

RHIC/RF-9

**RHIC Project**  
BROOKHAVEN NATIONAL LABORATORY

**RHIC/RF Technical Note No. 9**

**PP Collisions at Different Energies**

D. P. Deng

July 1993

# pp collisions at different energies

D.-P. Deng

Brookhaven National Laboratory

Upton, NY 11973

July 15, 1993

## Abstract

Attempt to answer how much energy difference between proton beams in two *RHIC* rings while maintaining collision between them. Suppose the circumferences of the two beams differ by  $2cm$ , the energy difference is approximately  $93GeV$ .

Each *RHIC* ring contains a beam of equally spaced bunches, the spacing between adjacent bunches is the same in both rings. After each meeting of these two beams of bunches, to ensure all bunches in each beam collide in the interaction region, it is necessary that each beam maintains the same revolution frequency around the ring. The common storage cavities next to one interaction region guarantee this condition.

If it is desired to have one proton beam at one energy to collide with the other proton beam at another energy. How much difference in energy (or momentum) is possible? Let's look at the fundamental equation

$$p = e\rho_0 \left( \frac{R}{R_0} \right)^{\frac{1}{\alpha_p}} B. \quad (1)$$

where  $e$  is the charge state of the particles,  $\rho_0$  is the bending radius of the dipoles,  $R_0$  the radius of the reference orbit,  $R$  is the radius of the orbit,  $\alpha_p$  is the momentum compaction factor and related to the transition energy  $\gamma_{tr}$  by  $\alpha_p = 1/\gamma_{tr}^2$ ,  $B$  is the magnetic dipole field; and the relationship between revolution frequency  $f$  and circumference  $C$  ( $C = 2\pi R$ ) of the orbit.

$$f = \beta c/C \quad (2)$$

where  $c$  is the speed of light. From these two equations, one can derive the following differential relationship

$$\frac{dp}{p} = \gamma^2 \frac{df}{f} + \gamma^2 \frac{dR}{R}. \quad (3)$$

One easily finds out that a relation between momentum difference and the orbit difference while holding the revolution frequency constant

$$\frac{dp}{p} = \gamma^2 \frac{dR}{R}. \quad (4)$$

The difference in orbits comes from moving the beam inward or outward to the beam pipe, which is limited by the physical size of the beam pipe. Given the same limit on orbit difference,

Suppose  $dC = 2\pi dR = 2.0cm$ , then  $\frac{dR}{R} \approx 5.2 \times 10^{-6}$ . At a reference top energy ( $\gamma = 268$ ),  $\frac{dp}{p} \approx 0.37$ . The momentum difference is  $dp = 250GeV/c \times 0.37 = 93GeV/c$ ; that is one beam at  $250GeV$ , the other beam at energy  $157GeV$ .

Figure 1 gives the difference variation as a function of the reference momentum.

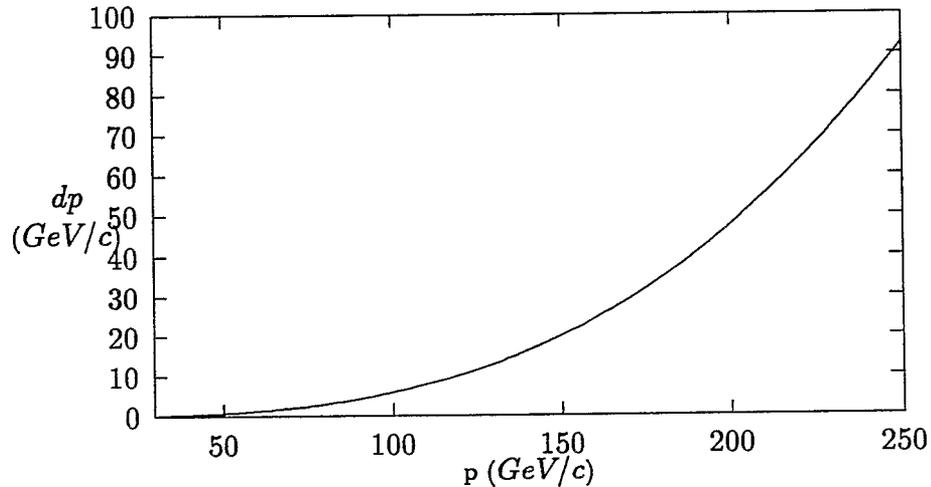


Figure 1: Momentum difference ( $GeV/c$ ) between two beams when  $dC = 2.0cm$

Another way is to let both beams travel on the designed orbit and their revolution frequencies to be in simple integer harmonic. The simplest ratio is 1:2. The revolution frequency of a beam at  $250GeV$  is approximately  $78.2kHz$ , the other beam could be at revolution frequency of  $39.1kHz$  which is to say the beam is at  $\beta = 0.5$  or  $1.1GeV$ .

## Acknowledgement

I would like to thank M. Harrison for bringing the question to me, and M. Harrison, J. Wei, S. Tepikian, S. Peggs, W. MacKay, D. Trbojevic for fruitful discussions.

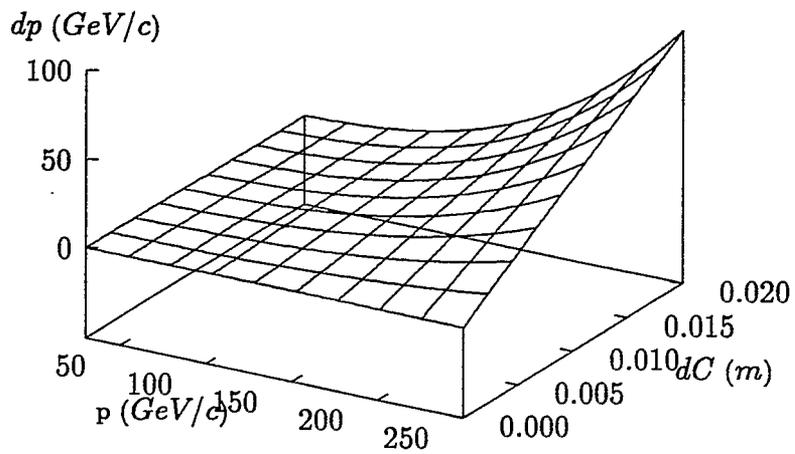


Figure 2: Momentum difference ( $\text{GeV}/c$ ) between two beams.