RHIC-AP-37

Tolerances on Systematic Magnet Errors

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Tolerances on Systematic Magnet Errors

$$\Delta\nu_n = b_n\beta \sum_i C^n_{2i+1} C^{2i+2}_{i+1} 4^{-i-1} A_\beta^{2i} (\eta\delta)^{n-2i-1}$$

with $C^i_j \equiv i! / j!(i-j)!$ $\quad i = 0 \div (n-1)$

n	Multipole	$\Delta\nu$
1	Quadrupole	$\beta b_1/2$
2	Sextupole	$\beta b_2\, \eta\delta$
3	Octupole	$3\beta b_3(\eta^2\delta^2/2+A_\beta^2/8)$
4	Decapole	$\beta b_4(2\eta^2\delta^2 + 3A_\beta^2/2)\eta\delta$
5	12th pole	$5\beta b_5(\eta^4\delta^4/2+3A_\beta^2\eta^2\delta^2/4+A_\beta^4/16)$
6	14th pole	$3\beta b_6(\eta^4\delta^4+5A_\beta^2\eta^2\delta^2/2+5A_\beta^4/8)\eta\delta$

β, η average for cell

For quadrupoles

$$(\Delta\nu)_{Quad} \approx (\Delta\nu)_{Dip} \cdot \frac{L_Q}{L_D}$$

Suggested Criterion in RHIC

$$\Delta\nu \lesssim 3 \times 10^{-3}$$

$$= \frac{1}{10} \text{ resonance free gap}$$

Saturation effects at high energy

h = odd $\quad A_\beta = 12.5$ mm $\quad \delta_E = 0$

$\quad = $ stability limit

h = odd, h = even $\quad \delta_E = 3.67 \times 10^{-3} \quad A_\beta = 0$

$\quad = \Delta_{Bucket}$ ("ultimate" Au with 5.5×10^9/bunch, $V = 2.4$ MV)

h = even $\quad \delta_E = 1.43 \times 10^{-3} \quad A_\beta = 12.5$ mm

Average orbit functions in RHIC

$$\langle \beta \rangle \cong 30 \text{ m}$$

$$\langle \eta \rangle \cong 1 \text{ m}$$

$$L_D = 9.7 \text{ m}$$

$$L_Q = 1.2 \text{ m}$$

Decapole in D

$$b_4 = 10 \times 10^{-4} \frac{1}{2.5^4 \, cm^4}$$

$$= 2.5 \times 10^{3} \; m^{-4}$$

$$\Delta\nu_4 = 2 \, \beta \, b_3 \, (\eta\delta)^3$$

$$= 7.4 \times 10^{-3}$$

$$= \beta b_4 \left(2 \, [\eta\delta]^2 + \frac{3}{2} A_\beta^2 \right) [\eta\delta]$$

$$= 25.6 \times 10^{-3}$$

Conclusion:

systematic b_4 required

Dodecapole in Q

$$b_5 = 6 \times 10^{-4} \frac{1}{2.5^5 \, cm^5}$$

$$= 6.1 \times 10^4 \; m^{-5}$$

$$\Delta_{\nu 5} = \frac{5}{16} \beta \, b_5 \, A_\beta^4 \cdot \frac{l_Q}{l_D}$$

$$= 1.7 \times 10^{-3}$$

$$\Delta_{\nu 5} = \frac{5}{2} \beta \, b_5 \, [\eta \delta]^4 \frac{l_Q}{l_D}$$

$$= 8 \times 10^{-4}$$

Conclusion: Borderline
need results from tracking