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Spin Note

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Possibilities of Accelerating Polarized D, ^{3}He and other ions in RHIC

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Polarized ions other than protons may be accelerated, stored and collided in a ring such as RHIC provided the techniques proposed for polarized proton operation can be adapted (or replaced by other strategies) for these ions.

To accelerate polarized particles in a ring, one must make provisions for overcoming the depolarizing resonances that occur at certain energies. These resonances arise when the spin tune (ratio of spin precession frequency to orbit frequency) resonates with a component present in the horizontal field. The horizontal field oscillates with the vertical motion of the particles (due to vertical focusing); its frequency spectrum is dominated by the vertical oscillation frequency and its modulation by the periodic structure of the accelerator ring. In addition, the magnet imperfections that distort the closed orbit vertically contain all integral Fourier harmonics of the orbit frequency.

The spin precession frequency in a plane machine is

$$\nu_{sp} = G\gamma \tag{1}$$

in the frame of reference turning with the particle, where G = (g-2)/2 is the magnetic moment anomaly of the particle. Resonances due to betatron oscillations ("intrinsic resonances") occur at those energies where

$$G\gamma = \pm\nu_y + kP \tag{2}$$

where ν_y is the vertical betatron oscillation tune, P is the periodicity of the lattice, and k is any integer. In addition "imperfection" resonances, associated with orbit distortion, occur at all integer values of $G\gamma$. Each resonance is characterized by a *strength* ε, which depends on the amplitude of oscillations and/or orbit distortion as well as on the details of the lattice and, of course, on G. The resonance strengths can be calculated by the program DEPOL .

For protons $G = 1.793$, and therefore the imperfection (integer) resonances are spaced by Mc^2/G = 523 MeV. In the RHIC lattice (6 insertions, the equivalent of about 81 FODO cells, P=3) intrinsic resonances for each sign in (2) are 3×523 MeV apart, but the strong ones only occur with spacing of about 81 units of $G\gamma$. The strongest imperfection resonances tend to occur at those integer values of $G\gamma$ which are near the ones for strong intrinsic resonances.

On passage through a resonance the vertical spin component is multiplied by the Froissart-Stora factor

$$F = 2\exp[-(\pi\varepsilon)^2 / \Delta] - 1 \tag{3}$$

where Δ equals the increment in $G\gamma$ per revolution. To avoid depolarization this factor should be close to +1 or -1 for the passage through each resonance, i.e. the resonance strength ε should be either small enough to make $F > .99$ or large enough to make $F < -.99$: either $\varepsilon < .0225\sqrt{\Delta}$ or $\varepsilon > .733\sqrt{\Delta}$.

To overcome the depolarization due to the resonances several methods may be employed:

For all resonances: "Siberian snakes". These are devices that rotate the spin by 180° about a horizontal axis. With two snakes, spaced exactly 180° apart in the ring, and having axes that differ by 90°, the effect of the snakes is that in the absence of depolarizing field the spin tune is exactly ½ *independent of energy*, so that resonances do not arise (assuming the oscillation frequency is not ½); the stable spin direction is vertical, up in half the ring and down in the other half. Siberian snakes may be made of interleaved horizontal and vertical transverse deflection magnets, or helical dipole magnets (as proposed for RHIC). They are attractive for high-energy machines because the magnet strength needed for a given amount of spin rotation is independent of energy (due to the factor γ in eq. (1)), so that they can be fixed-field, unramped devices. For protons a Siberian snake - depending on detailed design - typically requires a total of around 20 to 25 Tesla-meters of deflection magnets. The orbit deflection associated with a given spin rotation is $1/G\gamma$ times the spin rotation; therefore it can be quite small at high energy. On the other hand, Siberian snakes take up a minimum of 6-7 meters of space; this makes them impractical for machines such as the AGS where the longest straight section is 3 meters. Therefore other resonance-killing methods have to be, and are, employed there.

For imperfection resonances:

(a) Correct the magnet errors so well that the imperfection resonances become weak enough. This was the method originally used at the AGS; it was very laborious and time-consuming. This method has now been superseded by

(b) The "partial snake" - a solenoid which rotates the spin by a small fraction (2-10%) of a full rotation. This *enhances* all the imperfection resonances and makes them strong enough so that complete spin reversal occurs on passage through each resonance (i.e. the factor F of eq. (3) is close to -1).

For intrinsic resonances:

(a) Rapidly jump across the resonance (2) by energizing fast quadrupoles that change the value of the vertical tune ν_y. This worked at the ZGS and the AGS, but was awkward and required large power supplies; furthermore at each jump the beam emittance would grow irreversibly. This method has now been superseded at the AGS by

(b) With a radiofrequency dipole, excite coherent betatron oscillations at a frequency close to the resonant frequency, and with an amplitude large enough so that the strength of the resonance is in the spin-flip range. Then, as the resonance energy is traversed, the spin of all particles will reverse. After the resonance has been passed, modulate the rf excitation back down to zero; this should restore the beam emittance to its previous value. This method has been employed successfully first at IUCF and more recently in the latest run of Experiment E-880 at the AGS.

For acceleration of polarized ions other than protons, we have to investigate whether one or more of the above methods will work with these ions.

All nuclei with non-zero spin have magnetic moments, and if they are accelerated they again encounter spin resonances at appropriate values of $G\gamma$. The anomaly G for a nucleus whose magnetic moment is μ nuclear magnetons, atomic number Z , mass number A and spin S is

$$G = \mu \frac{Z}{2AS} - 1 . \tag{4}$$

A measure of the feasibility of Siberian snakes is the magnet strength (say in Tesla-meters) needed for a spin rotation of 180°, which can be seen to be

$$BL(\pi) = 10.48A/(ZG) \tag{5}$$

We tabulate parameters for a few light nuclei:

Nucleus	A	Z	S	μ (magnetons)	G	Resonance Spacing (GeV/u)	BL(π) (T-m)
p	1	1	½	2.793	1.793	.523	5.845
d	2	1	1	.8574	-.143	6.58	147.0
^{3}H	3	1	½	2.979	7.937	0.118	3.961
^{3}He	3	2	½	-2.1274	-4.191	.218	3.751
^{6}Li	6	3	1	.8219	-.178	5.27	117.7
^{7}Li	7	3	3/2	3.256	1.532	.612	15.96
^{10}B	10	5	3	1.801	-.40	2.347	52.42
^{14}N	14	7	1	.404	-.596	1.57	35.15
^{17}O	17	8	5/2	-1.893	-1.805	.726	12.34
^{19}F	19	9	½	2.6273	4.547	.256	4.866

We see that only ^{3}H, ^{3}He and ^{19}F have anomalous moments large enough to make Siberian snakes feasible with field strengths comparable to (in fact somewhat less than) the ones for protons.

Of these, ^{3}He is probably the one of greatest interest for physics. Here the helical snakes and rotators envisaged for protons in RHIC would work equally well if the field in each is reduced by the factor 3.751/5.845 = 0.642. So the helix whose field is 4.0 T in the proton case now needs a field of only 2.6 T, and the peak orbit excursion inside the helix, at injection energy, is 1.96 cm instead of 2.95 cm. On the other hand the resonances are more closely spaced, and somewhat stronger; with an invariant emittance of 10 π mm-mrad the strongest resonances have a strength of 0.7 units. Snakes should be capable of coping with this; however it will certainly help to make the emittance as small as possible.

The resonances in the AGS are also stronger and more closely spaced than with protons. But there should be no great difficulty in overcoming them with the partial snake for imperfections and the rf dipole for intrinsic resonances. - The rotators that bring the spin into the longitudinal direction also have to be reduced by the same 64% factor as the snakes, and should function as in the proton case.

Tritium and/or ^{19}F, if desired, can be handled the same way.

As for deuterons, the situation is entirely different. The anomalous moment of D is so small that the resonances are very few in number; in fact there is only one imperfection resonance - and no intrinsic resonance - in the AGS below the energy of injection into RHIC (presumably $\gamma \approx 14$). And in RHIC, with γ ranging from 13 to 140, there are only 12 intrinsic resonances and 19 imperfection resonances.

BUT

there is no way of getting rid of these with snakes. From the Table we see that the snake field that would be necessary for a given spin rotation is 25 times as strong as for protons, and clearly out of the question.

Intrinsic resonances can, however, be handled by the rf dipole method just as is done in the AGS; inducing coherent oscillations of amplitude 1 mm or so should be sufficient to make each intrinsic resonance strong enough for spin flip (the one at $\gamma = 120.15$ is anyway strong enough without help).

As for the imperfection resonances: With orbit corrections that bring the rms closed orbit error down to 0.6 mm DEPOL estimates that the first four imperfection resonances, up to $\gamma = 35$, are weak enough to give less than 10% depolarization; the higher ones are too strong, but not nearly strong enough for 100% spin flip. Therefore something analogous to the partial snake is needed. We have already seen that the existing helical snakes are much too weak to act as proper snakes. Even when they are fully energized, to 4 Tesla in each helix, they only act as partial snakes with a spin rotation of around 0.2°, corresponding to a resonance strength of approximately 3×10^{-4} units - not enough to overcome the imperfection resonances.

On the other hands, one may consider solenoids. Just as in the AGS, a solenoid of given strength BL rotates the spin by an angle

$$\vartheta = (1+G)BL/(B\rho) \tag{5}$$

producing integral resonances of strength

$$\varepsilon = \vartheta / 2\pi \tag{6}$$

at all integral values of $G\gamma$. If the value of ε is such that the factor F of eq. (3) is practically equal to -1 we get complete spin flip.

Now for deuterons $G = -.143$ from Table 1. With a 1 minute time T for accelerating deuterons from $\gamma = 15$ to 135, we have $\Delta = |G|(\gamma_f - \gamma_i)C/cT$ with $C = 3834$ m the circumference of RHIC, giving the vale of $\Delta = 3.66\times10^{-6}$. Thus to get $F < -0.99$ (99% spin flip) we need $\varepsilon > (\Delta \ln 200)^{1/2}/\pi = 1.4\times10^{-3}$; using (5) and (6) we find that at top energy ($B\rho = 840$ T-m) we need a solenoid of strength

$$BL = 2\pi\varepsilon(B\rho)/(1+G) = 8.7 \text{ Tesla-meters,}$$

i.e. just twice the "partial snake" solenoid in use at the AGS. Such a solenoid will give complete spin flip at each of the 19 integer resonances in the acceleration range, provided only that the imperfection resonances we would have without the partial snake are weaker than 1.4×10^{-3}. Calculations using the SYNCH and DEPOL computer programs show that, with one particular random-number generator seed, and with rms alignment errors of ¼ mm in the quads in the arc, the resulting closed orbit has an rms excursion of 2.5 mm, and the strongest depolarization resonance (at $\gamma = 119$) has strength 0.003, but the resonances below $\gamma = 100$ are weaker than 0.001. Thus our 8 T-m solenoid should be able to cope with all integer resonates below about 200 GeV. If the MICADO orbit correction scheme is used, the calculations show that the rms orbit excursion is reduced to 0.6 mm and the strongest integer resonance is well below the strength of .001; thus the partial-snake solenoid should work just fine.

With the partial-snake solenoid scheme. the spin is very nearly vertical in the whole machine when the energy is reasonably far (more than several times ε) from the resonance. But just exactly at the resonance energy the closed-orbit spin vector will precess in the horizontal plane, and just 180° from the snake it will be longitudinal. Therefore experiments with longitudinal spin will be possible at this discrete set of energies, $-G\gamma = 1,2,3,\ldots,19$ (energy at multiples of about 13.6 GeV).