Chapter 7

Collision of Polarized Protons in RHIC

7.1 Polarization Lifetime

In storage mode even a very small depolarizing resonance strength can in principle lead to significant depolarization. This was observed at the ZGS where the effect of high order depolarizing resonances were studied on a 1 second flat top as shown in Fig. 7.1[57]. In an accelerator without Snakes like the ZGS the spin tune is energy dependent ($\nu_{sp} = G\gamma$) and therefore if the resonance condition is within the energy spread of the beam each proton will cross the resonance condition repeatedly and eventually the whole beam can be depolarized. With Snakes, however, the spin tune is energy independent and therefore all

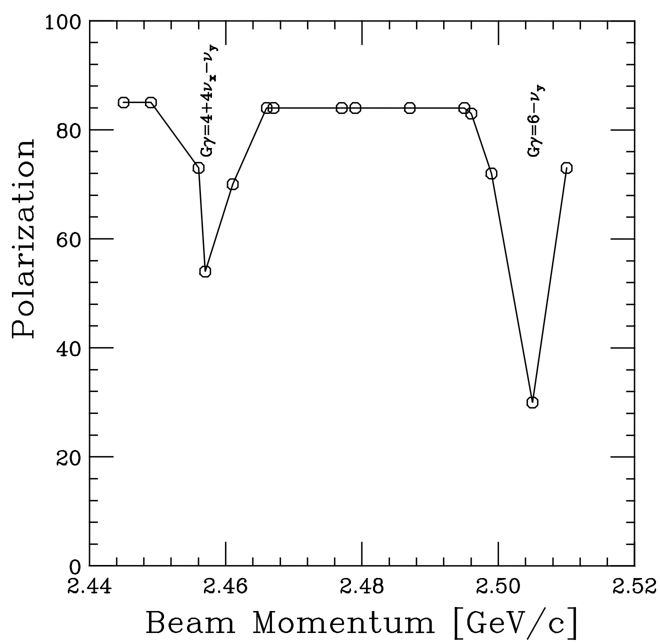

Figure 7.1: Depolarization on the 1 second flat top at the ZGS.

Snake resonance conditions are energy independent. As pointed out earlier and shown in Fig. 5.3 the beam can overlap higher order Snake resonances due to the betatron tune spread. Results of spin tracking calculations for a proton with $\Delta\nu_y = \frac{13}{16}$ and a Snake resonance strength of 0.15 showed that the spin vector is precessing about the vertical axis. [1] At a resonance strength of 0.15, the vertical projection is about 90%. This precession is in fact a spin closed orbit in a broader sense of 16 turns. Spin tracking calculations were calculated over 8×10^9 turns without significant deviations from this spin closed orbit. Only a small fraction of particles are located within the width of the resonance and therefore effective depolarization in the storage mode is small.

When the spin vector is acted on with an adiabatic modulation within the tolerable limit, the spin vector will follow the spin closed orbit adiabatically. Non-adiabatic processes, arising from rf noise at the spin precession frequency, can indeed cause beam depolarization. Let us consider that a single dipole with strength θ_k is modulated at $\nu_{sp} f_0$, which is about 39 kHz for RHIC. The corresponding induced spin precessing kick is $G\gamma\theta_k$. The number of turns that occur before the spin is perturbed to 80% of the original polarization is given by,

$$N_p = \frac{\cos^{-1}(0.8)}{G\gamma\theta_k}$$

Let us now consider the same angular kick to the orbital motion. If there is an rf source at $\nu_{sp} f_0$, one expects a similar angular kick at the frequency $q f_0$, where q is the fractional part of the betatron tune. The orbital survival turn number is given by

$$N_o = \frac{A}{\langle\beta\rangle\theta_k}$$

where $\langle\beta\rangle$ is the average betatron amplitude, and A is the dynamic aperture. Using $A = 0.01$ m, $\langle\beta\rangle = 20$ m for RHIC, we find that the orbital lifetime is only half as long as the polarization lifetime.

Indeed, any rf source at high frequencies around the synchrotron and betatron tunes, are dangerous to the orbital stability of particles in accelerators. Similarly, any rf source at the spin tune can cause beam depolarization. These high frequency rf sources should be addressed carefully in hardware design.

7.2 Beam-Beam Interactions

For round, Gaussian beams with rms beam size σ, the force experienced by a particle in one bunch due to its passage through an on-coming bunch is given by

$$F(r,s) = \frac{\lambda(s)\, e^2}{2\pi\epsilon_0}\left(1 + \frac{v^2}{c^2}\right)\frac{1 - e^{-r^2/2\sigma^2}}{r} \tag{7.1}$$

$$\approx \frac{\lambda(s)\, e^2}{2\pi\epsilon_0\sigma^2}\, r \tag{7.2}$$

[1] In 1995, the RHIC operating point was changed. Since the spin dynamics is symmetric about the half-integer tune, many results found in this report are for the old tune values.

where r is the transverse radial distance from the center of the bunch, $\lambda(s)$ is the local longitudinal particle density (particles per meter), and e is the proton charge. The last expression is for $r < \sigma$. The beam-beam interaction, for most of the protons in the center of the bunch, has a defocusing effect. Considering only one-dimensional (vertical, say) motion, a proton's trajectory would be altered by the other proton bunch by an amount

$$\begin{aligned}
\Delta y' &= \frac{\int (\partial F/\partial y) y \, ds}{pv} && (7.3)\\
&= \frac{e^2}{4\pi\epsilon_0 mc^2} \frac{2\int \lambda(s) ds}{\gamma(\sigma^*)^2} \cdot y && (7.4)\\
&= \frac{r_0 n}{\gamma(\sigma^*)^2} \cdot y && (7.5)\\
&= \frac{6\pi r_0 n}{\beta^* \epsilon_N} \cdot y && (7.6)\\
&\equiv \frac{1}{f^*} \cdot y && (7.7)
\end{aligned}$$

where n is the number of protons per bunch, r_0 is the classical radius of the proton, and σ^* is the rms beam size at the interaction point. The equivalent focal length, f^*, leads to the familiar beam-beam tune shift

$$\xi \equiv \frac{1}{4\pi} \frac{\beta^*}{f^*} = \frac{3 r_0 n}{2\epsilon_N}. \tag{7.8}$$

The beam-beam interaction will thus precess the spin of a typical (rms-amplitude) particle by an amount

$$\Delta\phi \approx G\gamma \, \Delta y' \approx \frac{G r_0 n}{\sigma^*} = G r_0 n \sqrt{\frac{6\pi\gamma}{\beta^* \epsilon_N}} \tag{7.9}$$

which, for 2×10^{11} particles per bunch, 20π mm mrad emittance, and $\beta^* = 1$ m, corresponds to $\Delta\phi =$ 5 mrad. This precession will contribute to the intrinsic depolarizing resonance strength at storage. Since the resonance strength is given by

$$\epsilon_k = \frac{1 + G\gamma}{2\pi} \oint \sqrt{\frac{\beta(s)\epsilon_N}{6\pi\gamma}} \frac{\partial B_x(s)/\partial y}{B\rho} e^{ik\theta(s)} ds \tag{7.10}$$

we can compare $\sqrt{\beta^*}/f^*$ with the value $\sqrt{\beta_{max}}/F$ for a standard RHIC arc quadrupole to check the magnitude of the effect. We find that the contribution to the intrinsic resonance strength of the beam-beam interaction is roughly 14% of the contribution from a single RHIC arc quad for the parameters above.

It is interesting to note that the beam-beam tune shift for the beam parameters used above is $\xi = 0.0073$ per crossing, or 0.015 for two IRs.

Analytical and numerical studies have been performed which indicate that the effect on spin of the beam-beam interaction is indeed small for RHIC.[58],[59]

7.3 Luminosity of Polarized Proton Collisions

The luminosity of RHIC is given by

$$\mathcal{L} = \frac{3 f_0 B N_B^2 \gamma}{2 \epsilon_N \beta^*}$$

where f_0 is the revolution frequency, B is the number of bunches in each ring, N_B is the number of particles per bunch, ϵ_N is the normalized emittance, β^* is the amplitude function at the interaction point, and $\gamma = E/mc^2$. For a β^* of 2 m, 20 π mm mrad emittance, 60 bunches, and 10^{11} protons per bunch (nominal RHIC proton parameters), the luminosity at 250 GeV is 1.5×10^{31} cm^{-2} s^{-1}. The above expression suggests a linear dependence of luminosity with beam energy. However, as the energy of the beam is decreased, the beam size in the interaction region triplet will increase. Hence, once a certain energy is reached (approximately 70 GeV in RHIC), the amplitude function at the interaction point must be optimized to maintain a maximum beam size in the low-beta triplet quadrupoles. With this scenario in mind, the luminosity at lower energies decreases more rapidly than linear.

The RHIC beam intensity can likely be raised to 2×10^{11} protons per bunch, and the interaction region optics is capable of generating a β^* of 1 m. In addition, the number of bunches can also be easily increased by a factor of two to 120. Thus the potential exists to produce a polarized proton luminosity at 250 GeV of 2×10^{32} cm^{-2} s^{-1}, which corresponds to about one interaction per crossing of the 120 bunches per ring. Even higher luminosity might possibly be achieved with lower emittance, or with higher beam current. The luminosity for polarized protons versus. energy is shown in Fig. 7.2. For this figure, we assume an emittance of 20 π mm mr emittance, 2×10^{11} protons per bunch, and β^* at high energy of 1 m. At 25 GeV, $\beta^* = 2.8$ m in order to maintain the required beam size in the triplet quadrupoles.

In the above analysis, the beam parameters were below the beam-beam tune shift limit of $\xi = 0.024$ typically observed in other hadron colliders (Tevatron, Sp$\bar{\text{p}}$S). If the beam emittance were improved to 10π mm mrad, for example, along with the "upgraded" parameters used above, the collider would become beam-beam limited. Fig. 7.3 shows the luminosity of the polarized proton collider at 250 GeV and for various emittances as a function of bunch intensity, assuming 120 bunches per ring, and $\beta^* = 1$ m. As the intensity is increased, the collider reaches the tune shift limit, and then the beam emittance would be increased in proportion to the bunch intensity to maintain this tune shift. The luminosity, at that point, would increase linearly with bunch intensity rather than quadratically.

7.4 Spin Reversal of Stored Beams

Since the proposed asymmetry measurements are high precision measurements, frequent polarization sign reversal is imperative to avoid systematic errors. Possible sources for systematic errors are luminosity variations, crossing angle variations, and detector efficiency variations. As mentioned earlier different bunches will have different polarization signs and therefore different bunch crossings will measure interactions with different combinations of incoming beam polarization signs. Although this will greatly reduce systematic

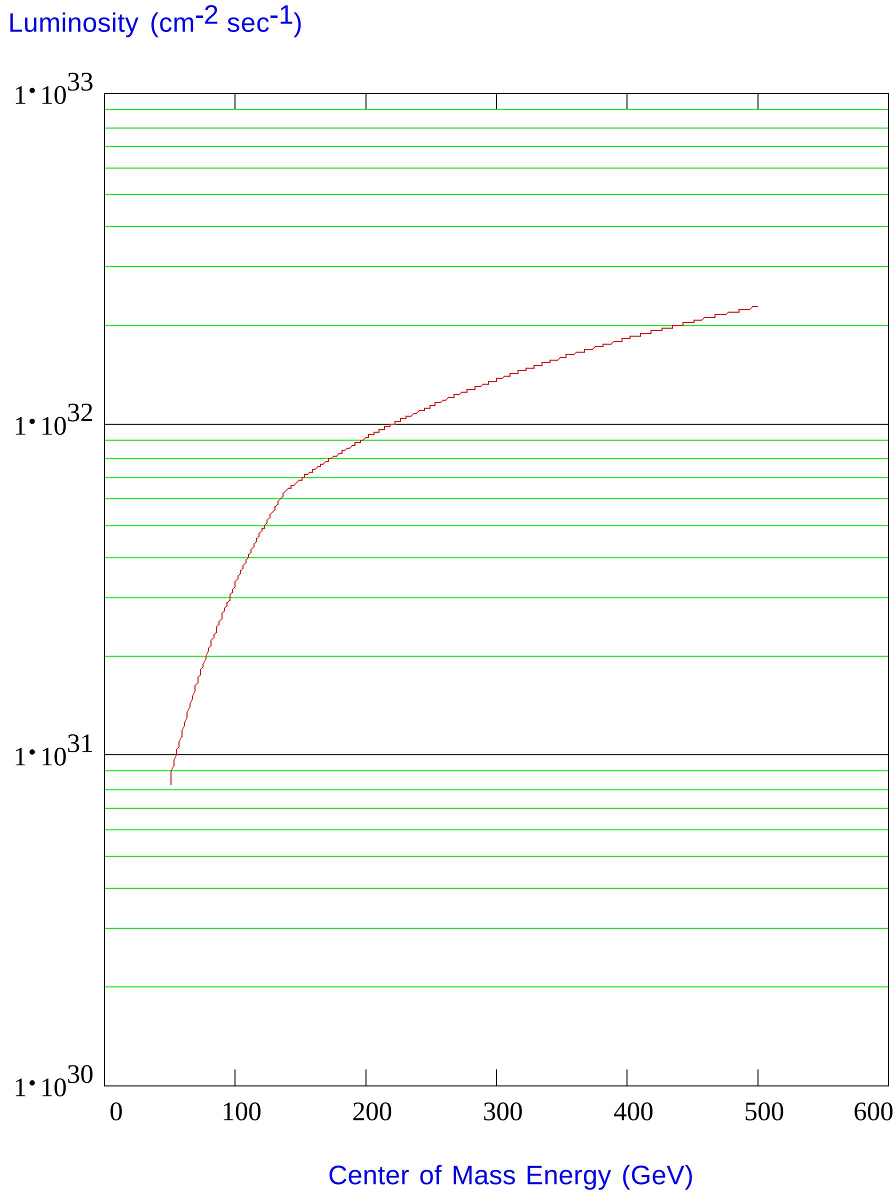

Figure 7.2: Polarized proton luminosity as a function of energy in RHIC. A normalized emittance of 20π mm-mr (95%), 120 bunches per ring, and a beam intensity of 2×10^{11} protons per bunch were used. A value of $\beta^* = 1$ m from 74 GeV to 250 GeV was assumed; at 25 GeV, β^* is 2.8 m.

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errors it is still true that one pair of bunches would always cross with the same combination of polarization signs during the whole lifetime of the stored beams which is at least several hours. To eliminate the possibility of systematic errors from this situation we propose to install a spin flipper in each ring which is capable of reversing the polarization sign of all bunches. The spin flippers consist of vertical dipole magnets[60] excited with about 40 kHz AC current. This would drive an artificial spin resonance which can be used to adiabatically reverse the polarization direction. Fig. 7.4 shows the result of a test of this

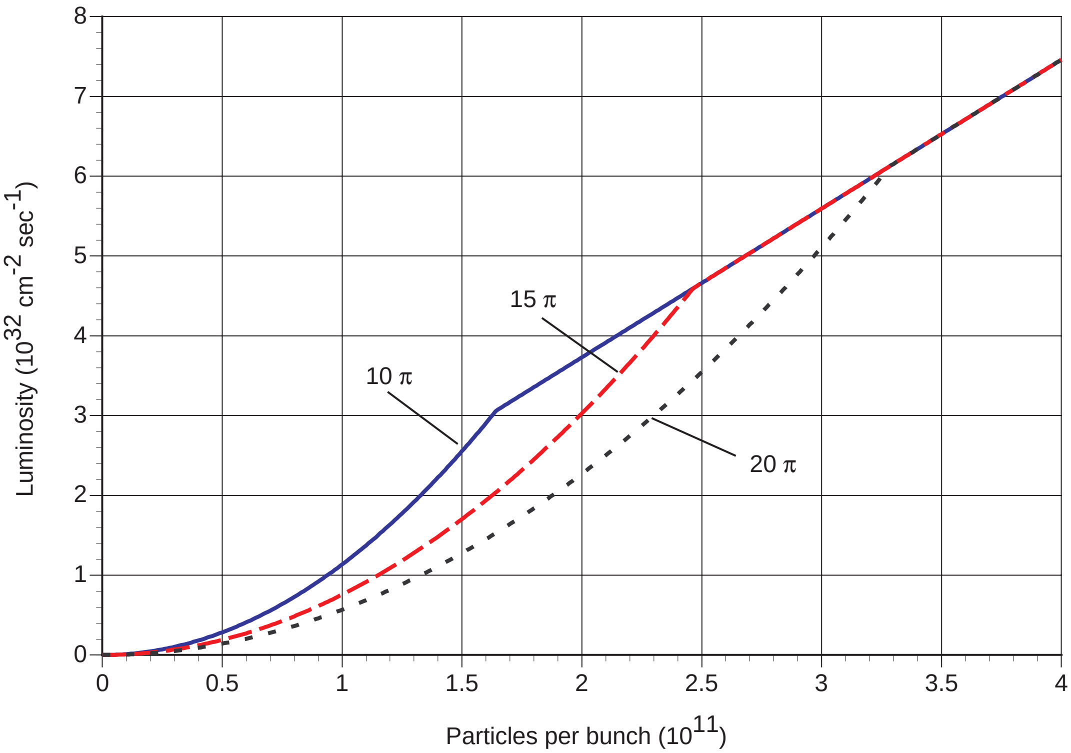

Figure 7.3: Polarized proton luminosity as a function of bunch intensity for normalized emittances of 10π, 15π, and 20π mm-mr (95%), using 120 bunches per ring, $\beta^* = 1$ m, and $E = 250$ GeV. A total (for two IRs) beam-beam tune shift limit of 0.024 was assumed.

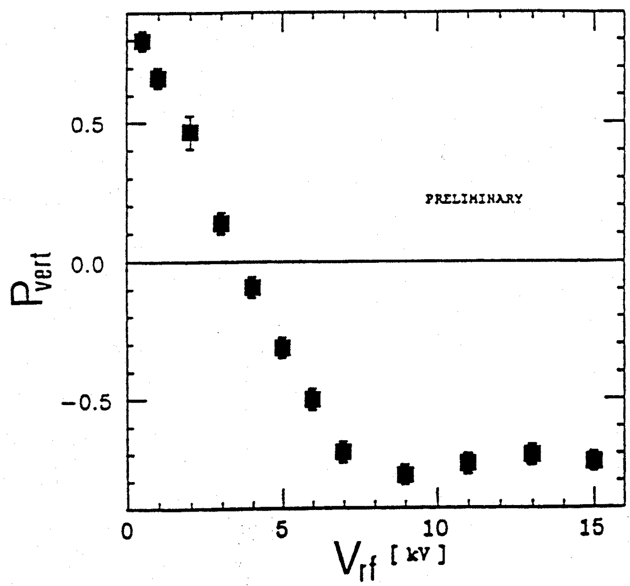

Figure 7.4: Adiabatic spin reversal of a stored beam of polarized protons at the IUCF Cooler ring. The RF voltage is proportional to the strength of the artificial spin resonance driving the spin reversal.

concept performed at the Indiana University Cyclotron Facility (IUCF)[61]. We estimate that complete spin reversal would take less than 1 second. The same device will be used to accurately measure the spin tune by measuring the spin reversal efficiency as a function of the frequency of the spin flipper excitation. This is instrumental to adjust the spin tune to 0.500.

In most cases such a simple oscillating driving field is very effective in driving an artificial resonance since the oscillating field can be thought of as the sum of two counter-rotating fields, only one of which is in resonance with the beam precession frequency. However, with a Snake the spin tune is a half-integer and therefore the two counter-rotating fields are both in resonance and interfere so that effectively only half of the beam around the ring circumference sees a driving field. By designing a true rotating field the beam polarization can be fully flipped even with a half-integer spin tune.

Several designs have been proposed, all involving two sets of AC magnets with a 90° phase difference between them. The first design uses three strong DC magnets interleaved with four independently driven vertical AC magnets. Table 7.1 shows the parameters of such a spin flipper, which could fit into a regular 12 m straight section. The DC magnets in this first design are quite strong and expensive. A less expensive design would utilize one of the two Siberian Snakes instead of special DC magnets. The two sets of vertical AC magnets are placed symmetrically around the Snake with one set generating a vertical bump of half a vertical betatron wavelength and the other one a full wavelength long. By driving the bumps with 40 kHz AC 90° out of phase a rotating driving field will be generated. Alternatively, and also least expensively,

Magnet	**Strength**	**Excitation**
1.Vertical magnet	0.01 Tm	$\sin\omega t$
2.Horizontal magnet	+4.2 Tm	DC
3.Vertical magnet	0.01 Tm	$-2\sin\omega t + \cos\omega t$
4.Horizontal magnet	−8.4 Tm	DC
5.Vertical magnet	0.01 Tm	$\sin\omega t - 2\cos\omega t$
6.Horizontal magnet	+4.2 Tm	DC
7.Vertical magnet	0.01 Tm	$\cos\omega t$

Table 7.1: Parameters for a spin flipper that produces a true rotating driving field. The excitation of the AC magnets indicates the relative phase shift between the four magnets.

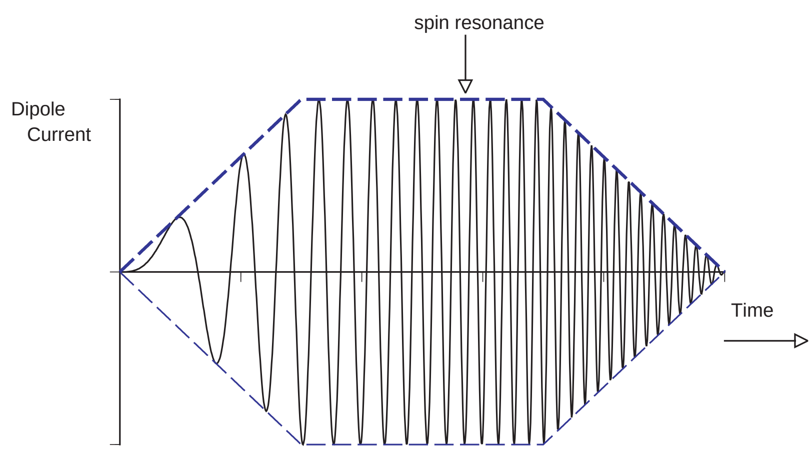

Figure 7.5: Schematic diagram of the excitation dipole current as a function of time. Note the increase of frequency with time. The trapezoidal curves are the amplitude envelopes to guide the eye.

the spin tune can be simply moved away from the value of 0.5 during the spin flip operation using the tunability of the Snakes. To achieve full spin flip with less than 0.01% polarization loss per flip a field integral for the AC magnet of $\int Bdl = 0.01$ Tm and a flip time of about 1.0 s is required[62].

To avoid emittance growth from the oscillating vertical deflections, the dipole needs to be turned on and off slowly (adiabatically) as shown in Fig. 7.5. This scheme was successfully tested in the AGS where adiabatic build up of betatron oscillation amplitudes in excess of two beam sigma, and subsequent adiabatic turn off, has been demonstrated without emittance growth.[63]