Crab cavity option for LHC IR upgrade *

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Abstract

Crab cavities, initially proposed by Palmer, are used to impart a z-dependent transverse kick to rotate particle bunches. An appropriate rotation of the bunch ensures head-on collisions to recover the geometric luminosity from the presence of a finite crossing angle of the IR.

INTRODUCTION

Crab cavities have been proposed starting 1988 by Palmer for linear colliders to compensate the geometric loss of the luminosity due to the presence of crossing angle. Subsequently, Oide and Yokoyo proposed this scheme for circular colliders in 1989. A deflecting cavity placed $\pi/2$ away from IP imparts a longitudinally dependent transverse kick to rotate the bunches and provide head-on collisions. A second cavity placed symmetric deflects the bunch back into the original orbit to make the crab compensation local.

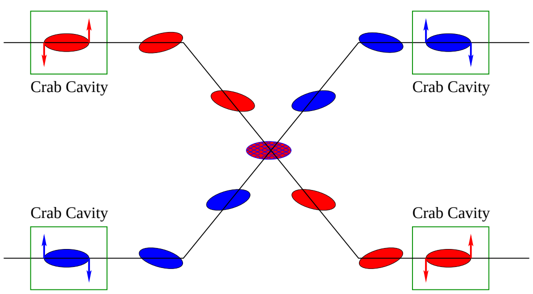

Figure 1: Local crab compensation scheme using transverse deflecting cavities to provide head-on collisions.

An alternate version of the crab compensation where cavities located elsewhere in the ring satisfy certain phase advance condition to the IP is being conceived at KEK-B, In this scenario, the head and the tail of the bunch oscillates around a reference closed orbit around the ring with a head-on collision at the IP.

The primary advantages of a crab scheme are:

- Reduction of geometric luminosity loss due to finite crossing angle at the collision point.
- Alleviation of long range beam-beam effects
- Simpler and a more flexible IR design with separate focusing channels

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- A further reduction of β^* to increase luminosity

A crab scheme for the LHC upgrade is under study and different options for crossing angle and technological issues and related technological issues will be discussed in the following sections.

VOLTAGE REQUIREMENT

For a finite full crossing angle θ_C, the transverse kick voltage required is given by

$$V_{crab} = \frac{cE_0 \tan(\theta_C/2)}{\omega_{RF}\sqrt{\beta_{crab}\beta^*}} \quad \{\sigma_z \ll \lambda_{RF}\} \tag{1}$$

where E_0 is the beam energy, ω_{RF} is the RF frequency of the cavity, β_{crab} and β^* are the beta-functions at the cavity location and the IP respectively. Fig. 2 shows a plot for the voltage dependence on the crossing angle for three different frequencies.

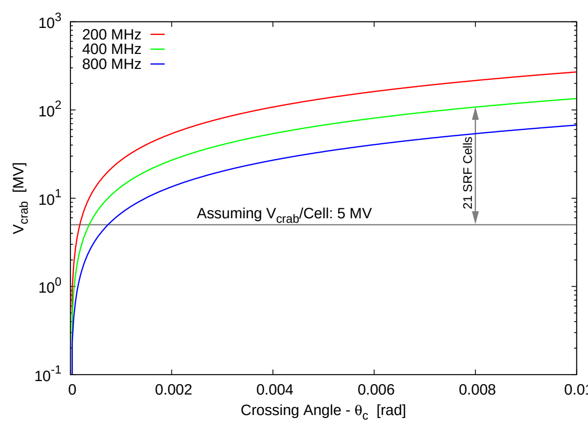

Figure 2: Crab voltage required for compensation as a function of crossing angle

Although the voltage and other RF tolerance issues (see following sections) prefer a higher frequency, the large bunch length (7.55cm) constrains the upper bound to 400 MHz. Since the dimensions scale inversely to the cavity frequency, a large crossing angle (∼8 mrad)is required to transversely accommodate cavities for the two beam lines near the IR. The voltage needed to compensate a 8 mrad angle is quite significant (see Table 1) which is to be compared to the current state of the art KEK-B cavity gradient of 2 MV/m [1, 2]. This will require a chain of superconducting structures spanning several tens of meters.

To realize such high voltage requirements, three options can be pursued:

Table 1: Crab cavity voltage required for three different crossing angles for three different possible RF frequencies.

X-Angle	1 mrad	5 mrad	8 mrad
200 MHz	27 MV	134 MV	216 MV
400 MHz	14 MV	67 MV	108 MV
800 MHz	7 MV	34 MV	54 MV

- Increase real estate gradient to 5-10 MV/m. The lower limit is within reach by carefully optimizing the elliptical body of the cavity and reduce surface fields. However, the upper limit will require multicell cavities, thus introducing complications due to higher order modes (HOMs) and their effective damping.
- An increase of the β-functions in the plane of the crossing at both the cavity location (β_x^{crab}) and the IP (β_x^*), while simultaneously decreasing it in the other plane to keep luminosity unaltered is very attractive. In addition to reducing the voltage required, a flat beam will entail a doublet type optics which is symmetric around the IP unlike triplet options [5]
- Shortening the bunch by a factor of 2 will allow a higher crab frequency cavity and significantly alleviate space, voltage and tolerance requirements. However, this option is not attractive due to the huge voltage required with bunch shortening system which is unfavorably proportional to $(\theta/\sigma_x^*)^4$ [3, 4].

LOCAL VS. GLOBAL COMPENSATION

An ideal crab scheme would involve a local compensation with two cavities on either side of the IP (Fig. 1), leaving the closed orbit in the rest of the ring unchanged. However, the transverse dimensions of the cavities (see next sections) require a large crossing angle of $\sim$8 mrad IR optics as described in Ref. [5]. In addition to being a risky venture, the beam dynamics and RF issues become more challenging for increasing crossing angles. A global crab

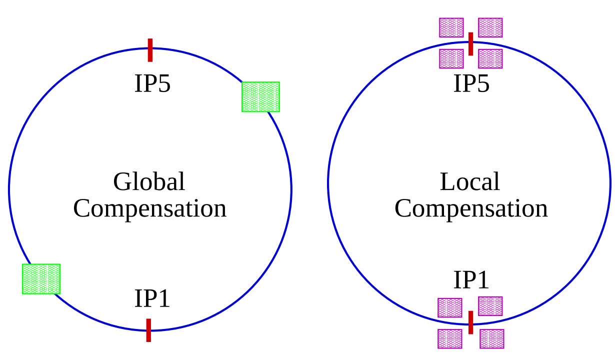

Figure 3: Schematic of global and local crab compensation at two IPs for LHC upgrade.

scheme offers the flexibility of placing the "rather large" cavities at a location with fewer space constraints than the IR. This feature may also relax the requirements on the magnetic channels to match the IR optics. For the case with two IP's, two cavities are sufficient to satisfy the phase advance constraint to achieve head-on collisions at both IP's. However, an oscillating closed orbit for particles away from the center of the bunch, although not catastrophic, can pose aperture and tune shift constraints that will limit the crossing angle. In addition, the motion of particles in the bunch head and bunch tail will become more sensitive to the magnetic errors in the entire ring. Fig. 4 shows closed orbit deviation and tune shift for the a particle that is 1σ away from the center of the bunch as a function of the crossing angle for the global scheme.

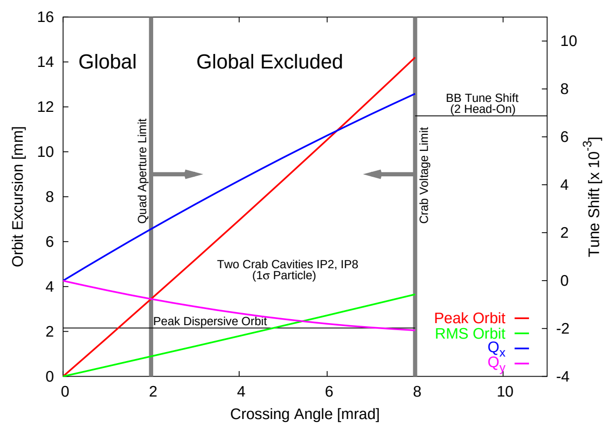

Figure 4: Orbit excursion and tune shift induced by two global crab cavities for a particle that is 1σ away from the center of the bunch.

Based on Fig. 4 and existing technological constraints, the compensation scheme can be divided into three categories:

- Small crossing angle (0.4-2 mrad): Since, the orbit excursion and tune shifts are small, global crab scheme is an ideal choice. This option is compatible with all quadrupole first and the D0 options [8] to recover the geometric luminosity loss. Technologically, these cavities are feasible using available technology, and the gradient requirements are minimal.
- Medium crossing angle (3-5 mrad): Beyond 2 mrad, the excursion and tune shifts are probably too large for a global scheme. There are additional constraints like cavity size and triplet aperture requirements which are not feasible with available technology and would require major R&D. However, this option offers great potential if designs for the cavities and triplets can be conceived. It is less risky than a large crossing angle while offering all the advantages of the crab compensation concept.
- Large crossing angle (7-8 mrad): If a local compensation with available technology is needed, a minimum

crossing angle of 7-8 mrad is required to accommodate the cavities, and provide sufficient aperture for the triplets. The voltage requirements for such crossing angles will need very long structures and the cost is perhaps prohibitive. Although this option is attractive, a very large crossing angle relies heavily on the cavities which is quite risky in the event of failures.

RF CURVATURE

For a linear transverse deflection, it is necessary that the RF wavelength (λ_{RF}) be much larger than bunch length (σ_z). Due to the finite wavelength ($\lambda_{RF} \sim 2.5\sigma$, 400 MHz), a particle displaced z for the center of the bunch will receive a deviation from the linear kick which is

$$\begin{aligned} \Delta x' &\approx \frac{1}{R_{12}} \sin\left(2\pi z/\lambda_{RF}\right) \\ &\approx \frac{2\pi}{\lambda}\left[z - \frac{2\pi^2 z^3}{3\lambda^2} + \cdots\right] \end{aligned} \qquad (2)$$

Fig. 5 shows the % deviation from the linear kick as a function of displacement z for three different frequencies.

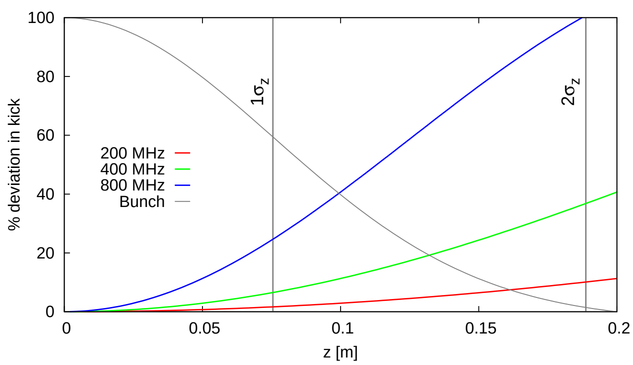

Figure 5: Percent deviation from a linear crab kick due to finite wavelength of the RF cavity for three possible frequencies.

The effect of the curvature is a distortion of the longitudinal profile of the bunch. This effect is non-negligible, thus resulting in a reduction of luminosity. In addition to the wavelength, the reduction factor is strongly dependent on the crossing angle. Using formula given in [3], an 8 mrad crossing angle with 400 MHz RF frequency yields a reduction factor $> 25\%$. This constrains the highest frequency to 400 MHz for medium and large crossing angles.

A 2^{nd} harmonic cavity can increase the span of the linear kick for particles at small displacements where the bunch is most populated. For small crossing angles ($<$ 1mrad), the reduction factor is $< 20\%$ (400 MHz), thus allowing higher frequencies like 800 MHz. Multiparticle simulations are underway to validate the analytical estimates and more accurately define the acceptable frequency regime as a function of crossing angle.

EMITTANCE GROWTH

Several sources of emittance growth due to imperfections of crab compensation are present. Amplitude and phase jitter of the RF sources are of major concern. Lattice properties such as incorrect betatron phase advance, linear and non-linear imperfections, and coupling need careful study. In addition the finite energy spread, chromatic and beam-beam effects require mutiparticle simulations to estimate the emittance growth. The effects of amplitude and phase noise are addressed using analytical estimate in this section.

Amplitude Noise

Voltage and phase fluctuations are present in any RF source. A small fluctuation in the voltage of the cavity induces a variation of the kick amplitude and effectively translates into a residual crossing angle at the IP as shown in Fig. 6.

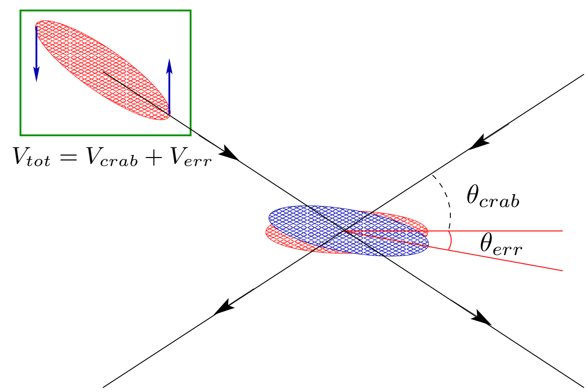

Figure 6: Effect of amplitude jitter on crab compensation which results in a residual crossing angle at the IP proportional to the jitter amplitude.

The jitter at the IP in terms of the voltage fluctuation is given by

$$\Delta x' = \frac{\theta_c}{2R_{12}}\left(\frac{\Delta V_\perp}{V_\perp}\right) z \qquad (3)$$

A tolerance for the voltage fluctuation can be estimated by requiring that the residual tilt error be much smaller than the diagonal angle of the bunch [6]

$$\frac{\Delta V_\perp}{V_\perp} \approx \frac{\theta_{err}}{\theta_c} \ll \frac{1}{\tan\left(\theta_c/2\right)} \frac{\sigma_x^*}{\sigma_z} \qquad (4)$$

For example, an 8 mrad crossing angle with a voltage jitter of 0.04% (feasible today) translates into 3 μrad residual crossing angle which appears negligible.

Phase Noise

A phase error in the RF wave causes an offset of the bunch rotation axis translating into an transverse offset at the IP as shown in Fig. 7 and is given by

$$\Delta x_{IP} = \frac{c\theta_c}{\omega_{RF}}\delta\phi \tag{5}$$

where θ_c is the full crossing angle and $\delta\phi$ is the phase error.

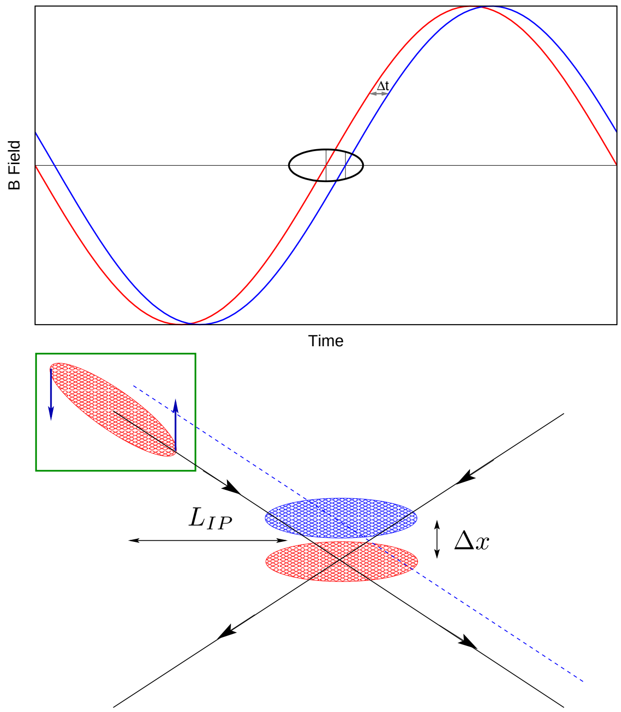

Figure 7: Effect of phase jitter on the crab compensation which results in a transverse offset of the bunch at the IP.

This random offset at the IP is potentially severe due to beam-beam effects. The emittance growth resulting from the beam-beam forces is estimated in [3]

$$\left(\frac{\Delta\epsilon_x}{\Delta t}\right)_{BB} \approx n_{IP} f_r \frac{8\pi^2\xi^2}{\beta_x^*}(\Delta x)^2 \tag{6}$$

An 8 mrad crossing angle with a maximum emittance growth of 10% per hour requires a relative crab to crab phase stability, $\Delta\phi_{BB} \leq 0.001°$.

Dipole kicks due to random crab phase jitter is an additional source of emittance growth which can be estimated as [7]

$$\left(\frac{\Delta\epsilon_x}{\Delta t}\right)_{Dip} \approx n_{IP}\frac{f_r}{2\beta^*}\left(\frac{c\theta_c}{2\omega}\Delta\phi_c\right)^2 \tag{7}$$

For nominal LHC upgrade parameters, and a maximum emittance growth of 10% per hour, the phase jitter $\Delta\phi_{Dip} < 10^{-4}$ deg. For comparison, the ratio of the relative displacement for required phase jitter to the transverse beam size $\Delta x/\sigma_x^* < 10^{-3}$ is rather tight. However, these errors can accumulate within the correlation time. Turn by turn transverse feedback system in LHC should alleviate this problem significantly. The available low level RF technology for a higher frequency (S-Band & X-Band) was demonstrated to reach a $\Delta\phi_{rel} \sim 0.003°$ [9]. Detailed multiparticle simulations are needed to confirm the analytical estimates. A preliminary analysis from K. Ohmi [10] compared in Table 2 suggest that these tolerances are more relaxed compared to analytical estimates.

Table 2: Amplitude and phase jitter tolerances required to control the emittance growth below 10% per hour. The tolenrance listed for simulations include both beam-beam and dipole kicks with feedback (θ_c = 8 mrad, f_{RF} = 400 MHz, and R_{12} = 31 m).

Jitter Estimate	Amp.	Phase	
		Beam-Beam	Dip. Kicks
Analytical	$< 0.04\%$	$< 0.001°$	$< 0.0001°$
Simulation	-	0.00023°	
Feasible Today	0.01%	0.003°	

BASELINE DESIGN & RF PARAMETERS

A baseline design using superconducting RF elliptical cavities similar to KEK-B design is considered. Taking the bunch length, and cavity dimensions into consideration, an RF frequency of 400 MHz is the most appropriate choice. By virtue of it size, a 8 mrad optics is needed to accommodate the cavities transversely into the IR for a local compensation scheme. The magnetic field of the TM_{110} transverse mode is used to deflect the particles. Fig. 8 shows a graphical representation of the TM_{110} mode in an elliptical cavity. The bunch receives a time dependent transverse kick with the zero crossing of the RF wave at the center as seen in Fig. 7.

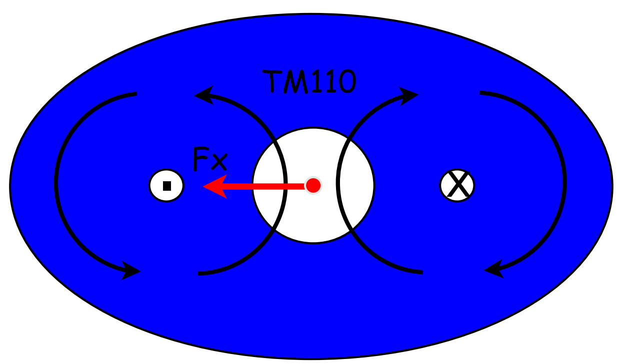

Figure 8: Schematic of squashed elliptical cavity. Magnetic field of the horizontal polarization of the TM_{110} mode giving a net horizontal kick.

A coupled two-cell cavity is being considered as a fundamental unit in the π mode to impart a total kick of 5 MV. A schematic of the two-cell cavity is shown in Fig. 9.

For large crossing angles, a four-cell or 2×2 cell superstructure will be needed to achieve the large gradient in the

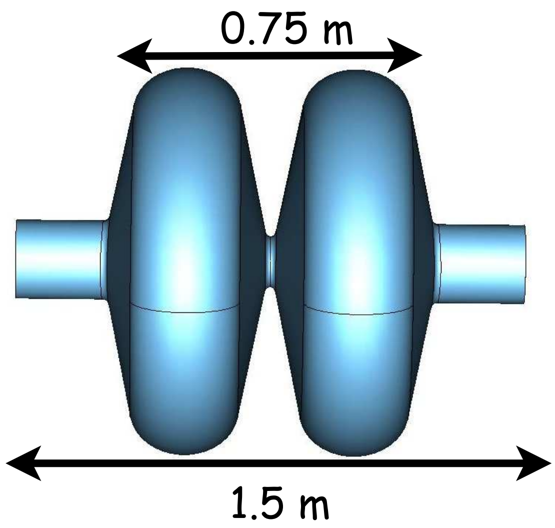

Figure 9: Graphic of the proposed two-cell 400 MHz cavity. The total length of the structure with couplers, ferrites and cryostat assembly (not shown here) is less than 2m.

available space ($\sim$ 25 m). Some geometric dimensions of the cavity design are shown in Table .

Table 3: Geometrical parameters of an elliptically squashed TM_{110} cavity at 400 MHz.

Half Cell Length, $L = \frac{\lambda\beta}{4}$	18.75 [cm]
Two Cells + Beam Pipe	$\sim$ 1.5 [m]
Horizontal Eq. Radius, R_{iris}	**53 [cm]**
Horizontal Eq. Radius, R_{iris}	37.5 [cm]
Squash Ratio	0.75
Beam Pipe Radius	15 [cm]
Wall Angle, α	$\sim$ 6 [deg]
Equator Dome Radius	12.0 [cm]
Cavity Beta, $\beta = \frac{v}{c}$	1.0

Since, the mode of choice is a dipole mode, the parasitic mode with the orthogonal polarization needs to be well separated in frequency and damped to avoid creating a crossing angle in the other transverse plane. The typical way of achieving this separation is to selectively polarize the mode by squashing the transverse shape to an elliptical profile. Fig. 10 shows frequencies of parasitic and other relevant modes in the cavity as a function of the squash ratio between the two transverse planes.

The maximum mode separation for this cavity shape is at a squash ratio of 0.75. Unfortunately, the major axis of the elliptical profile is in the plane of crossing, thereby further increasing the transverse space requirement. However, a 20 MHz mode separation is probably sufficient, thus relaxing the quash ratio and reducing the cavity dimensions. Table shows some relevant RF parameters. Further optimization of the cavity ellipses is required to reduce the peak surface field and increase the real estate gradient.

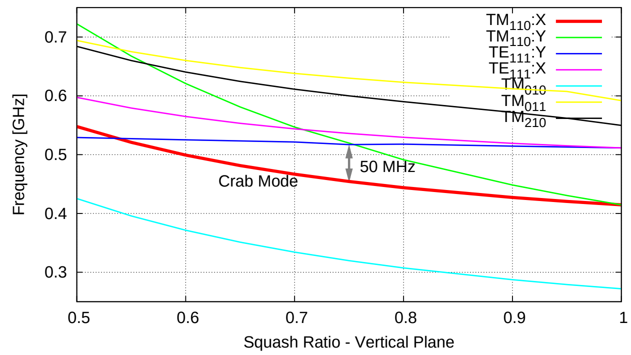

Figure 10: Frequencies of relevant TM and TE modes as a function of squash ratio for the proposed cavity. The maximum separation between the two polarizations of the TM_{110} mode (red & green) is found at a ratio of 0.75 for this geometry.

Table 4: Relevant RF parameters for a TM_{110} 400 MHz cavity.

Frequency	400 MHz
Q_0 (BCS 2K)	10^9
Voltage, $V_\perp$	5 MV
Number of cells	2$\times$2, 4$\times$2
Shunt Imp., $R_\perp/Q_0$	$\sim$ 90 Ω
Beam Power, P_B	$\sim$ 50 kW/mm
E_{peak}	$<$ 25 MV/m
B_{peak}	$<$ 150 Oe (Max Limit - 2200 Oe)

Several R&D and RF issues including fundamental and higher order couplers, longitudinal and transverse impedance, tuners and other relevant components are under investigation.

EXOTIC SCHEMES

In view of the transverse size problem with TM_{110} type elliptical cavities and reduced crossing angle, two exotic cavity concepts are also being pursued:

TM_{010} type cavity with beam pipes transversely displaced toward the equator to use the magnetic field for deflection.

The cross section is modified into a bi-elliptical shape as shown in Fig. 11 to have the peak magnetic field at the location of the beam pipe instead of the equator. Some pros and cons of a TM_{010} design are:

- A TM_{010} is the lowest order mode in a pillbox type cavity and hence this cavity will always be smaller than a corresponding TM_{110} cavity.
- TM_{010} mode will also have a higher R/Q_0 and smaller peak fields thus allowing for higher gradients.
- All modes in the cavity are of higher frequency

thereby allowing for much simpler coupler and HOM damping schemes than the TM_{110} cavity.

- The large transverse offset in the cavity will enhance the coupling of beam to HOMs and generate large wakefields which may be the major limitation.
- Other issues like multipacting and beam loading due to non-zero longitudinal electric field need evaluation.

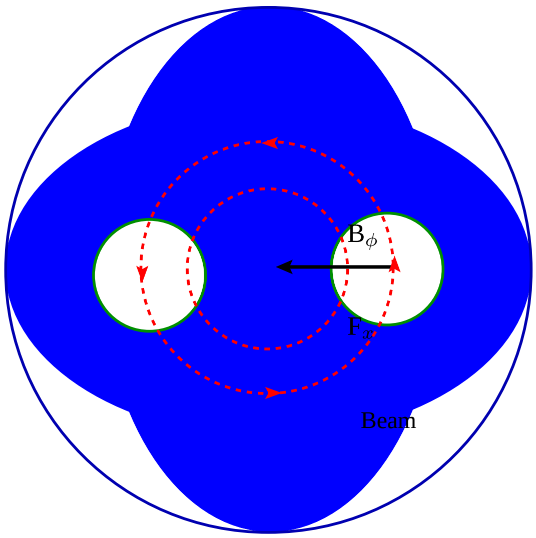

Figure 11: Conceptual design of a TM_{010} type deflecting cavity. The distinct bi-elliptical shape constrains the the boundary conditions for the maximum of the $\vec{B}$-field to occur closer to the center of the cavity.

Spoke type transmission line resonator. Fig. 12 shows a spoke cavity for conventional accelerating cavities. A deflecting spoke design does not exist but could be pursued as an alternate option. Some pros and cons of a spoke design are:

- The transverse dimension for the accelerating type is $\leq \lambda/2$ which is typically more compact than an elliptical counterpart.
- Spoke cavities are mechanically very stable by virtue of their design and having very relaxed tuning requirements.
- The design is very complicated (significant R&D),
- For accelerating cavities, the real estate gradient is typically smaller than its elliptical counterpart.
- Multipacting problems are severe and will need thorough analysis.

A deflecting spoke design does not exist but could be pursued as an alternate option.

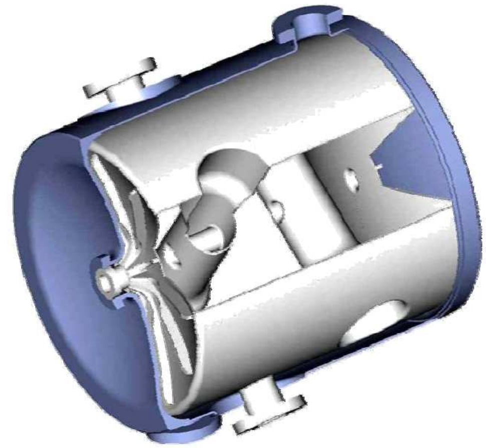

Figure 12: Graphic of an accelerating spoke cavity at 345 MHz with $\lambda/2$ transmission line transversely [11].

CONCLUSION

A crossing angle scheme using 400 MHz crab cavities to recover geometric luminosity loss and alleviate long range beam-beam effects has been proposed for LHC IR upgrade. Issues relating to both beam dynamics and cavity technology are discussed. Phase noise from cavities can be a severe source of emittance growth and detailed simulations are needed to more precisely define the tolerances required to control these effects. A preliminary cavity design has been described and further R&D is required to study many RF issues to reach an optimized design. Two exotic cavities are also outlined as possible alternatives to the conventional elliptical TM_{110} cavity.

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