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STOCHASTIC COOLING IN RHIC

S. Van der Meer

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Van der Meer Letter

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Geneva, 10th September 1984

Dear Dr Lee,

As promised, I had a further look at the cooling for your planned heavy-ion machine. Unfortunately, I am not quite sure of one of the parameters you gave me: the emittance ε. Is the initial value of 10π mm mrad the real emittance or the normalized value (i.e. multiplied by $\beta\gamma$)? In the first case, the power required for betatron cooling would be rather high. If it is the normalized emittance, this problem would not exist, although there are still a number of other difficulties.

In particular, it turns out that the power needed for longitudinal cooling would severely restrict the obtainable cooling rate.

We call τ_{min} the minimum cooling time that can be obtained for a given number of particles, a given bandwidth and given mixing conditions. (This assumes that there is no power limitation). The mean square energy spread obeys the equation

$$-\frac{1}{\overline{\Delta E^2}}\frac{d\overline{\Delta E^2}}{dt} = \frac{2}{\tau_{min}} \qquad (1)$$

The factor 2 occurs because τ_{min} refers to the rms energy spread $\sqrt{\overline{\Delta E^2}}$ rather than to $\overline{\Delta E^2}$ itself. By definition at τ_{min} the gain is adjusted so that the coherent (cooling) term is twice as strong as the incoherent term; the latter is therefore equal to (1) with opposite sign. From this we may find the mean square acceleration voltage that must be applied to the kicker:

$$\overline{V^2} = 2\overline{\Delta E^2}/f_0 \tau_{min}\, Z^2 \qquad (2)$$

if ΔE is expressed in eV, f_0 is the revolution frequency and Z the particle's charge compared to a proton. Note that for relativistic particles $\Delta E = \gamma M m_p\, \Delta p/p$, where M is the mass number and m_p the proton mass in eV/c^2. Therefore

$$\overline{V^2} = \frac{2}{f_0 \tau_{min}} \left(\frac{\gamma M m_p}{Z} \frac{\Delta p}{p}\right)^2 \qquad (3)$$

Now we will see that this will result in very high power indeed. We may reduce this by chosing a lower gain that will increase the cooling time to a value τ. In fact, if we multiply the gain by a factor $g(<<1)$, the power will vary with g^2, and the cooling rate with $2g - g^2 \approx 2g$ (the incoherent term becomes negligibly small). Thus, $g = \tau_{min}/2\tau$ and the required mean square voltage becomes

$$\overline{V^2} = \frac{\tau_{min}}{2f_0\ \tau^2} \left(\frac{\gamma M m_p}{Z} \frac{\Delta p}{p}\right)^2 \qquad (4)$$

What is the power needed to generate this acceleration voltage? We shall assume that we use a pair of matched quarter-wave coupling loops as kicker. The accelerating voltage seen by the the particles is $\sqrt{2}$ times the voltage applied to the splitter circuit feeding the loops, but this is in practice reduced by a geometrical factor because the particles see only partly the loops, and partly the vacuum chamber wall. Let us assume for simplicity that the particles see exactly the applied voltage (although this is only an order-of-magnitude estimate).

Then the total power is

$$P = \frac{\overline{V^2}}{n_k\ R_k} = \frac{\tau_{min}}{2f_o\tau^2 n_k R_k}\ \left(\frac{\gamma M m_p}{Z}\ \frac{\Delta p}{p}\right)^2 \qquad (5)$$

if n_k is the number of loop pairs and R_k their impedance. As an example we shall assume

$f_o = 78$ kHz	$Z = 79$
$n_k = 200$	$m_p = 938 \times 10^6$ eV/c^2
$R_k = 50\ \Omega$	$\Delta p/p = 1.5 \times 10^{-3}$ (rms value)
$\gamma = 100$	
$M = 197$	

This gives

$$P = 8 \times 10^7\ \frac{\tau_{min}}{\tau^2} \qquad \text{(P in Watt, } \tau \text{ in sec)} \qquad (6)$$

Clearly it is advantageous to have τ_{min} as short as possible. What can we obtain?

First of all, the mixing is rather bad for $\gamma_T = 26.4$. If we assume $\Delta p/p = 1.5 \times 10^{-3}$, and with

$$\eta = 1/\gamma^2 - 1/\gamma_T^2 = -1.33 \times 10^{-3},$$

we have for the revolution frequency spread $\Delta f_o/f_o = 2 \times 10^{-6}$. Therefore, the Schottky bands would overlap above harmonic number 2.5×10^5 (if $\Delta p/p$ is the half-width). Since the revolution frequency is about 78 kHz, the cooling bandwidth would have to start at 20GHz to get good mixing. Fortunately, you do not have to go quite as high as this to get reasonable cooling times.

To find the cooling time, we have to take the bunching into account. The bucket length is 1/342 of the circumference and I assume that the bunch length is about one half of this. Therefore, for 1.2×10^9 particles/bunch we would have about the same particle density vs time with an unbunched beam of $1.2 \times 10^9 \times 342 \times 2 = 8.2 \times 10^{11}$ particles (say 10^{12} particles since the density is peaked rather than flat within the bunch). It is this number that determines the cooling rate.

Now for a longitudinal cooling system with bad mixing, the cooling time is found from the spectral density:

$$\tau_{min} = \frac{dN}{df_0} / n_\ell \, n_{av} \qquad (7)$$

where n_ℓ is the total number of Schottky lines within the system bandwidth and n_{av} is the average harmonic number. With a band between 4 and 8 GHz we would have

$$n_\ell = 51 \times 10^3$$
$$n_{av} = 77 \times 10^3$$

and since $dN/df_0 \approx 10^{12}/4\times10^{-6} \times 78000$, we find $\tau_{min} = 820s$. Now I do not know what cooling time τ is still acceptable, but assuming that 2 hours would be good enough, we would find from (6) P = 1.3 kW. This is rather high (power amplifiers of this bandwidth cost a few hundred \$ per watt), but it is not impossible. Note that the rated power should be at least twice this to avoid overloading because of the noisy character of the signal. We have no experience in this frequency range, but we plan to install a 4-8GHz system in the AA this winter.

It would be better for cooling to increase η. This would, however, require a lower γ_T and therefore, presumably, a lower horizontal tune, so reducing the acceptance of the ring.

One further problem comes from the bunching. Each Schottky band splits up into satellite bands separated by the synchrotron frequency. Since for a bunch length equal to half the bucket length the synchrotron frequency is about the same for all particles, these satellites will not spread out much; they will be very narrow and dense and spoil the cooling completely. (In the time domain, this may be described by considering that with equal synchrotron frequency the same particles will meet each other again and again in the same sample). This problem can be overcome by adding a second-harmonic RF system with about half the voltage of the fundamental; this will spread out the synchrotron frequencies sufficiently.

The signal-to-noise ratio at the pick-ups will be no problem at all. With the high Z, even a single loop pair would probably be good enough. The number of kickers, however, as we have seen, must be much larger than unity to reduce the necessary power.

For optimum transverse cooling we have for the mean square deflection angle at the kicker

$$\overline{\alpha^2} = \varepsilon \,/\, 2 f_0 \tau_{min}\, \beta_k \qquad (8)$$

where ε is the non-normalized emittance (without the factor π) and β_k the lattice function β at the kicker. The power needed to make this deflection depends strongly on the ratio of the kicker half-aperture to the beam half-width $\sqrt{\varepsilon\beta_k}$. This should be at least equal to $\sqrt{\gamma/\gamma_{inj}} = 2.9$ to provide space for the injected beam (unless you would move the kicker electrodes inwards after acceleration), and presumably a safety factor of at least 2 would be needed.

For a pair of quarter-wave loop kickers, fed by a hybrid (to get push-pull operation), the voltage at the hybrid input is (for relativistic particles) given by

$$\frac{V}{\alpha} = \frac{\pi d \gamma m_p}{2\sqrt{2}\,\ell}\;\frac{M}{Z} \qquad (9)$$

where d is the half-aperture and ℓ the length of the loop. We shall now assume $d = F\sqrt{\varepsilon\beta_k}$ and (for an octave bandwidth) $\ell = \lambda/4 = c/6W$.

Combining all this, we find for the total power at optimum cooling

$$P = \frac{\overline{V^2}}{n_k R_k} = \frac{9\pi^2}{4f_o \tau_{min} n_k R_k} \left(\frac{F \varepsilon \gamma m_p W M}{cZ} \right)^2 \qquad (10)$$

and, as before, if the gain is reduced

$$P = \frac{9\pi^2 \tau_{min}}{16 f_o \tau^2 n_k R_k} \left(\frac{F \varepsilon \gamma m_p W M}{cZ} \right)^2 \qquad (11)$$

For non-overlapping Schottky bands (bad mixing) the minimum cooling time τ_{min} is the same for longitudinal and transverse cooling.

We now substitute

$\varepsilon = 10 \times 10^{-6}/100$ m (divided by γ, assuming it was normalized)
$F = 6$ $\qquad\qquad \tau = 7200$ s
$W = 4 \times 10^9$ Hz $\qquad R_k = 50\ \Omega$
$c = 3 \times 10^8$ m/s $\qquad n_k = 200$
$\tau_{min} = 820$ s

and find a power of 0.4 W. This is very low (so we could use smaller n_k (or get faster cooling), but if ε would be the non-normalized value, we would need 4kW ! And note that there is no safety factor included; at least a factor 2 should probably be adopted. Also, I have not taken into account the reduction of transverse kicker sensitivity by the geometrical factor, the losses in hybrids, etc.

I hope that this gives you some impression of what could be done. Of course, this is not a serious design study, but rather an order-of-magnitude estimate.

If there are any problems, or if I have misunderstood something or made mistakes, please do not hesitate to write!

With best regards,

S. van der Meer