitude points at $\Delta p/p = 2.5\ \sigma_p$, $m = 4$, $n = 2$ and 3 is believed to be caused by a technical problem (currently under investigation) in the TEALEAF program. The lost particle at 7 σ betatron amplitude (with $\Delta p/p = -2.5\ \sigma_p$) indicates that the perturbative approach fails when the amplitude is larger than 7 σ in the case of injection lattice.

4. Conclusion

Our study shows that the agreement between the HARMON and the TEAPOT tracking on the tune shifts for the bare lattice is within about 20%. Combined with the HARMON bare-lattice result, the tune shifts calculated by the perturbation method can be compared to that from TEAPOT tracking. With the $\beta^* = 1$ m storage lattice, the agreement is within about 30% when the betatron amplitude is less than 5 σ, provided that the IR triplets are corrected by the insertion of tuning shims. With the injection lattice, these two methods are comparable up to 7 σ betatron amplitude.

References

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Table 1: Multipole errors in the triplet quadrupole magnets used for tracking studies in 10^{-4} primed quadrupole units.[2]

Table 2: R.m.s. difference between the perturbation method and TEAPOT tracking on the tune shifts for the $\beta^* = 1$ m storage lattice. The Tune shifts are shown in Fig. 2.

$\Delta p/p$ (σ_p)	n	r.m.s. $\Delta\nu_x$ ($\times 10^{-3}$)	r.m.s. $\Delta\nu_y$ ($\times 10^{-3}$)
-2.5	0	0.98	0.98
-2.5	2	0.46	0.39
-2.5	3	0.47	0.44
-2.5	4	0.60	0.34
-2.5	5	75[a]	71[a]
0	0	0.37	0.39
0	2	0.28	0.41
0	3	0.37	0.49
0	4	0.44	0.57
0	5	0.87	0.68
2.5	0	0.091	0.17
2.5	2	0.63	0.32
2.5	3	0.68	0.40
2.5	4	0.91	0.54
2.5	5	1.7	1.0

a) Assume that the particle sitting on the $\nu_x = \nu_y$ coupling line at the end of the 1024–turn tracking is eventually lost.

Table 3: R.m.s. difference between the perturbation method and TEAPOT tracking on the tune shifts for the injection lattice. The Tune shifts are shown in Fig. 4.

$\Delta p/p$ (σ_p)	n	r.m.s. $\Delta\nu_x$ ($\times 10^{-3}$)	r.m.s. $\Delta\nu_y$ ($\times 10^{-3}$)
-2.5	2	1.4	1.1
-2.5	3	1.8	1.3
-2.5	4	1.8	1.3
-2.5	5	1.5	1.1
-2.5	6	1.1	0.63
-2.5	7	76[a]	77[a]
0	2	1.3	0.90
0	3	1.6	1.1
0	4	1.7	1.1
0	5	1.4	0.93
0	6	1.1	0.52
0	7	0.93	0.33
2.5	2	1.3	5.9[b]
2.5	3	1.6	2.0[b]
2.5	4	1.6	0.96
2.5	5	1.4	0.79
2.5	6	1.0	0.49
2.5	7	0.58	0.24

a) One particle lost before the end of the 1024–turn tracking.

b) Large error due to problems with TEALEAF.

Figure 1: Tune shift of particles of momentum deviation $\Delta p/p = 0, \pm 2.5\ \sigma_p$ and betatron amplitude from 0 to 5 σ with the bare storage lattice calculated a) by using the output from HARMON subroutine of the MAD program; b) from the TEAPOT tracking data.

Figure 2: Tune shift of particles of momentum deviation $\Delta p/p = 0, \pm 2.5\ \sigma_p$ and betatron amplitude from 0 to 5 σ with the storage lattice and the multipole error in Table 1 when the tuning shims are inserted, calculated a) by using the perturbation methods with HARMON output; b) from the TEAPOT tracking data.

Figure 3: Similar to Figure 2, except the tuning shims are not inserted.

Figure 4: Tune shift of particles of momentum deviation $\Delta p/p = 0, \pm 2.5\sigma_p$ and betatron amplitude from 0 to 7 σ with the injection lattice and the multipole error for MAC93 tracking run seed #0, calculated a) by using the perturbation methods with HARMON output; b) from the TEAPOT tracking data. Tuning shims are inserted, but octupole and decapole corrections are not.