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Using Skew Quadrupoles in the Arc

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ABSTRACT

Effects of skew quadrupoles in the arc are analyzed. A strategy for skew quadrupole correction is suggested.

I. Introduction

In RHIC, random and systematic skew quadrupoles may be important to the beam dynamical problem through the following resonances:

(1) $$\nu_x - \nu_y = 0$$

(2) $$\nu_x + \nu_y = 58, 57 \text{ for } Q_x \simeq Q_y > 28.5$$
$$57, 56 \text{ for } Q_x \simeq Q_y < 28.5$$

(3) $$\nu_y = 29, 28$$

where $\nu_y = 29$ and 28 corresponds to the vertical dispersion.

A set of skew quadrupoles at the QD's in the arc, Q5 and Q3 in the insertions have been proposed.[1,2] In this note, we shall study the effect of these skew quadrupoles on the resonances.

II. Effect of the Correctors in One Sextant

There are 12 cells in each arc. Thus there are 12 skew quadrupoles in each arc. Within each arc, the following properties are relevant to these resonances.

(1) $\nu_x - \nu_y = 0$:

In RHIC, we have $\psi_x - \psi_y = 0$ at each arc quadrupole, where ψ_x and ψ_y are betatron phases. The resonance width derived from the skew quadrupoles is proportional to the arithmetic sum of the sum of the skew quadrupoles.

(2) $\nu_x + \nu_y = n = 58$ or 57:

The cell length is 29.622 m, the radius of the circumference is 610.18 m and the betatron phase advance per cell is $0.246 \times 2\pi$. Thus we obtain the phase angle of the adjacent skew quadrupole correctors as

$$\phi_{58} = 178.10^o \qquad n = 58$$
$$\phi_{57} = 175.32^o \qquad n = 57.$$

[1] G. Parzen, AD/RHIC–AP–72 (1988); AD/RHIC–AP–81 (1989).
[2] H. Hahn & G. Parzen, AD/RHIC/RD–20 (1990).

Depending on the magnitude and the sign of the 12 skew quadrupole correctors, the resulting resonance width is a vector sum of these 12 correctors. The adjacent correctors are nearly 180° out of phase.

(3) $\nu_y = n = 29$ or 28:

Due to the horizontal dispersion function, the skew quadrupoles can also give rise to vertical dispersion. Fortunately, this is also a measurable quantity. The nearby skew quadrupole correctors are shifted in a phase angle of

$$\phi_{29} = 89.107^o \qquad n = 29$$
$$\phi_{28} = 86.022^o \qquad n = 28.$$

Thus the adjacent skew quadrupoles are nearly 90° out of phase.

Based on the above analysis, it is useful to excite the arc skew quadrupoles as following:

$$3 \times \left(\overline{S} + \Delta_+ + \Delta_1;\ \ \overline{S} - \Delta_+ + \Delta_2;\ \ \overline{S} + \Delta_+ - \Delta_1;\ \ \overline{S} - \Delta_+ - \Delta_2\right)$$

Their effect on the resonance width can be calculated as following:

(1) $\nu_x - \nu_y = 0$:

$$J_{\nu_x - \nu_y} = \frac{\sqrt{\beta_x \beta_y}}{4\pi}\, 12\, \overline{S} \tag{1}$$

(2) $\nu_x + \nu_y = n = 58$ or 57:

$$\begin{aligned} J_{58} = \frac{\sqrt{\beta_x \beta_y}}{4\pi} \Big[& 0.1978\, \overline{S}\, e^{i79.55^o} + 11.92\, \Delta_+\, e^{-i10.45^o} \\ & + 0.1978 \left(\Delta_1 + \Delta_2\, e^{i178.10^o}\right) e^{i80.50^o} \Big] \end{aligned} \tag{2}$$

$$\begin{aligned} J_{57} = \frac{\sqrt{\beta_x \beta_y}}{4\pi} \Big[& 0.4711\, \overline{S}\, e^{i64.26^o} + 11.52\, \Delta_+\, e^{-i25.74^o} \\ & + 0.4722 \left(\Delta_1 + \Delta_2\, e^{i175.32^o}\right) e^{i66.60^o} \Big] \end{aligned} \tag{3}$$

(3) $\nu_y = n = 29$ or 28:

$$\begin{aligned} J_{29} = Q_y \frac{\sqrt{\beta_y\, X_p}}{4\pi} \Big[& 0.1331\, \overline{S}\, e^{i130.09^o} + 0.1310\, \Delta_+\, e^{i40.09^o} \\ & + 5.992 \left(\Delta_1 + \Delta_2\, e^{i89.11^o}\right) e^{-i4.465^o} \Big] \end{aligned} \tag{4}$$

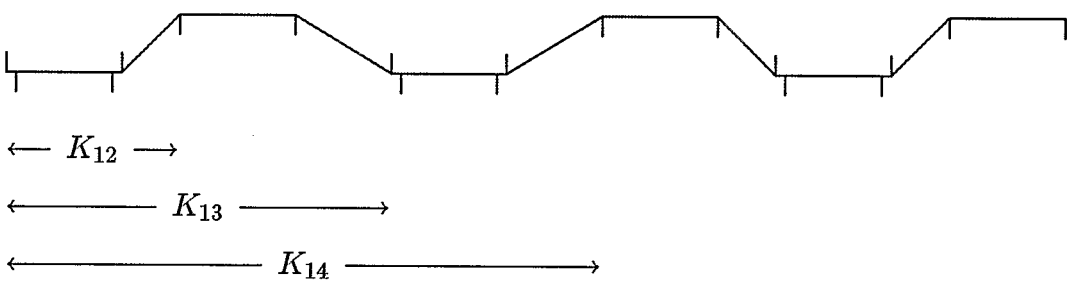
$$\begin{aligned} J_{28} = Q_y \frac{\sqrt{\beta_x\, X_p}}{4\pi} \Big[& 0.5931\, \overline{S}\, e^{i113.12^o} + 0.5534\, \Delta_+\, e^{i23.12^o} \\ & + 5.833 \left(\Delta_1 + \Delta_2\, e^{i86.02^o}\right) e^{-i19.89^o} \Big] \end{aligned} \tag{5}$$

Note that $\overline{S}$ drives weakly the sum resonances and the dispersion resonance, Δ_+ affects the sum resonance and Δ_1 and Δ_2 modify mainly the dispersion resonance.

Let us study the phase relation between the adjacent arcs.

III. Phase Relation Between Different Arcs

There are six arcs. Let us start from the arc of QF to QF shown in the following:



where K_{1i} correspond to the phase relation between the stopband integrals.

For $\nu_x - \nu_y = 0$ resonance, $K_{1i} = 0$. The total contribution is the algebraic sum of all the skew quadrupole strength.

For $\nu_x + \nu_y = n$, we obtain the following Table 1.

Table 1: K_{1i} Phase

i/n	58	57
1	0°	0°
2	150.95^o	92.34^o
3	$120.^o$	0°
4	270.95^o	92.34^o
5	$240.^o$	0°
6	30.95^o	92.34^o

Let Δ_{+i}, $i = 1, 2, ...6$ be the Δ_+ family strength. Let

$$\Delta_{+i} = a_{57} + c_{58} \cos(i2\pi/3) \qquad i = 1, 3, 5 \tag{6}$$

$$\Delta_{+i} = b_{57} + d_{58} \cos([i-1]\, 2\pi/3) \quad i = 2, 4, 5\ . \tag{7}$$

Such an arrangement would separate the effect of the 57 and 58 harmonic completely.[2]

[2] S. Tepikian, Booster Technical Note Nos. 125, 132 and 149 (1989).

Combining the coupling resonance and the systematic 57th harmonic, we obtain the

$$\frac{4\pi}{\sqrt{\beta_x\beta_y}}\begin{pmatrix} J_0^R \\ J_{57}^R \\ J_{57}^I \end{pmatrix} = \begin{pmatrix} 72 & 0 & 0 \\ -0.68 & 31.15 & 13.74 \\ 1.83 & -15.02 & 31.74 \end{pmatrix}\begin{pmatrix} \overline{S} \\ a_{57} \\ b_{57} \end{pmatrix} \tag{8}$$

where J_{57}^R and J_{57}^I are respectively the real and imaginary parts of $\nu_x + \nu_y = 57$ resonance width and J_0^R is the real part of $\nu_x - \nu_y = 0$ resonance resonance width.

Inverting the matrix, we obtain

$$\begin{pmatrix} \overline{S} \\ a_{57} \\ b_{57} \end{pmatrix} = \begin{pmatrix} 0.823 & 0 & 0 \\ 0.032 & 1.573 & -0.681 \\ -0.032 & 0.745 & 1.544 \end{pmatrix} \times 10^{-2} \begin{pmatrix} J_0^R \\ J_{57}^R \\ J_{57}^I \end{pmatrix} \tag{9}$$

Eq. (9) give us needed relation to excite the systematic correction system. Based on the realistic RHIC lattice ($\beta^* = 2$m), we expect $|J_0| \leq 2.7 \times 10^{-2}$ and $|J_{57}| \leq 2.7 \times 10^{-2}$ out of the worst of 10 random seeds. Thus the needed correction strength is

$$|a_{57}| \leq 5 \times 10^{-4}\ \mathrm{m}^{-1} = \frac{0.42\ \mathrm{T}}{840\ \mathrm{Tm}} \tag{10}$$

Using the matrix in Eq. (9), we can set the excitation strength a_{57}, b_{57} by requesting change in the resonance strength of the 57th harmonics.

For $\nu_y = n$ resonances, Table 2 shows the relative phases of each sextant.

Table 2: Phase K_{1i} for $\nu_y = n$ Resonances

i/n	29	28
1	0°	0°
2	255.45^o	195.45^o
3	$240.^o$	120°
4	135.45^o	-44.55^o
5	$120.^o$	240°
6	15.45^o	75.45^o

Let Δ_{1i} and Δ_{2i} be two skew quadrupole families for sextant i. Note first that the nearby sextants are about 180° out of phase with each other for the harmonic 28. As an example, we can use sextants 1 and 2 to devise a 29–th harmonic correction without using 28th harmonics, i.e.,

$$\left(\Delta_{11} + \Delta_{12}e^{i86.02}\right) e^{-i19.89^o} + \left(\Delta_{21} + \Delta_{22}e^{i86.02}\right) e^{i(i95.45-19.89^o)} = 0 \tag{11}$$

where the phase angles are obtained from previous table and Eqs. (4) and (5). Here Δ_{21} and Δ_{22} can be related to Δ_{11} and Δ_{12} as

$$\Delta_{21} = 0.9824\ \Delta_{11} + 0.2671\ \Delta_{12}$$
$$\Delta_{22} = -0.2671\ \Delta_{11} + 0.9453\ \Delta_{12}$$

Using the sextants 1 and 12, we obtain the resonance width as

$$J_{29} = \frac{Q_y\sqrt{\beta_y}X_p}{4\pi} \times 5.992\,[(0.4257\ \Delta_{11} + 0.8951\ \Delta_{12}) + i\,(-0.9157\ \Delta_{11} + 0.4173\ \Delta_{12})] \tag{12}$$

Similarly using sextants 3, 5 and 6, the harmonic 28 can be corrected without affecting the harmonic 29. Since the sextants 3 and 5 are separated by 120° in phase, these two sextants are set to have identical skew quadrupole excitations $\Delta_{51} = \Delta_{31}$ and $\Delta_{52} = \Delta_{32}$.

The orthogonality condition reduces to

$$\Delta_{61} + \Delta_{62}e^{i89.11} = e^{i15.45^o}\left(\Delta_{31} + \Delta_{32}\ e^{i89.11^o}\right) \tag{13}$$

or

$$\Delta_{61} = 0.964\ \Delta_{31} - 0.266\ \Delta_{32}$$
$$\Delta_{62} = 0.266\ \Delta_{31} + 0.964\ \Delta_{32}$$

The final stopband width becomes

$$J_{28} = \frac{Q_y\sqrt{\beta}_yX_p}{4\pi}5.833\,[(-0.604\ \Delta_{31} - 1.310\ \Delta_{32}) + i\,(1.310\ \Delta_{31} - 0.532\ \Delta_{32})]\ . \tag{14}$$

IV. Random Correction System

The example shown at the end of Section III for the dispersion would also excite weakly the 57th harmonic. To avoid such a cross talk between the correction systems, we can use the scheme of Eqs. (16) and (17), i.e.

$$\begin{cases} \Delta_{i1} = \Delta_{11}\cos(i2\pi/3) & \text{i=1,3,5} \\ \Delta_{i2} = \Delta_{12}\cos(i2\pi/3) & \text{i=1,3,5} \\ \Delta_{i1} = \Delta_{21}\cos([i-1]\cdot 2\pi/3) & \text{i=2,4,6} \\ \Delta_{i2} = \Delta_{22}\cos([i-1]\cdot 2\pi/3) & \text{i=2,4,6} \end{cases} \tag{15}$$

In this scenario, the resonances $\nu_x + \nu_y = 58$, $\nu_y = 29$, $\nu_y = 28$ are treated together. Let us define the scaling factor

$$A = \frac{\sqrt{\beta_x\beta_y}}{4\pi}$$
$$B = Q_y\frac{\sqrt{\beta_y}X_p}{4\pi}\ .$$

We obtain then

$$\begin{pmatrix} J_{58}^R/A \\ J_{58}^I/A \\ J_{28}^R/B \\ J_{28}^I/B \\ J_{29}^R/B \\ J_{29}^I/B \end{pmatrix} = \begin{pmatrix} 17.586 & -13.80 & 0.049 & -0.059 & -0.185 & 0.192 \\ -3.243 & 11.375 & 0.293 & -0.291 & -0.232 & 0.226 \\ 0.763 & -0.649 & 8.227 & 3.540 & -8.723 & -1.281 \\ 0.326 & -0.518 & -2.977 & 8.001 & 0.677 & -8.655 \\ 0.150 & 0.085 & 8.960 & 0.839 & -2.928 & 8.450 \\ 0.127 & -0.177 & -0.700 & 8.948 & -8.497 & -3.060 \end{pmatrix} \begin{pmatrix} c \\ d \\ \Delta_{11} \\ \Delta_{12} \\ \Delta_{21} \\ \Delta_{22} \end{pmatrix} \tag{16}$$

or more useful relation as

$$\begin{pmatrix} c \\ d \\ \Delta_{11} \\ \Delta_{12} \\ \Delta_{21} \\ \Delta_{22} \end{pmatrix} = \begin{pmatrix} 7.33 & 8.90 & -0.29 & 0.60 & 0.13 & -0.10 \\ 2.10 & 11.34 & -0.39 & 0.55 & 0.16 & 0.03 \\ -0.46 & 0.06 & 8.66 & 6.81 & 4.69 & -9.96 \\ 0.03 & 0.04 & -7.32 & 9.37 & 10.20 & 4.74 \\ 0.01 & 0.05 & -4.78 & 10.88 & 7.33 & -8.53 \\ 0.34 & -0.32 & -10.11 & -4.40 & 8.38 & 7.14 \end{pmatrix} \times 10^{-2} \begin{pmatrix} J_{58}^R/A \\ J_{58}^I/A \\ J_{28}^R/B \\ J_{28}^I/B \\ J_{29}^R/B \\ J_{29}^I/B \end{pmatrix} \tag{17}$$

where c, d are c_{58} and d_{58} in Eqs. (6) and (7). The relation in Eq. (17) gives us the required strength for the random resonances corrections. We estimated that $|J_{58}| \leq 2.7 \times 10^{-2}$ thus

$$|c \text{ or } d| < 0.0015 \text{ m}^{-1} = \frac{1.26 \text{ T}}{840 \text{ Tm}}$$

and

$$|J_{28,29}| < 1.3 \times 10^{-1}$$

and

$$|\Delta_i| \leq 0.0011 \text{ m}^{-1} = \frac{0.94 \text{ T}}{840 \text{ Tm}}$$

The correction strategy is to excite Δ_{11}, Δ_{12}, Δ_{21}, Δ_{22} such that the resonance strengths J_{29} or J_{28} can be changed according to Eq. (17).

V. Conclusion

In conclusion, we have analyzed the skew quadrupole effect on the resonances. A scheme for the skew quadrupole correction in the arc can be used to correct the dispersion and the sum resonances. The skew quadrupole wiring can be

$$3\left(\overline{S} + \Delta_+ + \Delta_1, \overline{S} - \Delta_+ + \Delta_2, \overline{S} + \Delta_+ - \Delta_1, \overline{S} - \Delta_+ - \Delta_2\right) ,$$

where $\overline{S}$ is used for coupling resonance, Δ_+ is used for the sum resonances and Δ_1 and Δ_2 is used for dispersion correction. Since $\overline{S}$ correction is mainly real, we can replace it by the skew quadrupole correctors in the insertions. Using proper excitation of skew quadrupoles in different sextants, we can correct sum and dispersion resonances.