Spin Dynamics in AGS and RHIC

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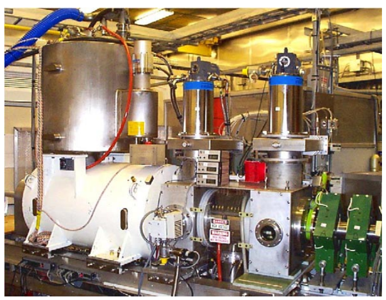
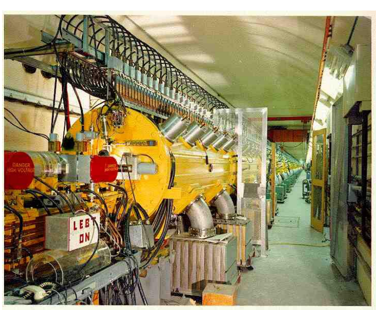
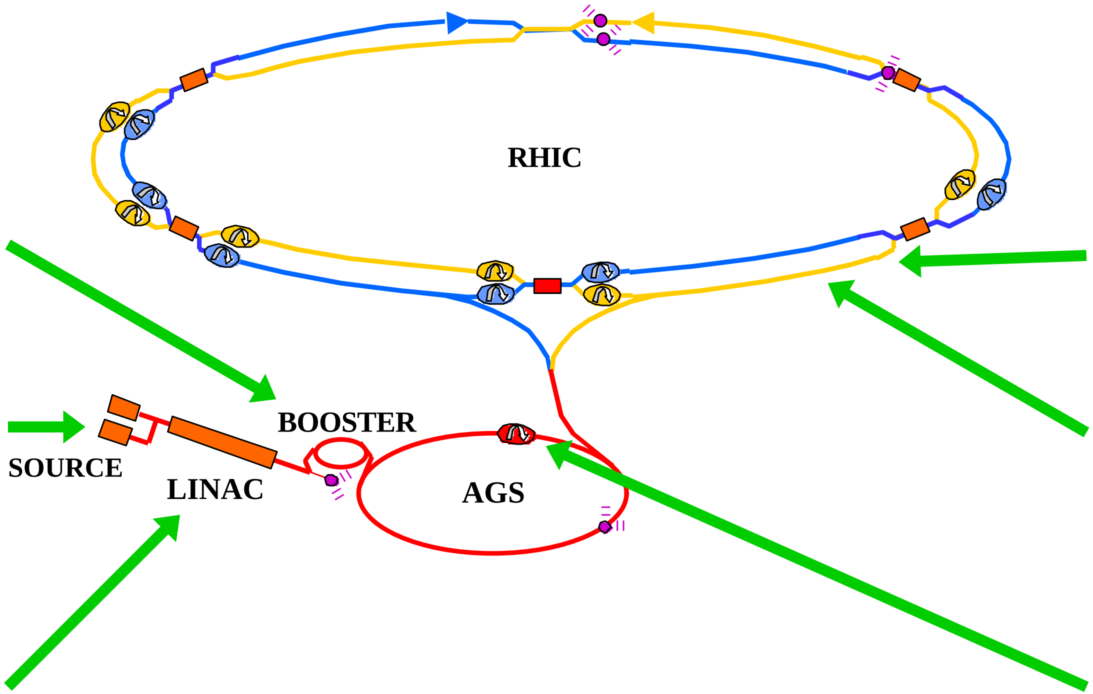
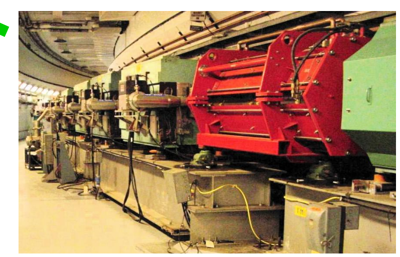
Accelerator Complex (Pol. Protons)









LINAC: Linear Accelerator
AGS: Alternating Gradient Synchrotron
RHIC: Relativistic Heavy Ion Collider

Thomas—Frenkel (BMT) Equation

In the local rest frame of the proton, the spin precession of the proton obeys the Thomas-Frenkel equation:

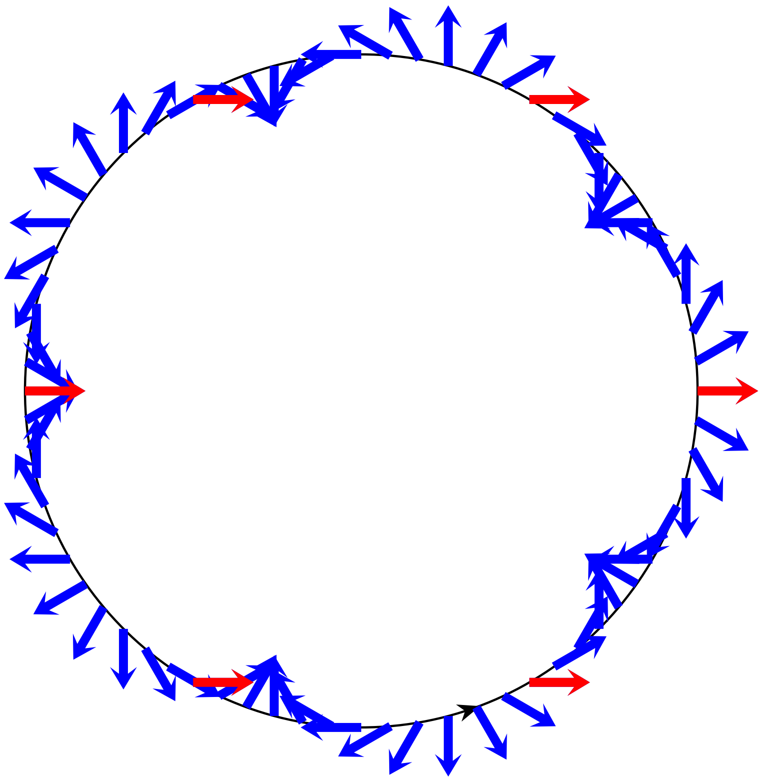
$$\text{Torque}: \quad \frac{d\vec{S}^\diamond}{dt} = \frac{q}{\gamma m}\vec{S}^\diamond \times \left[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel\right] \qquad \text{TF}$$

$$\text{Force}: \quad \frac{d\vec{p}}{dt} = \frac{q}{\gamma m}\vec{p} \times \vec{B}_\perp \qquad \text{Lorentz}$$

(This is a mixed description: t, and $\vec{B}$ in the lab frame, but spin $\vec{S}^\diamond$ in local rest frame of the proton.)

$$G = \frac{g-2}{2} = 1.7928, \qquad \gamma = \frac{\text{Energy}}{mc^2}.$$

Spin Precession in a Ring



Example with 6 precessions of spin in one turn:

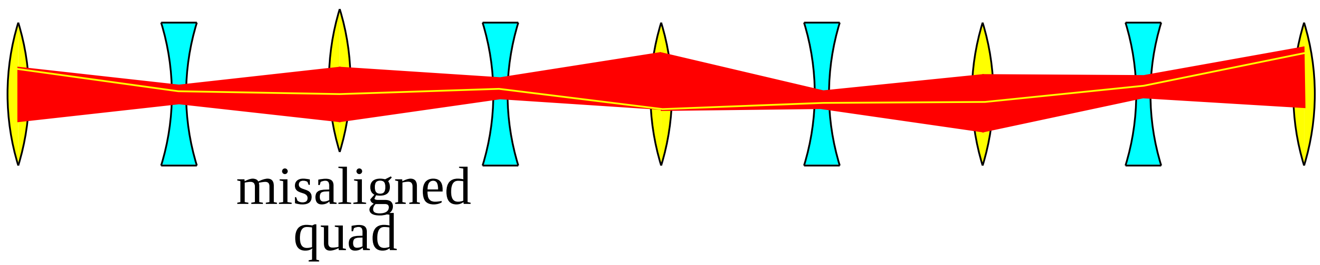
$$G\gamma + 1 = 6.$$

Spin tune: number of precessions per turn relative to beam's direction.
So we subtract one:

$$\nu_{\text{spin}} = G\gamma \quad \propto \quad \text{energy},$$

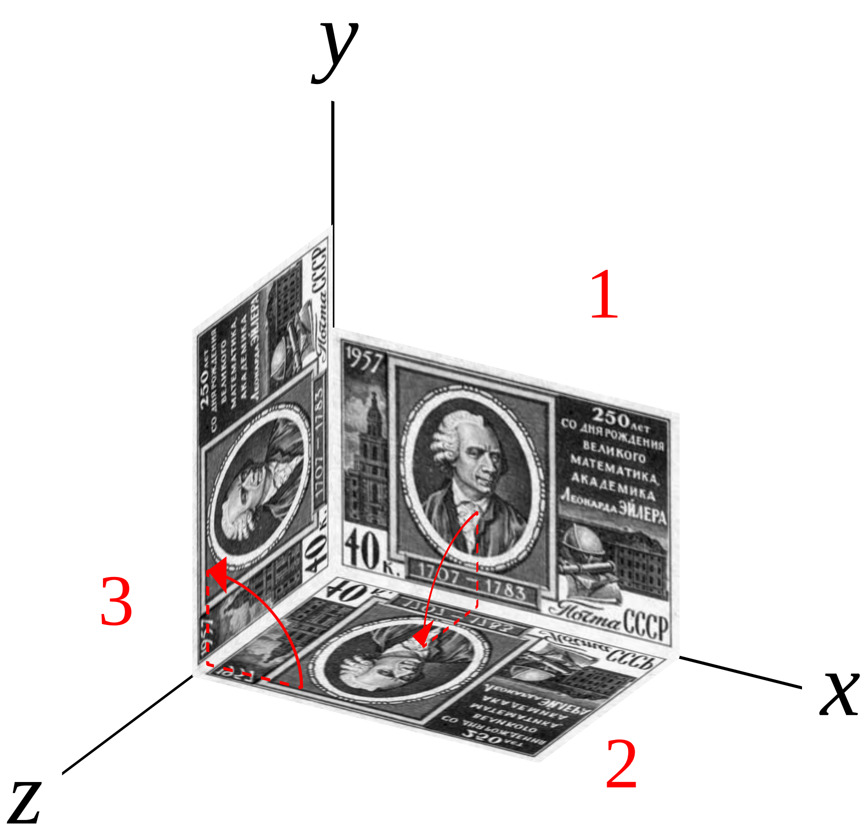
i.e., 5 in this example.

Misalignments or Imperfections

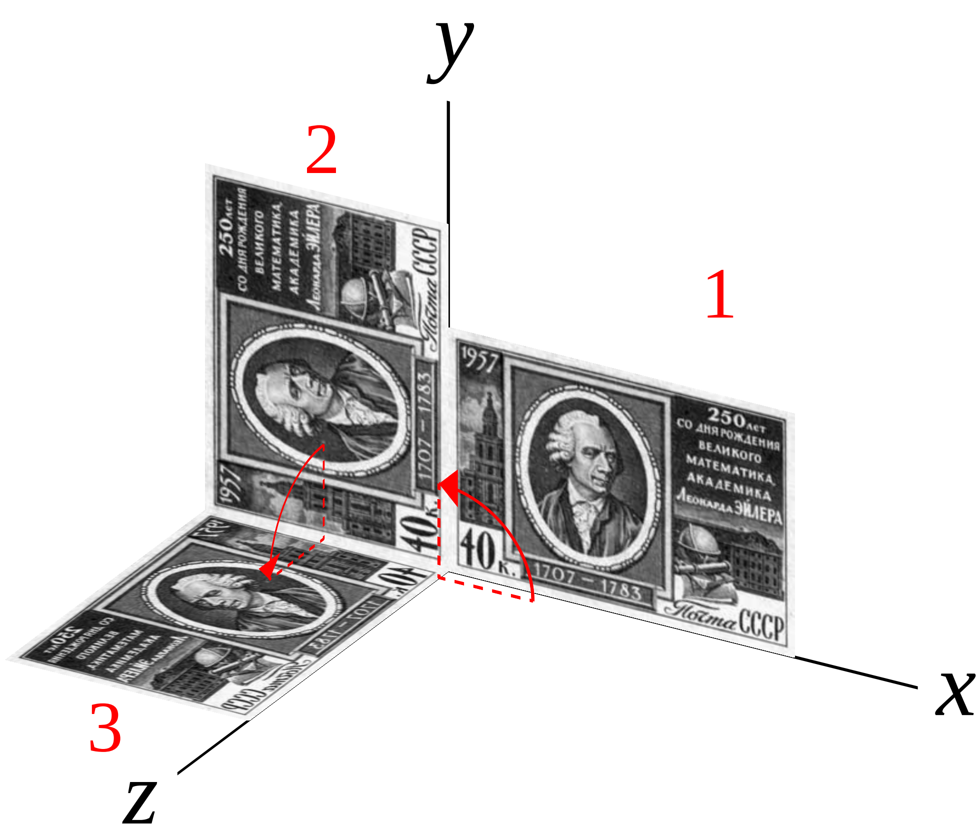


- A misaligned quadrupole creates a steering error which propagates through the lattice.
- For an accelerator ring, this shifts the closed orbit away from the design trajectory.
- If the misalignment is vertical, then the design trajectory will have a periodic set of small vertical bends interspersed with the normal horizontal bends of the bending magnets.
- This leads to an integer resonance condition for the spin tune.

In general, rotations don't commute.



$R_z(90°)\,R_x(-90°)$



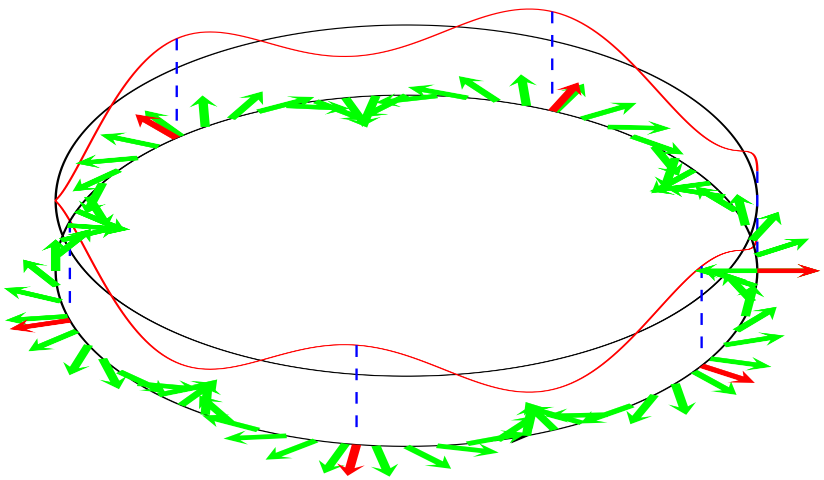
$R_x(-90°)\,R_z(90°)$

Depolarizing Resonances

Simple Resonance Condition:

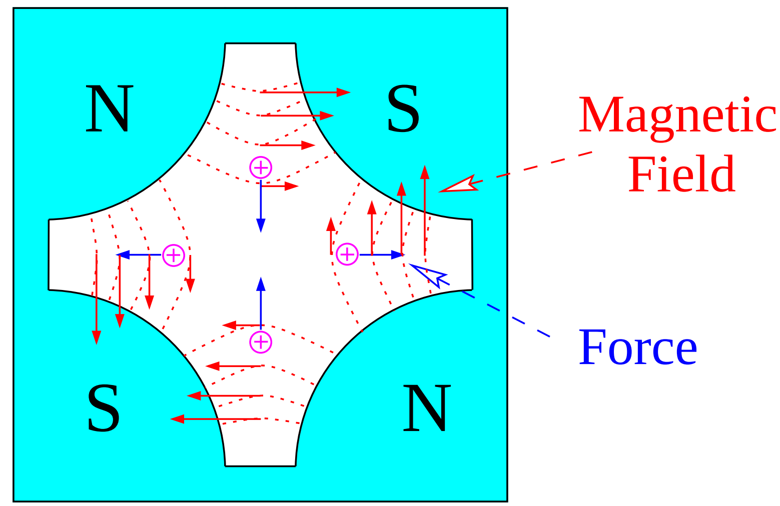
$$\nu_{\text{spin}} = \underset{\text{(imperfection)}}{N} + \underset{\text{(intrinsic)}}{N_{\text{v}} Q_{\text{v}}},$$

where N and N_{v} are integers.



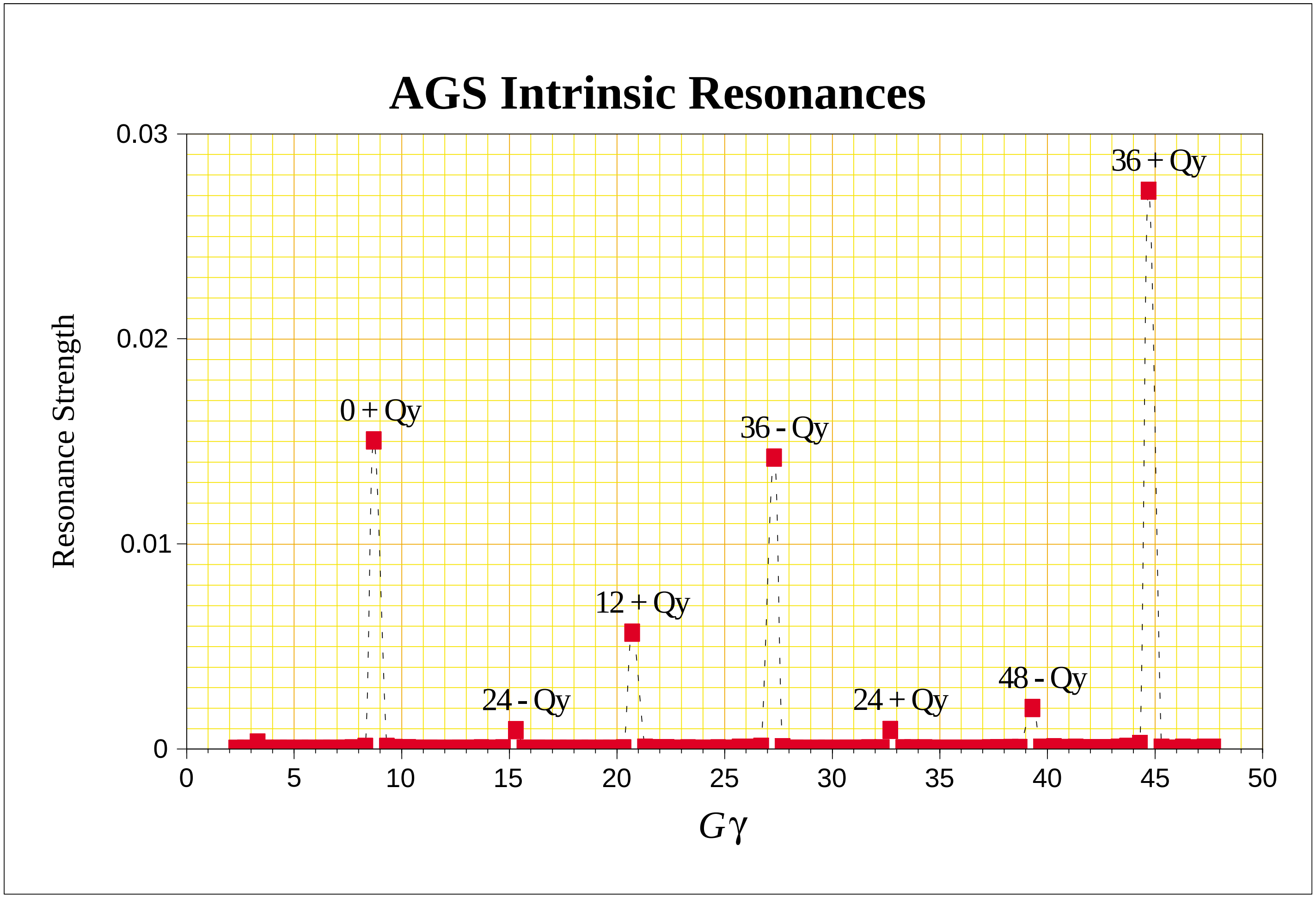
Magnetic Lens (quadrupole)

Vertically focusing

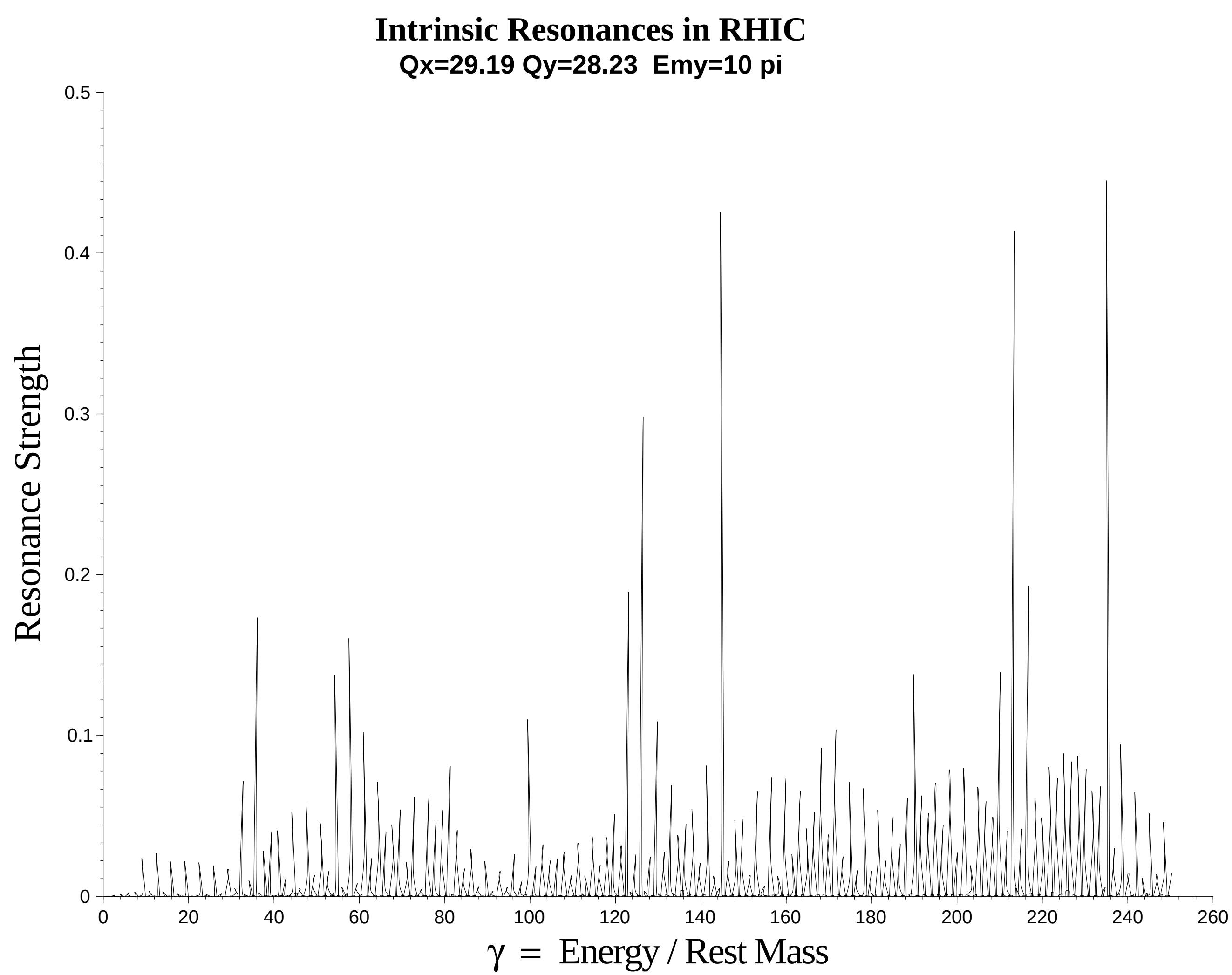


⊕ Protons moving into screen

AGS Intrinsic Resonances



Depolarizing Resonances



Will depolarize beam during acceleration.

Increase in strength with energy.

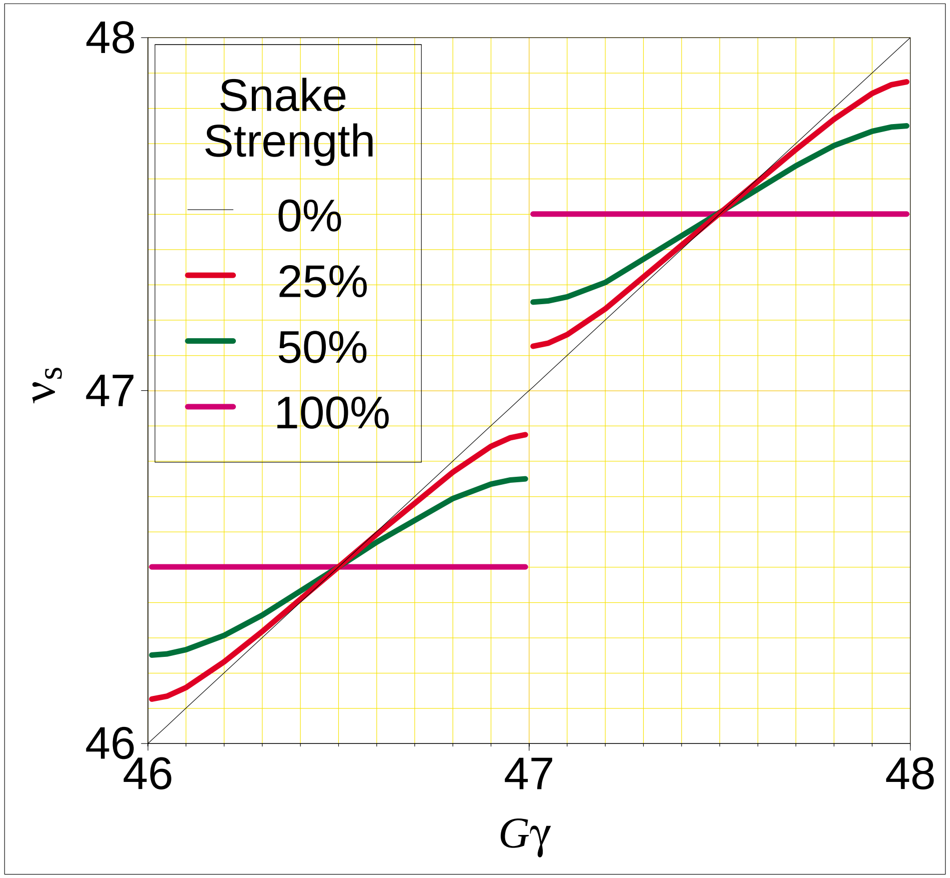
Partial Snakes

Adding a partial snake opens up stop bands around the integer imperfection resonances.

At the snake the stable spin direction points along the snake's rotation axis when $G\gamma =$ integer.

Partial snake strength: $\frac{\mu}{\pi}$

$$\cos \pi \nu_{\rm s} = \cos(G\gamma\pi) \cos \frac{\mu}{2}$$



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Crossing an Isolated Spin Resonance

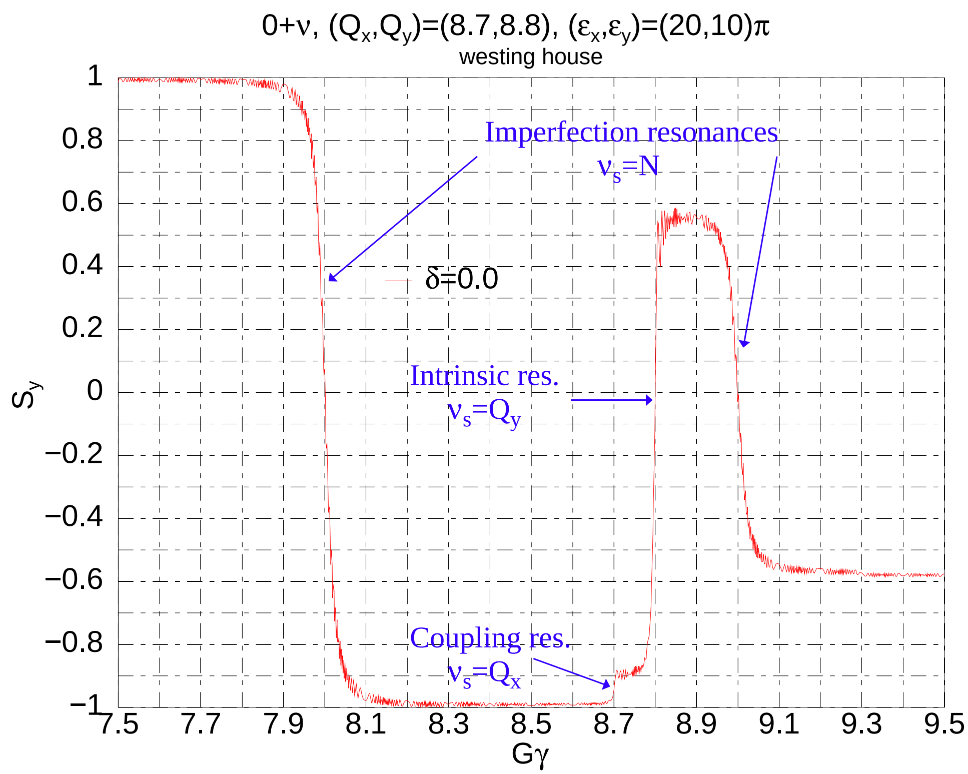
Froissart—Stora Formula:

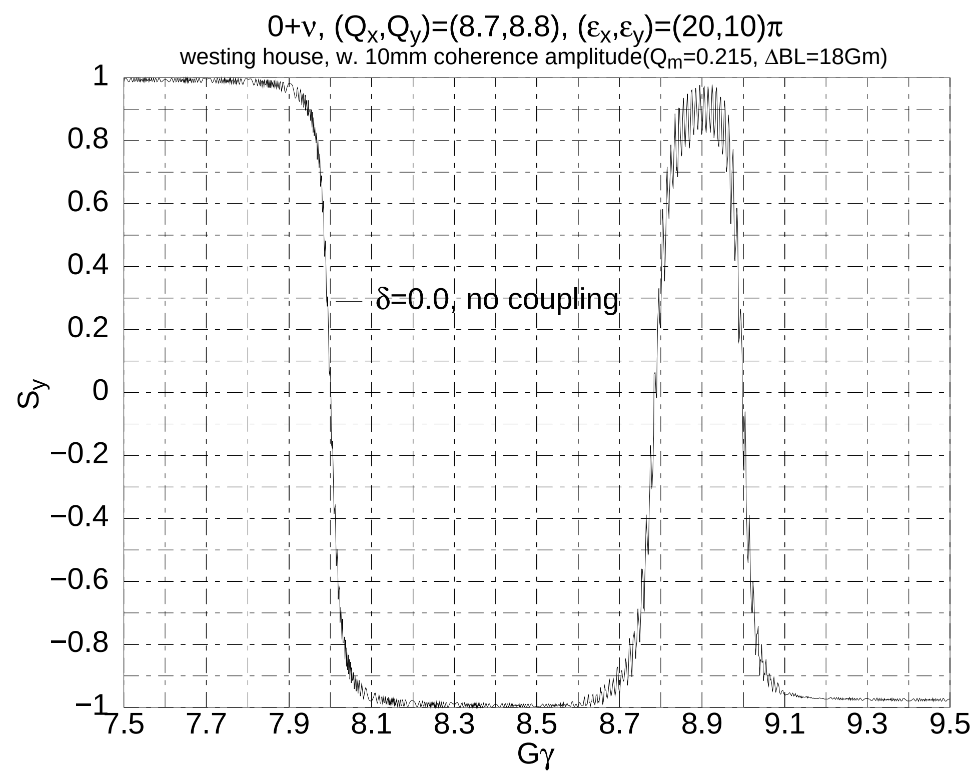
$$\frac{P_\mathrm{f}}{P_\mathrm{i}} = 2\exp\left(-\frac{\pi|\epsilon|^2}{2\alpha}\right) - 1.$$

Ramp rate: $\alpha = \frac{dG\gamma}{d\theta}$, $(\theta :\ 2\pi/\mathrm{turn.})$

Resonance strength: ϵ =Fourier amplitude.

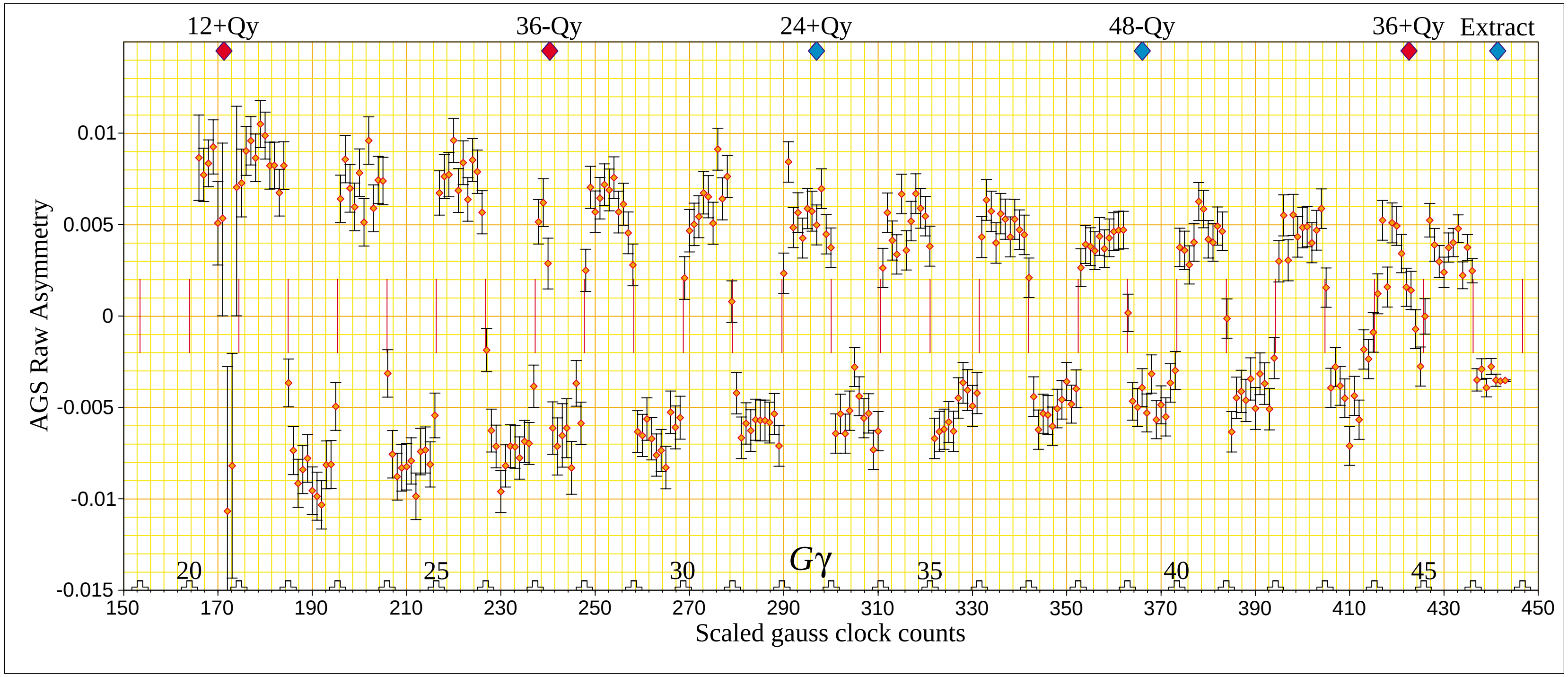
Simulation of Resonance Xing in AGS





AC dipole used to increase strength of $\nu_s = Q_y$ resonance.

AGS Raw Asymmetry during Ramp



AGS has 12 superperiods.
Vertical betatron tune: 8.7
Snake strength: 5%

AC dipole pulses at resonances:

- $0 + Q_y$
- $12 + Q_y$
- $36 - Q_y$
- $36 + Q_y$

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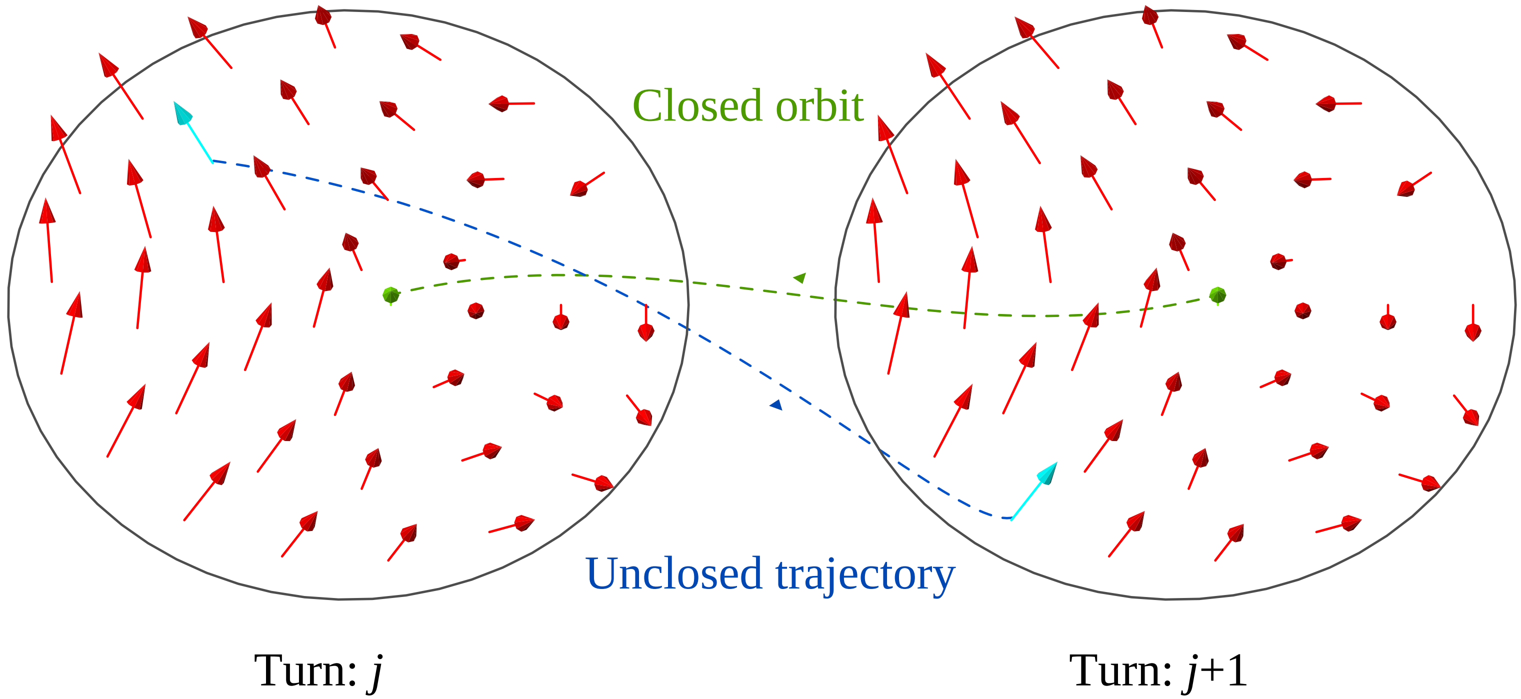
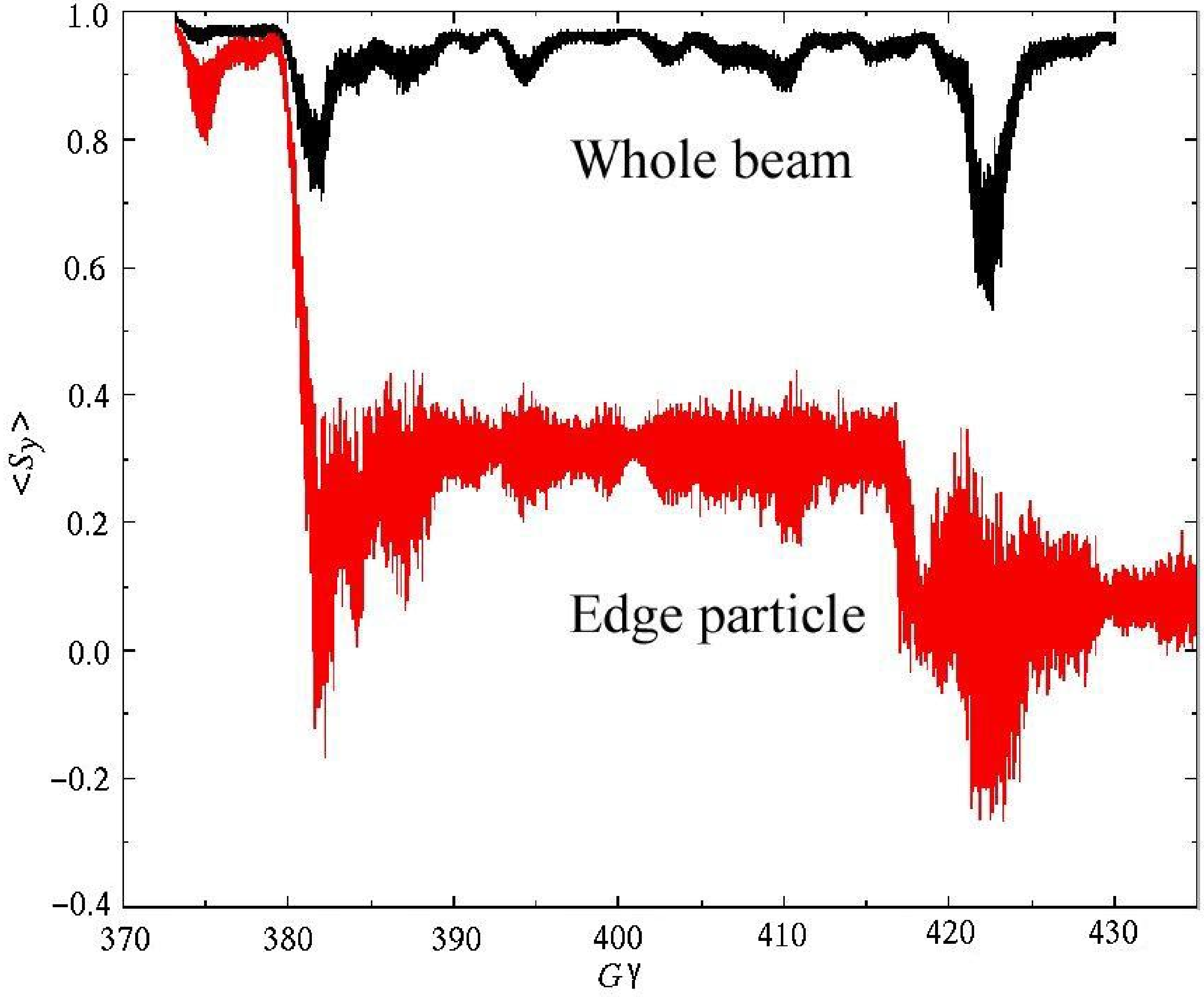
Invariant Spin Field



- For the closed orbit: $\vec{n}_0(s) = \vec{n}_0(s+L)$,
  with $\vec{q}_0(s) = \vec{q}_0(s+L)$ and polarization $\vec{P}_0(s) = \vec{P}_0(s+L)$.
- For other locations in phase space: $\vec{n}(\vec{q},\vec{p},s) = \vec{n}(\vec{q},\vec{p},s+L)$,
  even though in general $q(s+L) \neq q(s)$ and $\vec{P}(s+L) \neq \vec{P}(s)$.

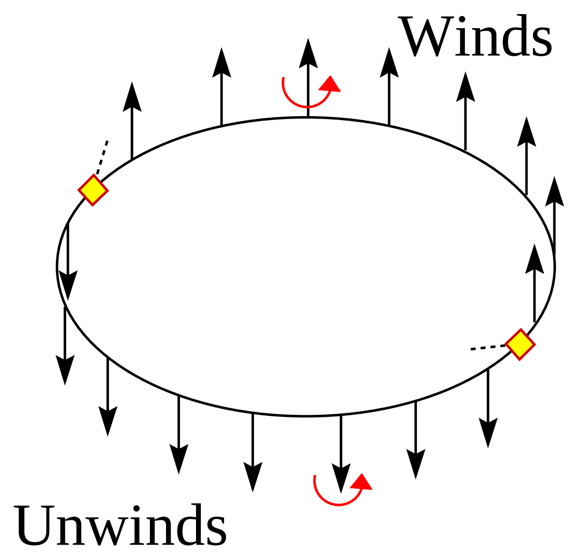
Spin Tracking in RHIC



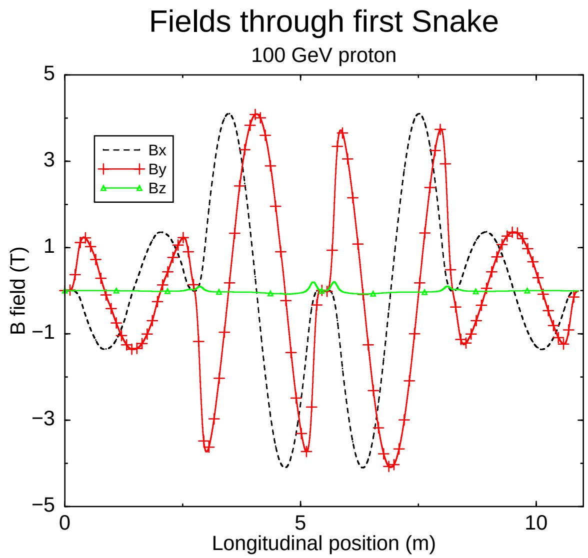
Particles with larger amplitude betatron oscillations may experience more precession away from the stable spin direction of the center of the beam

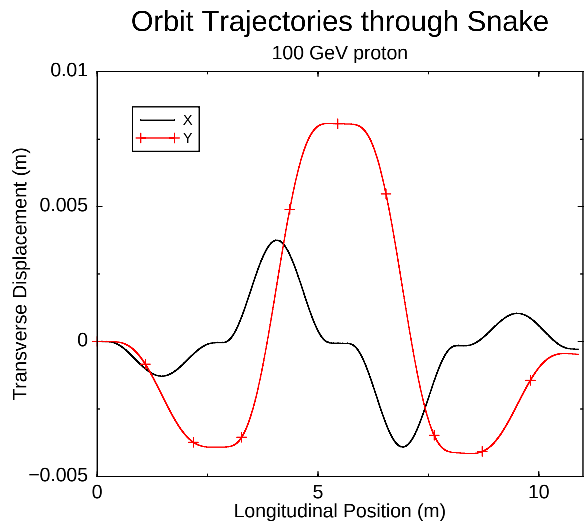
Snake Charming

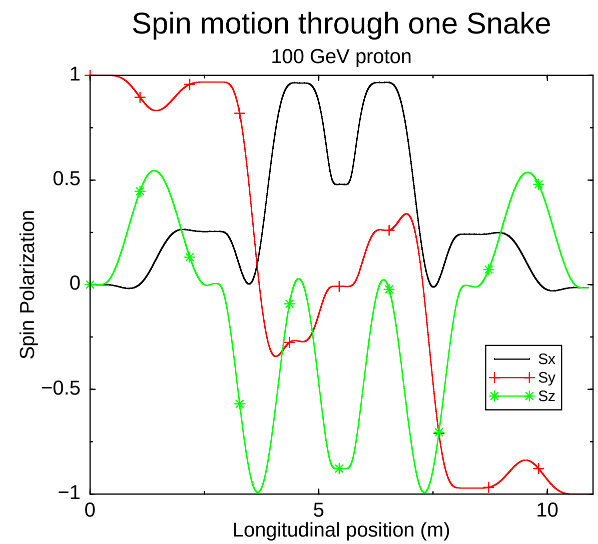
- 2 snakes: spin is up in one half of the ring, and down in the other half.
- Spin tune: $\nu_{\text{spin}} = \frac{1}{2}$
  (It's energy independent.)
- "The unwanted precession which happens to the spin in one half of the ring is unwound in the other half."

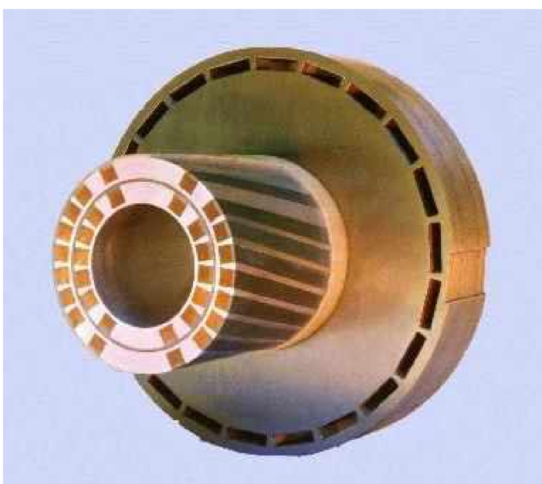


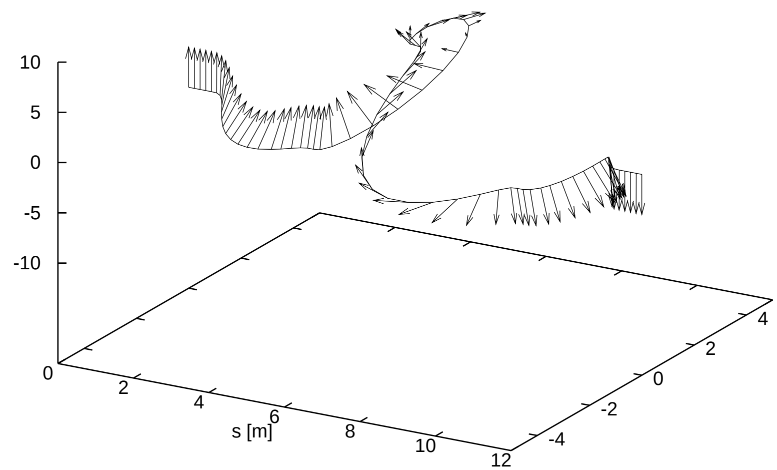
Trajectory and Spin through Snakes



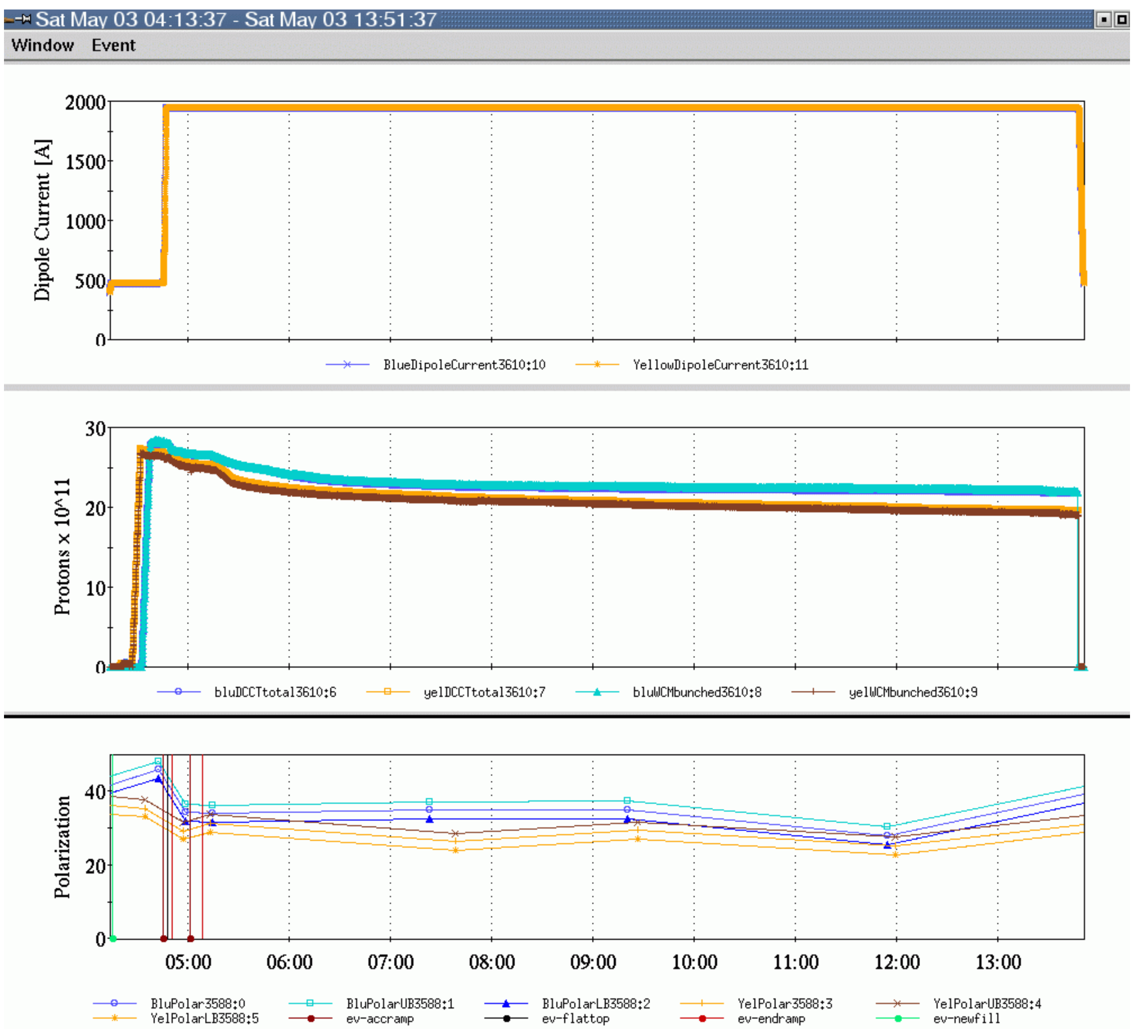


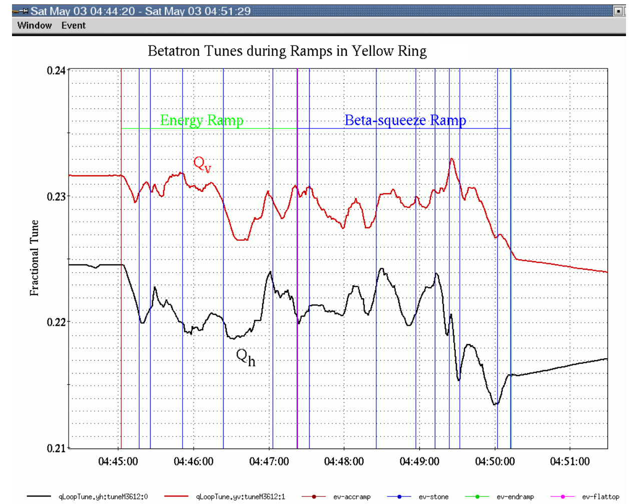


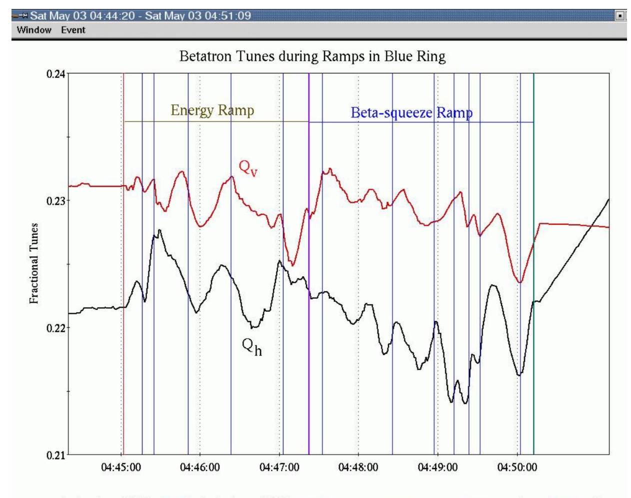




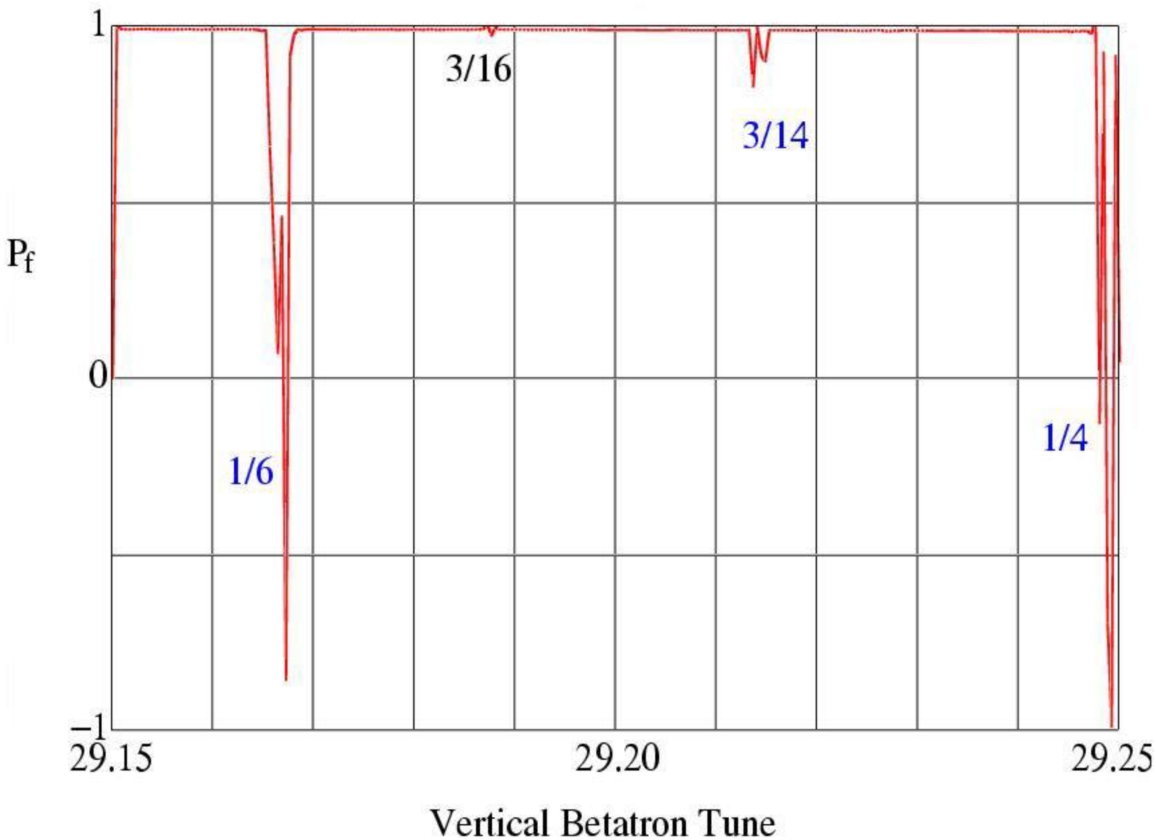
RHIC Beam Polarization



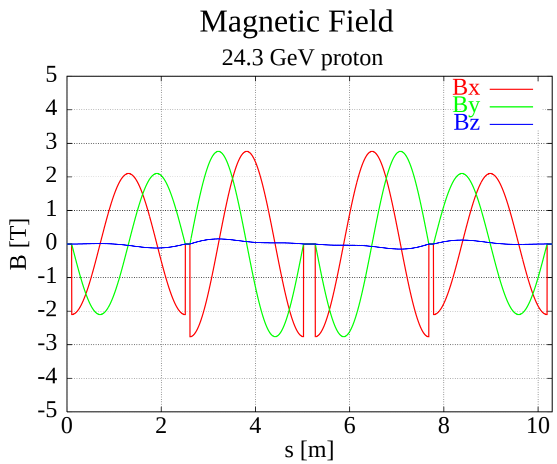


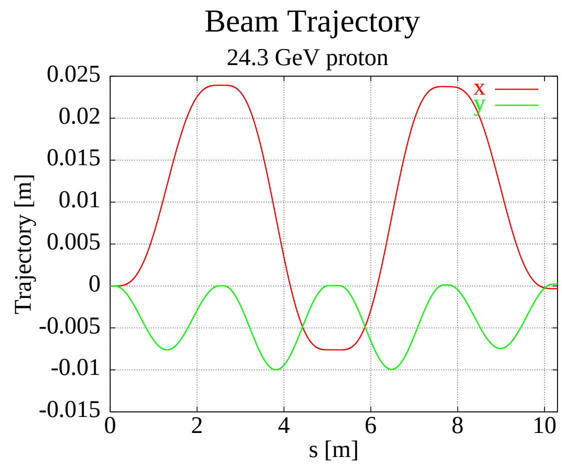


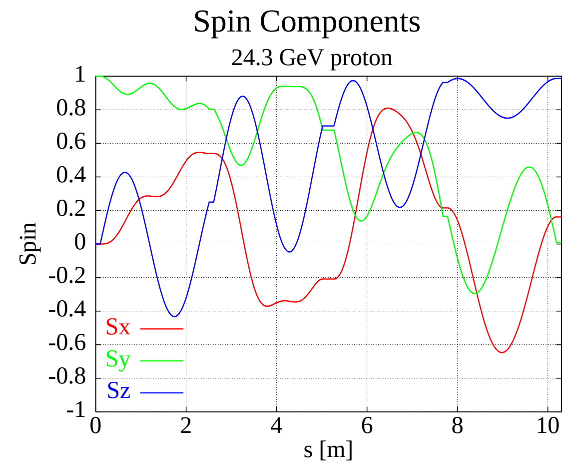
Snake Resonances

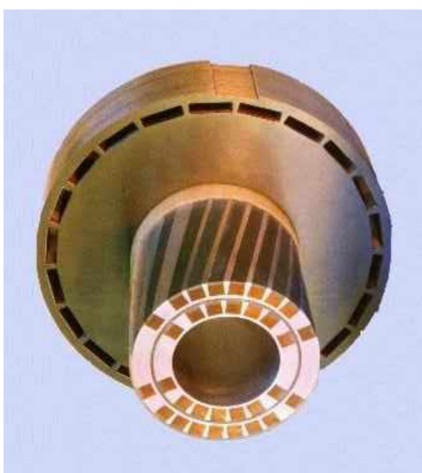


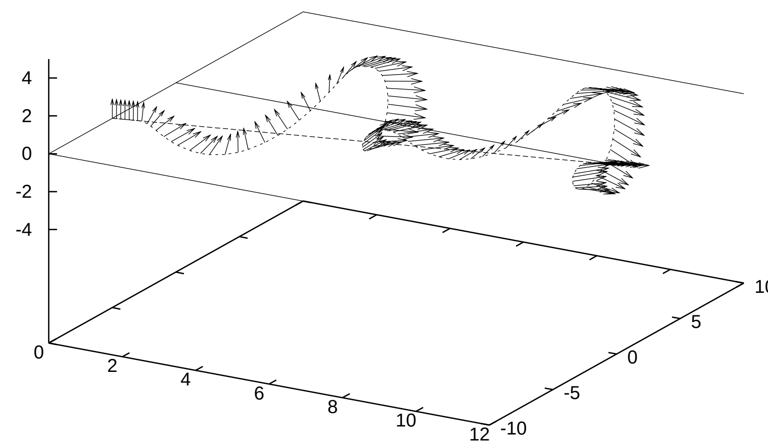
Rotators for Longitudinal Polarization



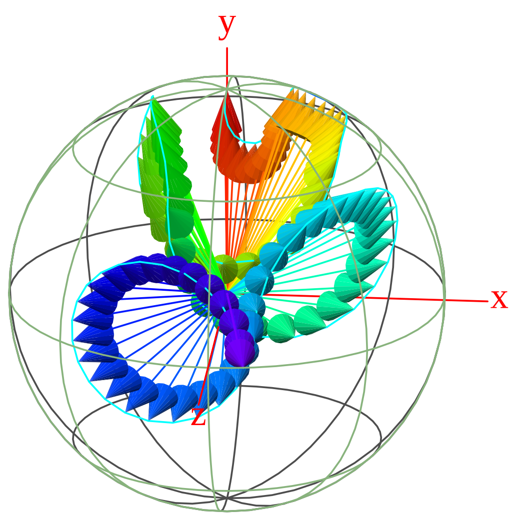


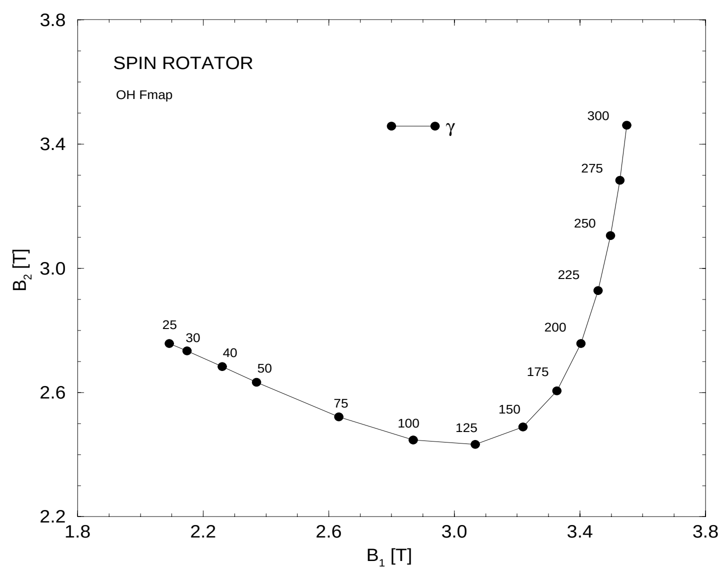


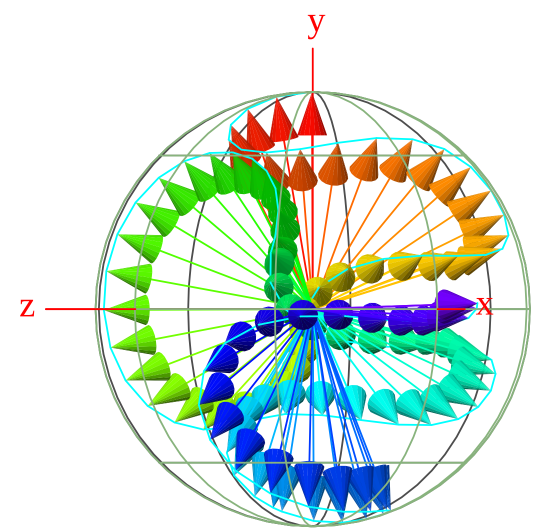




Compensation for D0-DX Bends

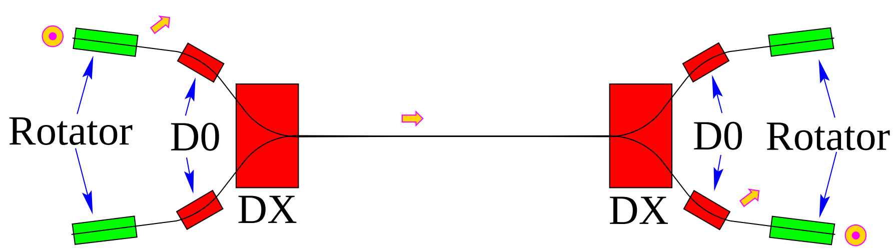




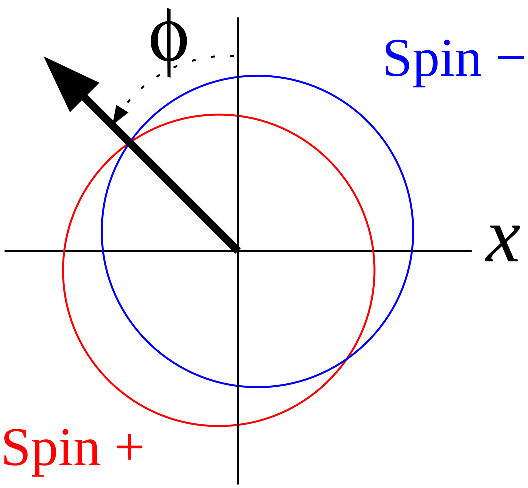


$E = 25$ GeV
D0DX: 10° precession

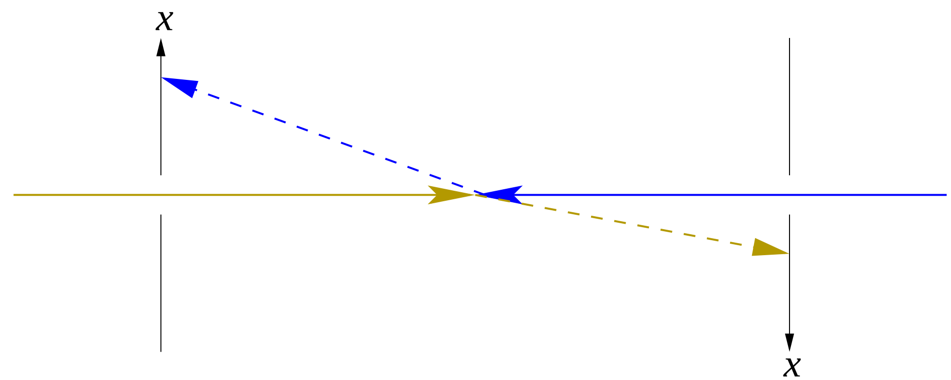
$E = 250$ GeV
D0DX: 100° precession



Orientation of PHENIX Polarimeters



"Left−Right" Asymmetry
(Tilted at 45°)

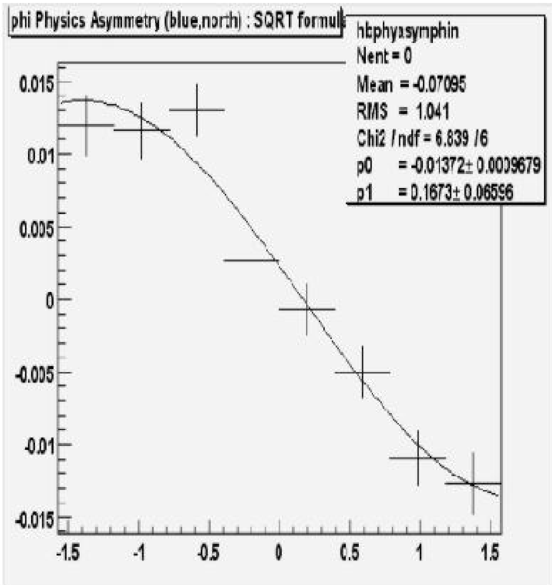


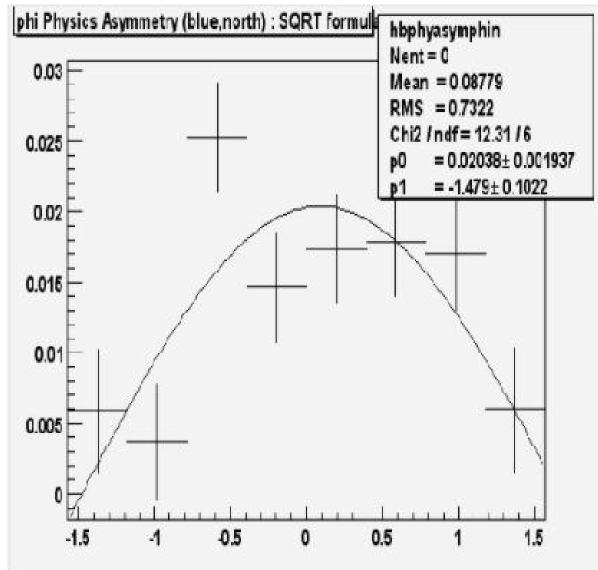
Schematic layout of PHENIX polarimeters
Yellow from left. Blue from right.

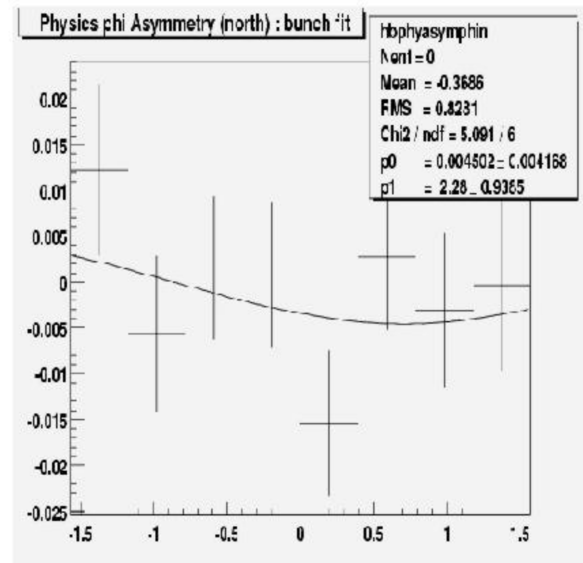
The PHENIX Local Polarimeter measures an asymmetry in small angle scattered neutrons which is proportional to transverse polarization.

$$A_{\mathrm{LR}} = \frac{\sqrt{L^+R^-} - \sqrt{L^-R^+}}{\sqrt{L^+R^-} + \sqrt{L^-R^+}} \propto P_y$$

Tale of the Blue Ring







Vertical polarization with rotators off.

Spin is down.

Rotators on

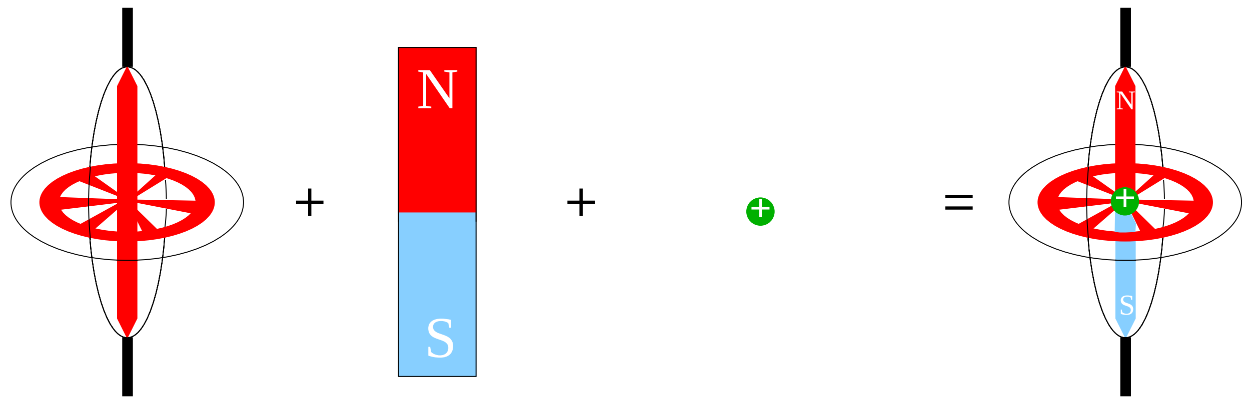
Spin is radially inwards!

OOPS!

Reverse all rotator power supplies and try again.

YES!

Simple Model of Proton

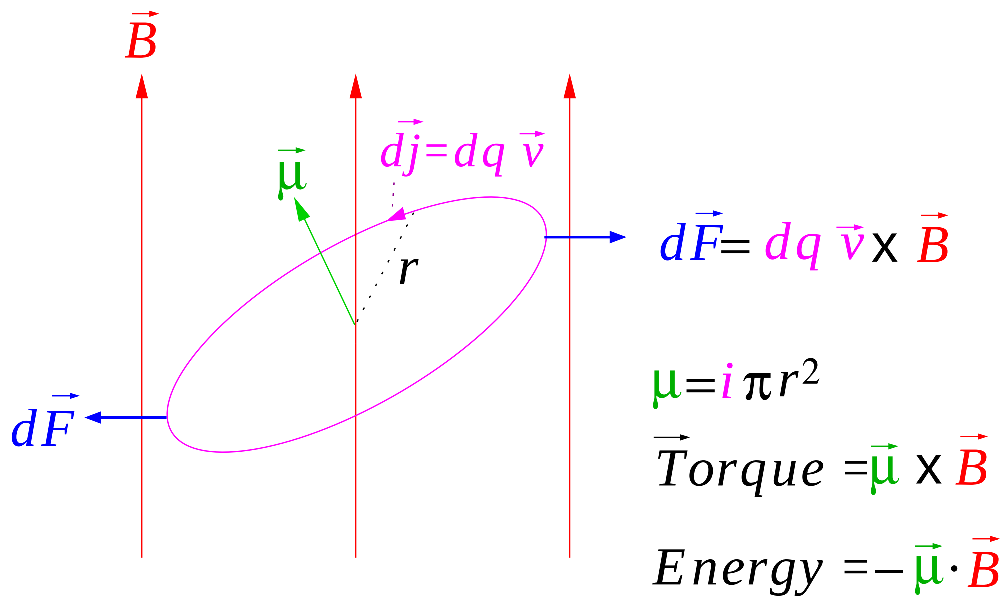


Gyroscope + Bar magnet + Charge = "proton"

Spin — Magnetic Dipole Moment

Polarization: Average spin of the ensemble of protons.

Torque on Classical Magnetic Moment



- Semiclassical model:
  - The spin $\vec{S}$ has a constant magnitude in the rest frame.
  - The magnetic moment $\vec{\mu} \propto \vec{S}$.
    - $\vec{\mu}$ has a constant magnitude in the rest frame.
      (Sort of like a loop of infinite inductance.)

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Thomas—Frenkel (BMT) Equation

In the local rest frame of the proton, the spin precession of the proton obeys the Thomas-Frenkel equation:

$$\frac{d\vec{S}^\diamond}{dt} = \frac{q}{\gamma m}\vec{S}^\diamond \times \left[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\| + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right].$$

This is a mixed description: t, $\vec{B}$, and $\vec{E}$ in the lab frame, but spin $\vec{S}^\diamond$ in local rest frame of the proton.

$$G = \frac{g-2}{2} = 1.7928, \qquad \gamma = \frac{\text{Energy}}{mc^2}.$$

Hamiltonian with Spin

(Here I drop the "$\diamond$" superscript on $\vec{S}$.)

$$
\begin{aligned}
\frac{d\vec{S}}{dt} &= \vec{W} \times \vec{S} \\
H(\vec{q}, \vec{P}, \vec{S}; s) &= \mathcal{H}_{\mathrm{orb}} + \mathcal{H}_{\mathrm{spin}} \\
&= \mathcal{H}_{\mathrm{orb}} + \vec{W} \cdot \vec{S} + O(\hbar^2)
\end{aligned}
$$

Group symmetries:

- Orbital motion without spin: Sp(6,r).
- Spin by itself: $\mathrm{SU}(2, \mathrm{c}) \cong \mathrm{SO}(3, \mathrm{r})$ (homomorphic).
- Full blown symmetry: $\mathrm{Sp}(6, \mathrm{r}) \oplus \mathrm{SU}(2, \mathrm{c})$.
  - Spin dependence on orbit (Thomas-Frenkel).
  - Orbit dependence on spin (Stern-Gerlach Force)—Usually ignored.

Representation of Rotations

SU(2) with usual spinor notation:

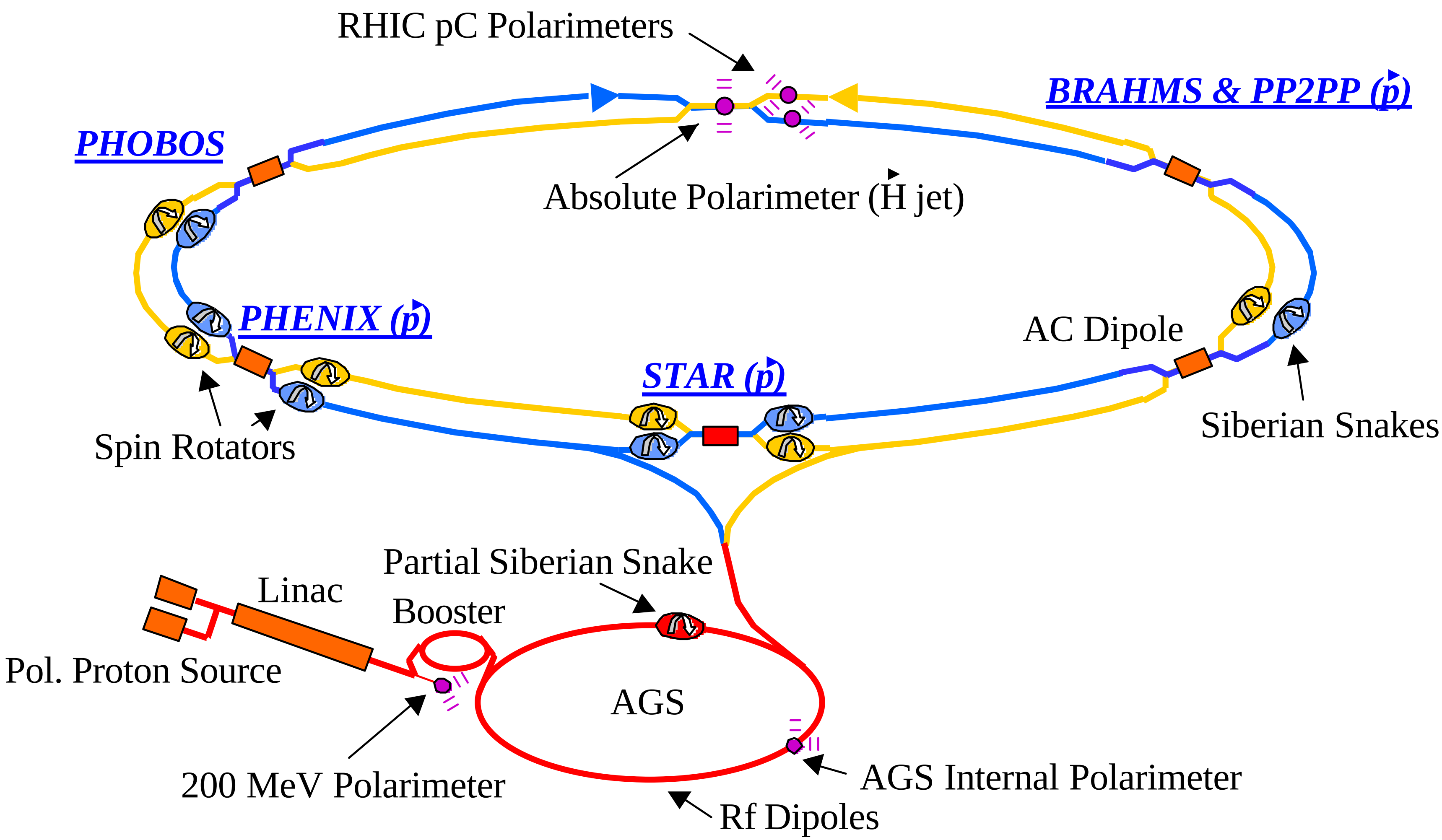
Pauli matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

$$\mathbf{R}_{\hat{n}}(\theta) = e^{i\,\hat{n}\cdot\vec{\sigma}\,\theta/2} = \begin{pmatrix} \cos\frac{\theta}{2} + i n_z \sin\frac{\theta}{2} & (n_y + i n_x)\sin\frac{\theta}{2} \\ (-n_y + i n_x)\sin\frac{\theta}{2} & \cos\frac{\theta}{2} - i n_z \sin\frac{\theta}{2} \end{pmatrix}.$$

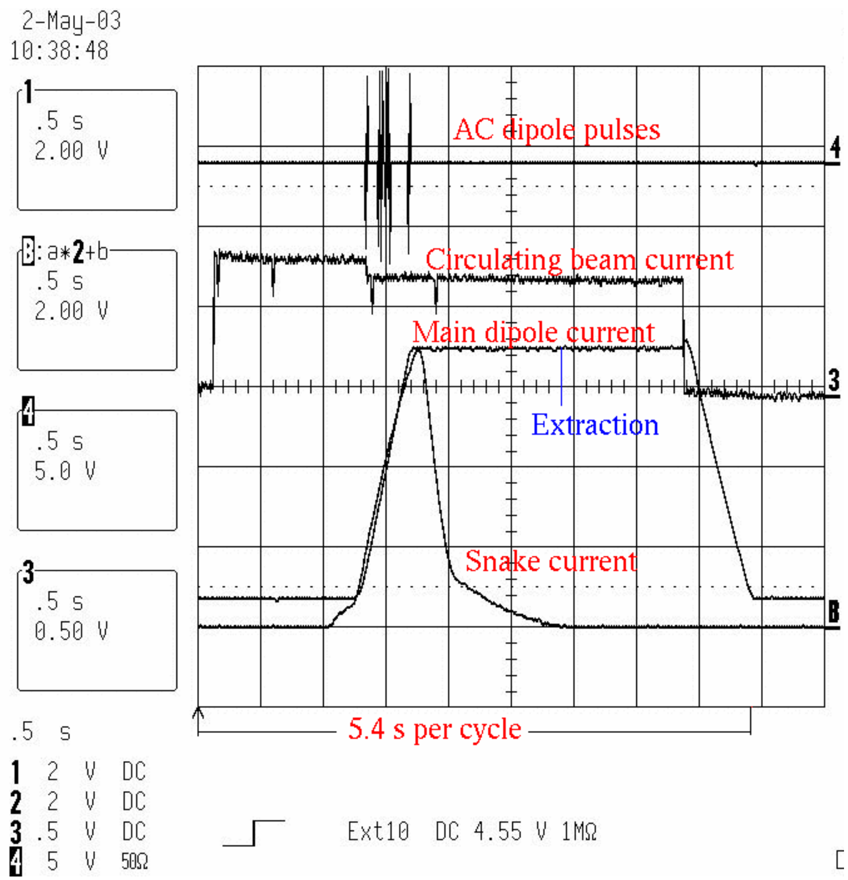
SO(3) :

$$\mathbf{R}_{\hat{n}}(\theta) = \mathbf{I}\cos\theta + \begin{pmatrix} 0 & n_z & -n_y \\ -n_z & 0 & n_x \\ n_y & -n_x & 0 \end{pmatrix}\sin\theta + \begin{pmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_x n_y & n_y^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{pmatrix}(1-\cos\theta).$$

Accelerator Complex for Protons



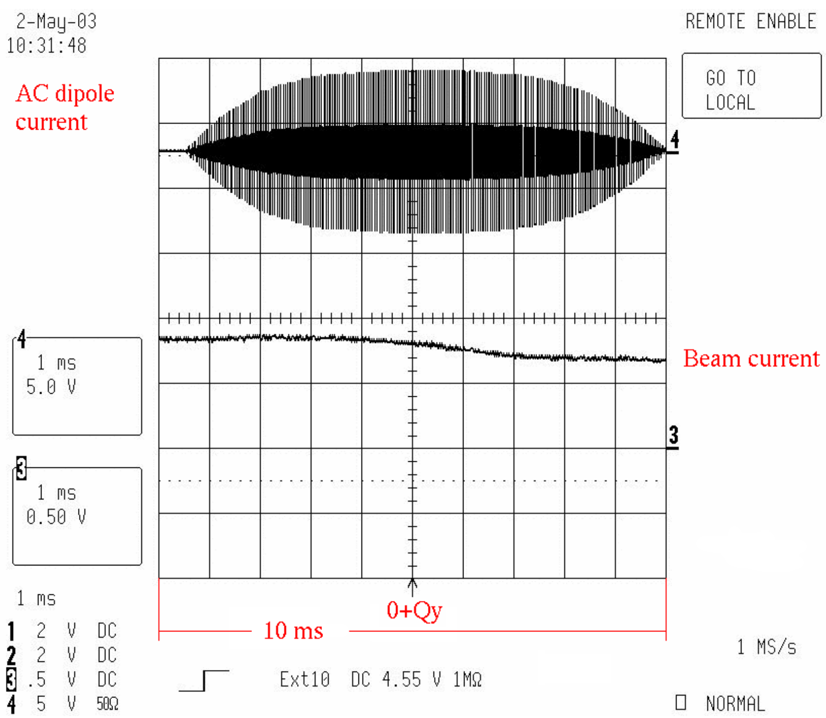
AGS Cycle

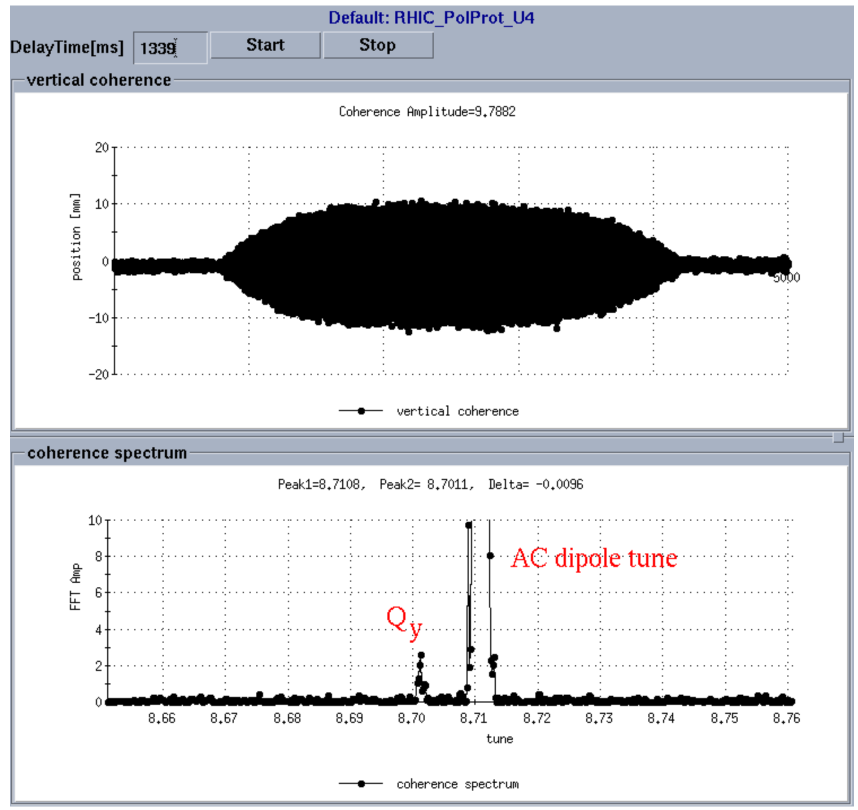


- Snake at 5% up the ramp.
- 4 ac dipole pulses.
  Notice beam loss at first pulse.
- Transition occurs at 1.426ms between the first two ac dipole pulses.

Note: This cycle was not extracted.

AC Dipole pulse at $G\gamma = 0 + Q_y$



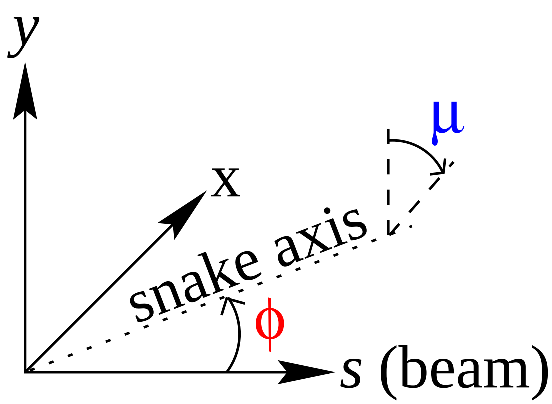


Top: AC dipole pulse amplitude (current)

Bottom: Beam current. (Just scrapes the beam pipe.)

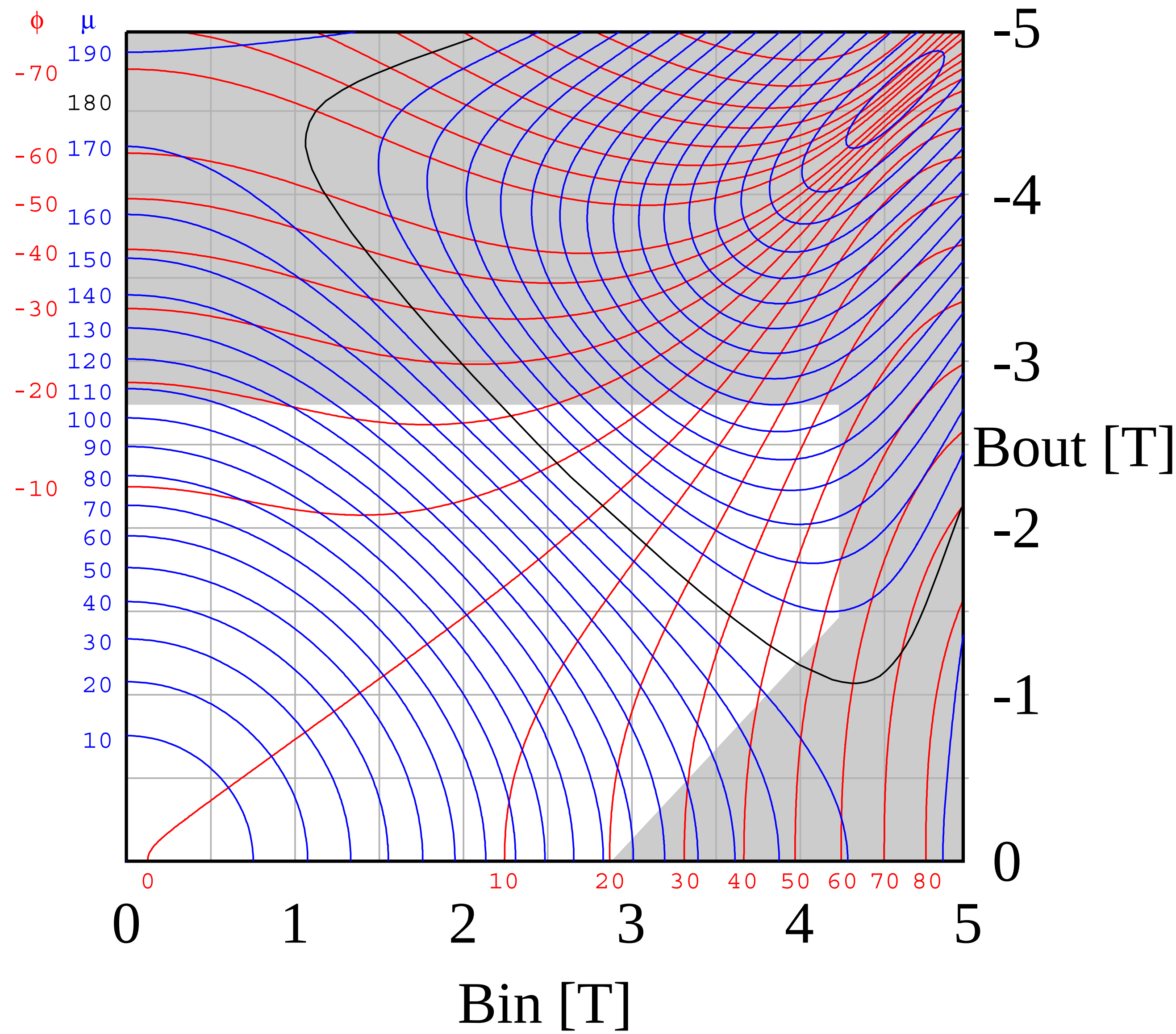
Top: Beam coherence

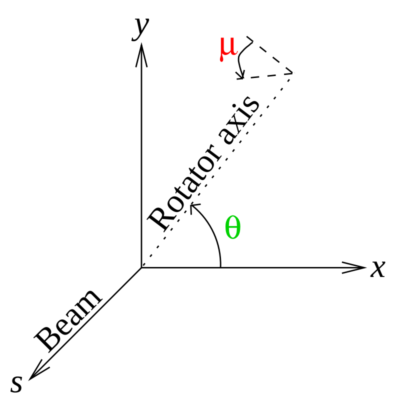
Bottom: Tune spectrum



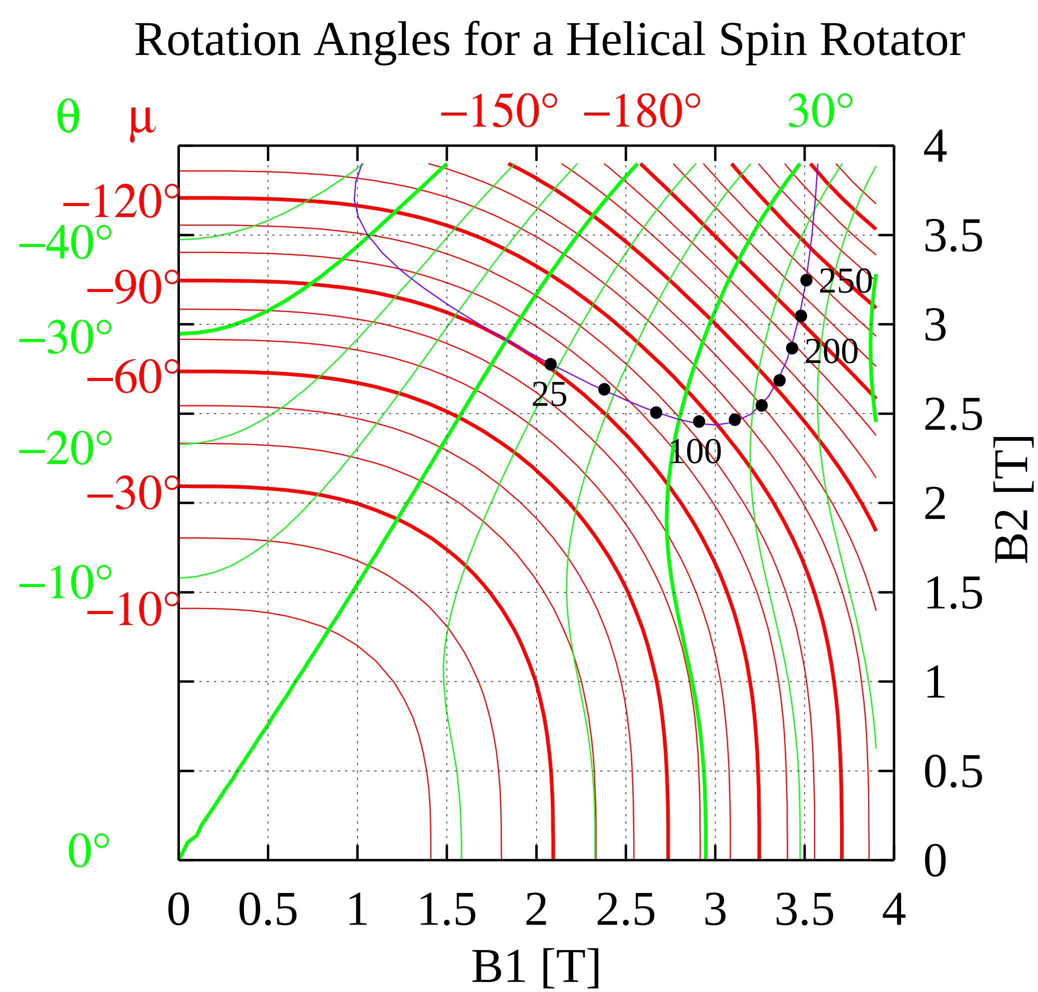
The rotation axis of the snake is ϕ, and μ is the rotation angle.

The shaded region shows approximate restrictions for field limit (right) and injection aperture limits (top and lower right).





The rotation axis of the spin rotator is in the x-y plane at an angle θ from the vertical. The spin is rotated by the angle μ around the rotation axis.



Note: Purple contour for rotation into horizontal plane.
Black dots show settings for RHIC energies in increments of 25 GeV from 25 to 250 GeV.

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