Recent Advances of Strong-Strong Beam-Beam Simulation

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In this paper, we report on recent advances in strong-strong beam-beam simulation. Numerical methods used in the calculation of the beam-beam forces are reviewed. A new computational method to solve the Poisson equation on nonuniform grid is presented. This method reduces the computational cost by a half compared with the standard FFT based method on uniform grid. It is also more accurate than the standard method for a colliding beam with low transverse aspect ratio. In applications, we present the study of coherent modes with multi-bunch, multi-collision beam-beam interactions at RHIC. We also present the strong-strong simulation of the luminosity evolution at KEKB with and without finite crossing angle.

I. INTRODUCTION

The beam-beam interaction puts a strong limit on the luminosity of the high energy storage ring colliders. At the interaction points, the electromagnetic fields generated by one beam focus or defocus the opposite beam. This can cause beam blowup and a reduction of luminosity. An accurate simulation of the beam-beam interaction is needed to help optimize the luminosity in high energy colliders. In this case, the self-consistent strong-strong beam-beam simulation provides an invaluable tool for the study of the colliding beams.

In the strong-strong beam-beam simulation, a number of simulation particles (macroparticles) are used with the same charge-to-mass ratio as the real particles. Outside the interaction region, the macroparticles are transported through the simulated lattice using transfer maps associated with external elements, radiation damping, and quantum excitation. At the interaction point, the electromagnetic fields from the beams are calculated and applied to the particles of the opposing beam.

The soft Gaussian approximation is sometimes used to obtain the electromagnetic fields of the beams at the collision point [1–3]. While this approximation has the advantage of computational speed, it is not self-consistent because it assumes a Gaussian distribution for the macroparticles even when the actual distribution might differ substantially from the Gaussian shape. To take into account the effects of the beam distribution self-consistently, one has to solve the Poisson equation numerically during each collision for the actual macroparticle distribution at that instant. A number of methods have been used to solve the Poisson equation. The five-point finite difference method with Fourier analysis and cyclic reduction (FACR) has been used by Krishnagopal [4] and Cai et al. [5]. This method solves the Possion equation efficiently with finite domain boundary conditions. For the open boundary conditions, which are appropriate in typical beam-beam simulations, the method requires finding an effective boundary condition on the problem boundary; this can be computationally expensive. In addition, this method is not efficient to handle the case with two widely separated beams, where the domain of the source particles (particle domain) and the domain of the electric field (field domain) are different. Another method based on the fast

multipole expansion has been used by Herr et al. [6] to solve the Poisson equation. In this method, the computational cost scales linearly with the number of particles or with the number of total mesh points for the open boundary condition. The efficiency of this method is independent of the distribution of the source particles and the field domain, which makes it suitable to handle the situation with two separated beams. However, this method is an approximate algorithm in the sense that the accuracy of the expansion depends on the radius of convergence. The computational speed depends on the number of polynomials required in the multipole expansion.

The widely used method to solve the Poisson equation in beam-beam simulations is the Green function method with fast Fourier transform (FFT) on uniform grid. This method uses an FFT to calculate the cyclic summation on a doubled computational grid [7–10]. The computational cost scales as $N^2 \log(N)$, where N is the number of grid points in one direction. By defining a new shifted integrated Green function, this method can handle the separated beams, and beams with large aspect ratio.

During the beam-beam interaction, when the bunch length is large compared with the beta function value or the beam-beam forces are strong, finite bunch length effects are not negligible. In this case, a multiple slice model has to be used. The computational cost scales as the square of the number of slices. For a hadron collider with small radiation damping, it is required to track the beams for many millions of turns to study the dynamics on the time scale of the lifetime of the beams. To study the beam-beam interaction fully self-consistently for both beams (i.e. a strong-strong formulation), and to include all the physical processes of long range off-centroid interactions, finite beam bunch length effects, and crossing angle collisions, requires computational resources far beyond the capability of current serial computers. A parallel beam-beam simulation code, Beam-Beam3D, with both weak-strong and strong-strong capabilities, that can simulate these physical processes accurately using high performance computers has been developed at Lawrence Berkeley National Laboratory [12]. In this paper, we present recent advances in the numerical method to calculate the beam-beam forces and in applications to the studies of beam-beam interactions at RHIC and KEKB.

The organization of the paper is as follows: The computational methods are described in Section 2. Applications to the studies of beam-beam interactions at RHIC and KEKB are given in Section 3. We summarize our results in Section 4.

II. COMPUTATIONAL METHODS

In strong-strong beam-beam simulation, the electric fields generated by the opposite moving beam can be obtained from the solution of Poisson's equation. In Cartesian coordinate system, the solution of Poisson's equation can be written as

$$\phi(x, y) \;=\; \int G(x, \bar{x}, y, \bar{y})\rho(\bar{x}, \bar{y})\; d\bar{x}d\bar{y} \tag{1}$$

where G is the Green's function, ρ is the charge density, and (x, y) represent the coordinates in the plane perpendicular to the direction of motion of the beam. For the case of transverse open boundary conditions, the Green's function is given by:

$$G(x, \bar{x}, y, \bar{y}) \;=\; -\frac{1}{2}\ln((x-\bar{x})^2 + (y-\bar{y})^2) \tag{2}$$

Now consider a simulation of an open system where the computational domain containing the particles has a range of $(0, L_x)$ and $(0, L_y)$, and where each dimension has been discretized using

N_x and N_y points, the electric potentials on the grid can be approximated as

$$\phi(x_i, y_j) \;=\; h_x h_y \sum_{i'=1}^{N_x} \sum_{j'=1}^{N_y} G(x_i - x_{i'}, y_i - y_{j'}) \rho(x_{i'}, y_{j'}) \tag{3}$$

where $x_i = (i-1)h_x$ and $y_j = (j-1)h_y$. This convolution can be replaced by a cyclic convolution expression in a double-gridded computational domain. The cyclic convolution can be computed efficiently using an FFT as described by Hockney and Eastwood [13].

The method described above involves use of the FFT on a uniform computational grid. In high energy colliders, the colliding beams normally have a non-uniform transverse charge density distribution. A nonuniform grid will help resolve the charge density distribution more efficiently. To use a nonuniform grid, we have transformed the charge density distribution from the Cartesian coordinates (x, y) into a cylindrical coordinates (r, θ). Then, we define another transform between radial r and a new variable s as:

$$s = \frac{1}{k_1} \log(\frac{r}{k_2}) \tag{4}$$

where the constants k_1 and k_2 control the scale and the rate of the function variation between r and s. Using a uniform grid along s will generate a nonuniform grid along r since $dr = k_1 k_2 r ds$. For a uniform computational grid in (s, θ) coordinates, we can use the FFT based method to calculate the convolution for electric potential. A similar transform has been used in calculation of the gravitational potential in a disk galaxy system [14]. The new Green function in the (s, θ) coordinate is:

$$G(s, \theta) \;=\; -\frac{1}{2} \log(e^{2k_1 s} - 2e^{k_1 s} \cos(\theta) + 1) \tag{5}$$

In the (s, θ) coordinates, both the Green's function and the charge density distribution are periodic functions of θ. Hence, we do not need to double the computational domain along θ to use the Hockey's algorithm. This reduces the computational cost and the storage by a factor of two compared with the standard FFT based Green method on uniform Cartesian coordinate.

As an example of the above algorithm, we have computed the radial electric field distribution generated by a round beam with a Gaussian density distribution. The left plot of Fig. 1 shows the analytical solution of the radial electric field E_r as a function of radial distance r. The right plot of Fig. 1 shows the absolute error of E_r as a function of r using the nonuniform grid Green function method and the standard uniform grid Green function method. It can be seen that using the nonuniform grid Green function method, the numerical error of E_r is about half of that using the uniform grid Green function method. In this example, the transverse aspect ratio of the colliding beam is one. This is true for most hadron collider where radiation damping is negligible. For electron-positron colliders such as KEKB and PEPII, the colliding beam can have a very large transverse aspect ratio. To test the applicability of the above algorithm, we have also calculated the electric field for a Gaussian charge density distribution with an aspect ratio of 30. The relative error of E_x on the x axis is given in Fig. 2 together with that calculated from using the integrated Green function method on uniform grid. Here, three relative errors of E_x from using the nonuniform grid Green function method on a computational grid of 256×512, 512×256 and 1024×512 are given. The relative error from the integrated uniform Green function method uses a computational grid of 256×256. Since the nonuniform grid Green function does not need to double the computational domain in the θ direction, it has the same computational cost on a 256×512 grid as the uniform grid Green function method does on a 256×256 grid. It is seen that the integrated Green function method on a 256×256 uniform grid gives the least error. This suggests that the integrated Green function method might be more efficient for a beam with large aspect ratio.

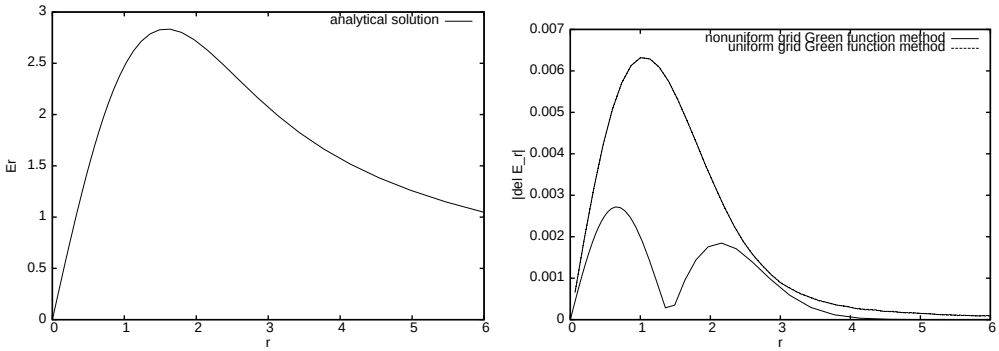

FIG. 1: Radial electric field E_r and the absolute error of E_r as a function of r from the nonunifrom grid Green function method and from the uniform grid Green function method.

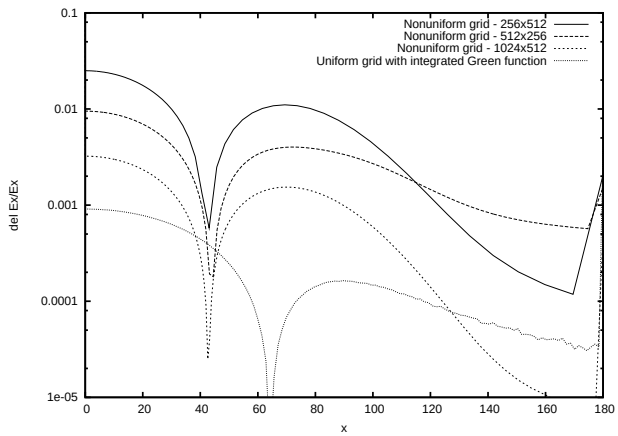

FIG. 2: Relative error of E_x as a function of x from using the nonuniform grid Green function method and from using the integrated uniform grid Green function method.

III. APPLICATIONS

In recent applications, we have studied the coherent modes of multi-bunch collisions at RHIC through a strong-strong beam-beam simulation [15]. Fig. 3 gives a schematic plot of two colliding beams at RHIC. Here each beam has three bunches. The six bunches couple with each other through collisions at four interaction points, IP2, IP6, IP8 and IP10. Table 1 gives a list of the physical parameters used in the simulation. All bunches are assumed to have the same physical parameters listed in the table. Fig. 4 shows power spectra of horizontal centroid motion of three bunches. There are only two distinct eigenmodes, the π mode (180 degree out of phase) and the σ mode (in phase), which are observable in this example. The other four modes are degenerated and buried into the incoherent continuous spectra. The π mode tune shift is 4.918ξ which is about of a factor of 4 times the single bunch π tune shift 1.21ξ. This is in agreement with the analytical calculation of Yokoya et. al. [16]. The large tune shift of the π mode due to the multi-bunch collisions presents a potential instability since it can not be damped out by the continuous spectra through the Landau damping. In above example, we have assumed that the two beams have the same parameters. In reality, the parameters of two rings can be controlled so that the two colliding beams have different tunes. Fig. 5 gives power spectra of horizontal centroid motion of three bunches with the horizontal tune of the second beam set as 0.2 while the first beam is set as

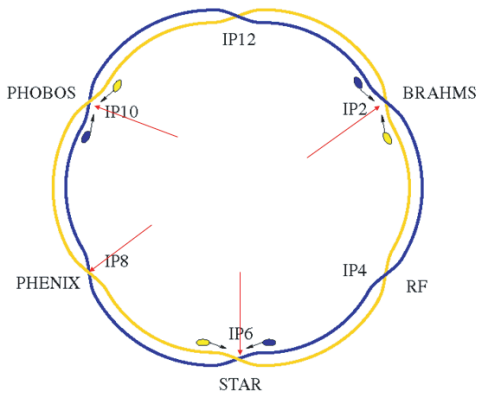

FIG. 3: A schematic plot of two colliding beams at RHIC.

beam energy (GeV)	100
protons per bunch	10.0×10^{10}
β^* (m)	1.0
RMS spot size at the IP (mm)	0.176
betatron tunes (ν_x, ν_y)	(0.22, 0.23)
synchrotron tune ν_z	3.7e-4
RMS bunch length (m)	3.6
momentum spread	1.6e-3
beam-beam parameter ξ	0.00366

TABLE I: RHIC physical parameters for beam-beam simulations

0.22. The two colliding beams lose the coherent motion and the dipole mode disappears into the continuous spectra.

In another application, we have studied the time evolution of luminosity at KEKB. The physical parameter used in the simulation is given in Table 2. Fig. 6 shows the time evolution of single bunch collision luminosity with (left plot) and without (right plot) a finite crossing angle. With 11 mrad of finite crossing angle, the luminosity has dropped by about a factor of two after about 1000 turns. This suggests that using a head-on collision (e.g. by using crab cavity at interaction point) will significantly improve the luminosity at KEKB.

IV. SUMMARY

In this paper, we have reported on some recent advances of strong-strong beam-beam simulation. The new nonuniform grid Green function method for calculating the beam-beam forces has the advantage of better accuracy and less computational cost for low aspect ratio beam in hadron collider. The application to the study of the multi-bunch coherent modes at RHIC shows a much larger dipole mode tune shift than that of the single bunch collision. This mode can be removed with asymmetric tunes of two colliding beams. In the KEKB application, the collision with 11 mrad crossing angle shows a significant decrease of the luminosity compared with the head-on collision. This suggests that using a crab cavity to correct the crossing angle collision will improve the luminosity of the future machine operation.

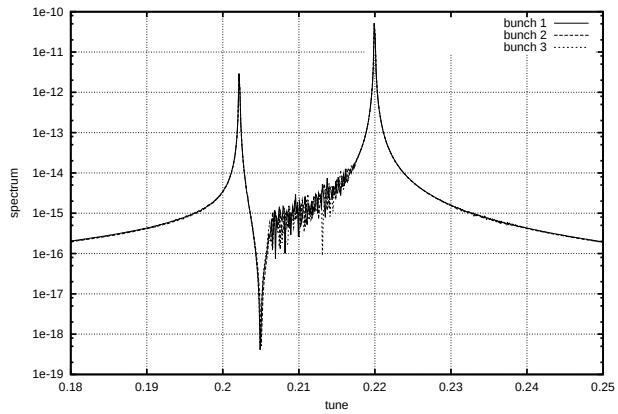

FIG. 4: Power spectra (arbitrary normalization) of the horizontal centroid motion of three bunches at RHIC.

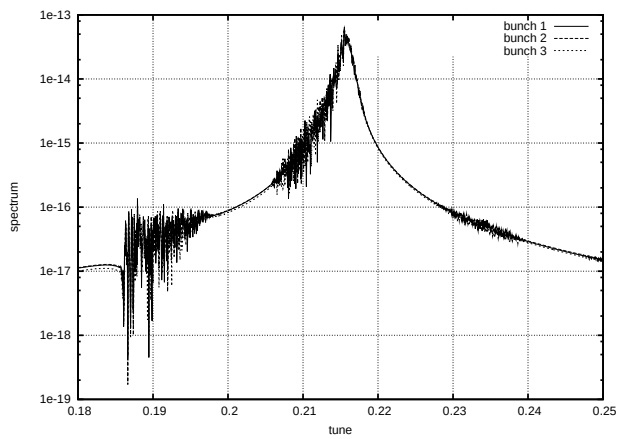

FIG. 5: Power spectra (arbitrary normalization) of the horizontal centroid motion of three bunches with different tune in each ring of RHIC.

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e^-/e^+ beam energy (GeV)	8.0/3.5
e^-/e^+ per bunch	$4.375 \times 10^{10}/10.0 \times 10^{10}$
β^* (horizontal, vertical, longitudinal) (m)	(0.6, 0.007, 10.0)
Emittance (horizontal, vertical, longitudinal) (m-rad)	(1.8×10^{-18}, 1.8×10^{-18}, 4.8×10^{-6})
betatron tunes (ν_x, ν_y)	(0.5151, 0.5801)
synchrotron tune ν_z	0.016
damping time (horizontal, vertical, longitudinal) (/turn)	(2.5×10^{-4}, 2.5×10^{-4}, 5.0×10^{-4})

TABLE II: KEKB physical parameters for the beam-beam simulation

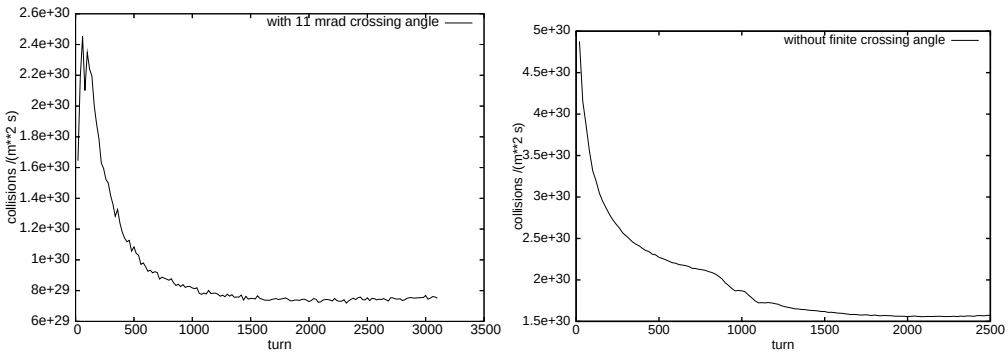

FIG. 6: Evolution of single bunch collision luminosity at KEKB with and without a finite crossing angle.

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