Linear Beam Dynamics

&

Ampere Class Superconducting Cavities

@RHIC

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Outline

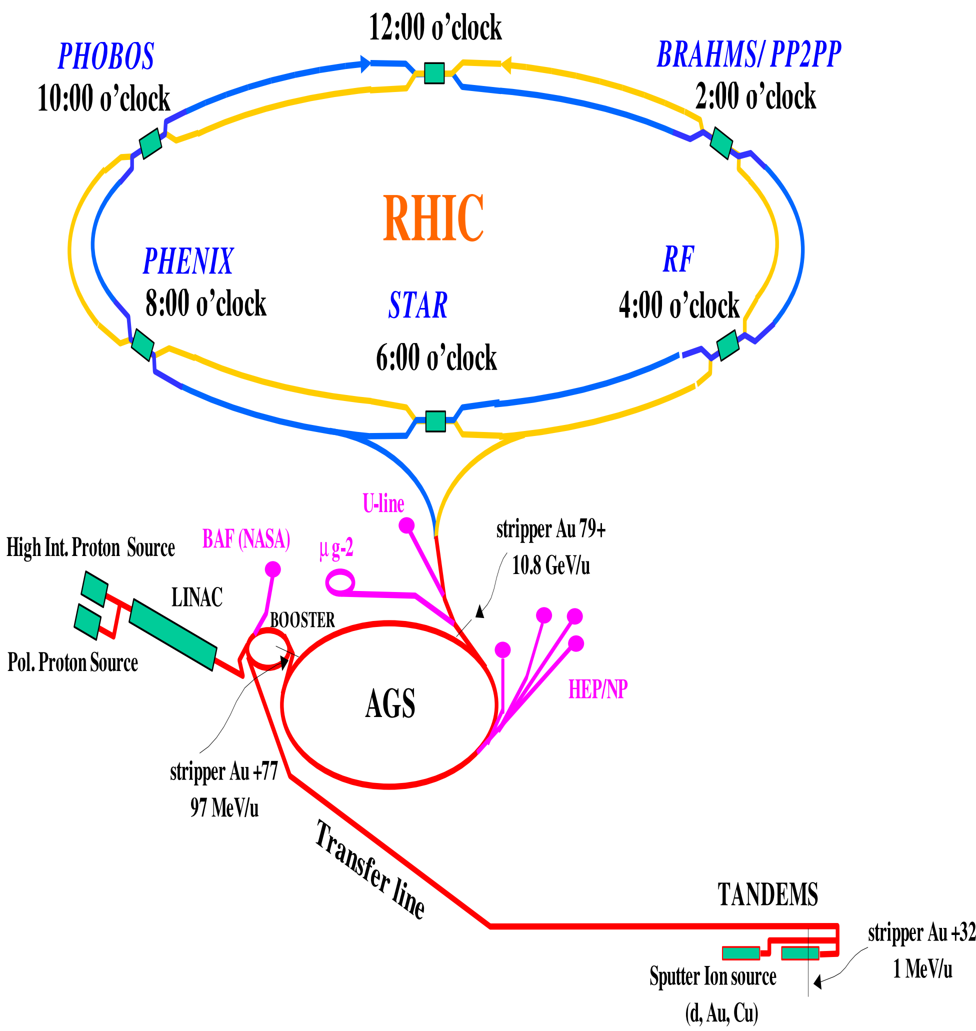
- Part I
  - Introduction
  - Principle Component Analysis & SVD
  - Optics Measurements
  - Linear Coupling - Formalisms & Measurements

- Part II
  - Electron Cooling @RHIC
  - Why Superconducting RF & Energy Recovery
  - Five-Cell SRF Cavity
  - $\frac{1}{2}$ Cell SRF Gun

- Conclusions

Relativistic Heavy Ion Collider

- Collide Ion
  - Au^{+79}-Au^{+79} - 100 GeV/n
  - p^{+}-p^{+} - 250 GeV
- Six Arcs (Bending + Focusing Magnets)
- Six Interaction Regions (IR's)
- Transverse Kickers, AC Dipoles
- Beam position monitors (BPMs)
  - 72 dual-plane (IR's)
  - 176 single-plane (Arcs)
- WCMs, IPMs, BLMs...



Some Basic Concepts

$$x(s) = x_0(s) + x_\beta(s) + D_x(s)\delta$$

Hill's Equations:

$$x'' + K_x(s)x(s) = 0$$
$$y'' + K_y(s)y(s) = 0$$

Solution:

$$x(s) = \sqrt{2J\beta(s)}\cos(\psi(s) + \phi)$$

CS Invariant:

$$2J = \gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$
$$\alpha = -\beta'/2, \quad \gamma = (1+\alpha)/\beta$$

Phase Advance:

$$\psi(s_1 \to s_2) = \int_{s_1}^{s_2} \frac{1}{\beta(s)}\, ds$$

Tune:

$$Q = \frac{1}{2\pi}\psi_C = \frac{1}{2\pi}\oint \frac{ds}{\beta}$$

One Turn Map:

$$\begin{bmatrix} x \\ x' \end{bmatrix}_2 = \mathcal{M}_C \begin{bmatrix} x \\ x' \end{bmatrix}_1$$

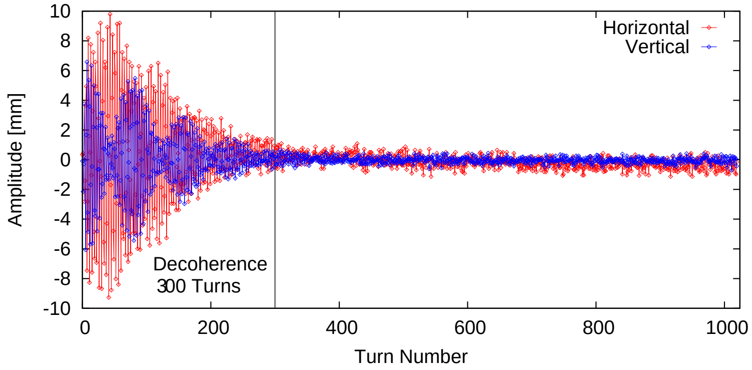
$$\mathcal{M}_C = \mathbf{I}\cos(\psi_C) + \mathbf{J}\sin(\psi_C).$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{J} = \begin{bmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{bmatrix}$$

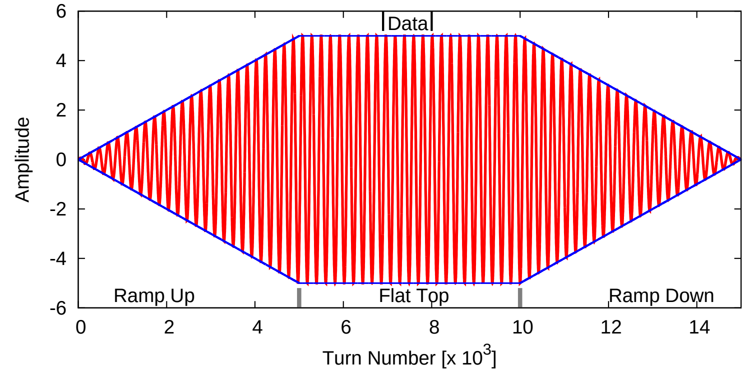
Stable Motion: $|\mathrm{tr}\mathcal{M}_C| \le 2$

How Does One Measure ??

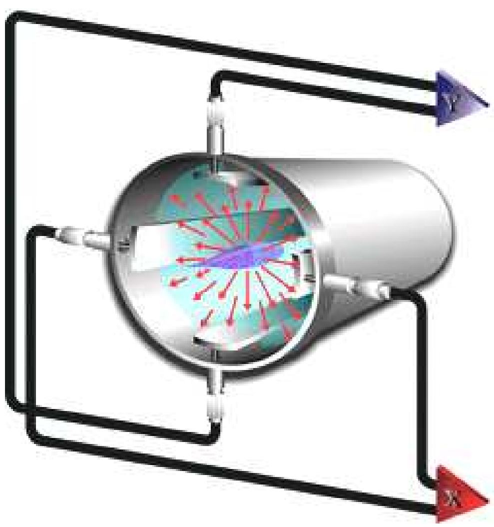
Transverse Kickers



AC Dipoles



Beam Position Monitors



- Avg. Closed orbit
- Turn-by-turn data

$$x \approx \frac{w}{2}\left[\frac{U_+ - U_-}{U_+ + U_-}\right]$$

Linear Optics

$\beta_{x,y}$ & $\psi_{x,y}$

Physical Base Decomposition

Turn-by-turn BPM data matrix can be decomposed:

$$B = \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 & \cdots \\ b_1^2 & \ddots & & \\ b_1^3 & & & \\ \vdots & & & \end{pmatrix}_{n\times m} = \underbrace{\begin{bmatrix} q_1^1 & \cdots & q_d^1 \\ \vdots & \ddots & \\ \vdots & & \\ q_1^t & & \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} f_1^1 & \cdots & f_d^1 \\ \vdots & \ddots & \\ f_1^d & & \end{bmatrix}^T}_{F^T} + N$$

$d \rightarrow$ rank of the matrix

$[q_1 \ldots q_d] \rightarrow$ temporal series, $[f_1 \ldots f_d] \rightarrow$ spatial series

Singular Value Decomposition:

$$\boxed{B = U\Sigma V^T = \sum_i \sigma_i u_i v_i^T}$$

Σ - Singular Values

U - Temporal Vector

V - Spatial Vector

Transverse Motion - 1D (J. Irwin et al.)

Assume only betatron motion (no coupling):

$$x(m) \;=\; \sqrt{2J\beta_m}\,\cos(\phi_t + \psi_m)$$

$$B = U\Sigma V^T$$

$$\begin{bmatrix} u_1^+ & u_1^- & \cdots \\ \vdots & \vdots & \\ \vdots & \vdots & \\ u_t^+ & u_t^- & \cdots \end{bmatrix} \begin{bmatrix} \sigma_+ & 0 & \cdots & \cdots \\ 0 & \sigma_- & & \\ \vdots & & \ddots & \\ \vdots & & & \sigma_m \end{bmatrix} \begin{bmatrix} v_1^+ & v_1^- & \cdots \\ \vdots & \vdots & \\ \vdots & \vdots & \\ v_m^+ & v_m^- & \cdots \end{bmatrix}^T$$

$$u_+ \;=\; \sqrt{\frac{2J_t}{T\langle J\rangle}}\cos\left(\phi_t - \phi_0\right)$$

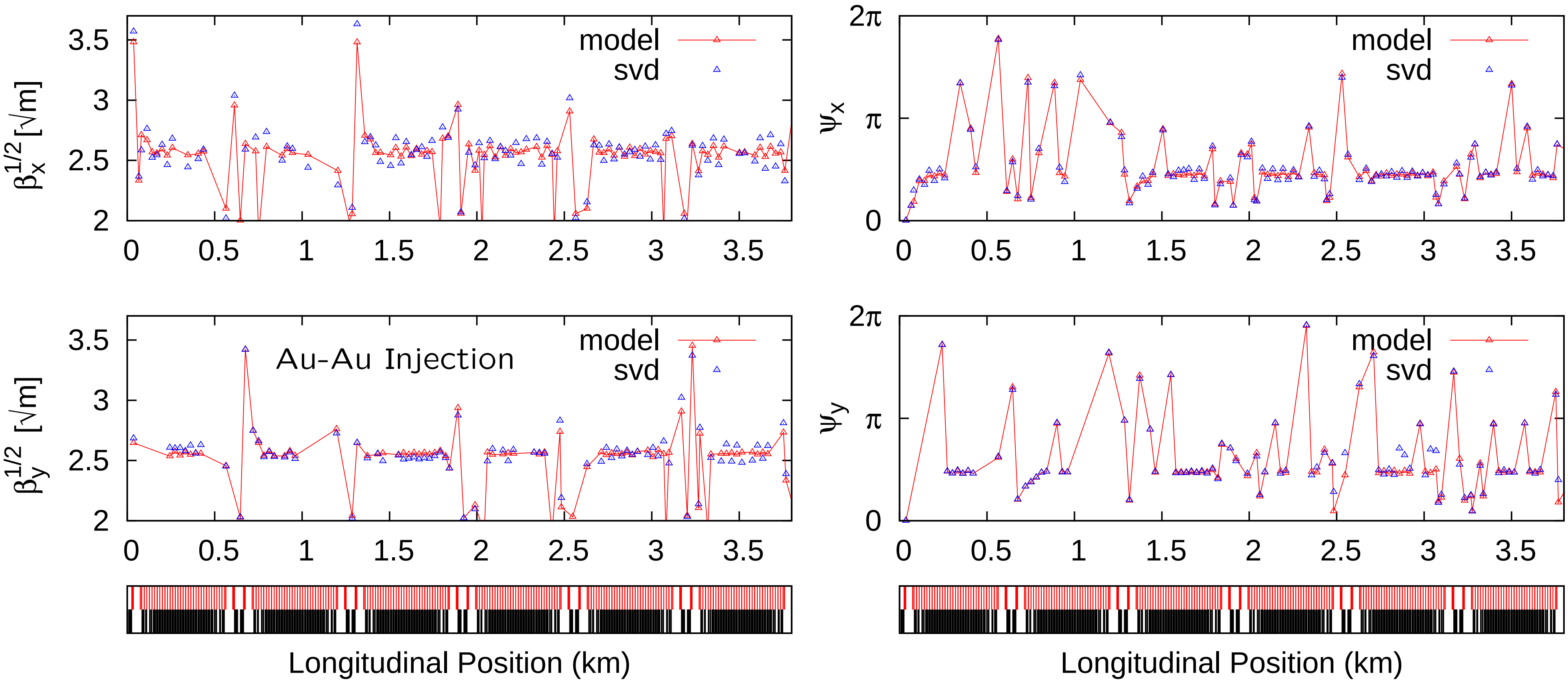
$$u_- \;=\; -\sqrt{\frac{2J_t}{T\langle J\rangle}}\sin\left(\phi_t - \phi_0\right)$$

$$v_+ \;=\; \frac{1}{\sqrt{\lambda_+}}\left[\sqrt{\langle J\rangle\beta_m}\cos\left(\phi_0 + \psi_m\right)\right]$$

$$v_- \;=\; \frac{1}{\sqrt{\lambda_-}}\left[\sqrt{\langle J\rangle\beta_m}\sin\left(\phi_0 + \psi_m\right)\right]$$

RHIC Optics Measurements

$$\psi = \tan^{-1}\left(\frac{\sigma_- v_-}{\sigma_+ v_+}\right); \qquad \beta = \langle J \rangle^{-1}(\sigma_+^2 v_+^2 + \sigma_-^2 v_-^2)$$



*** Beta-beating evident (10-15 %)

Linear Coupling (x-y)

Linear Betatron Coupling

Coupled Oscillators:

$$\begin{aligned} x'' + Q_x^2 x &= -ky \\ y'' + Q_y^2 y &= -kx \end{aligned}$$

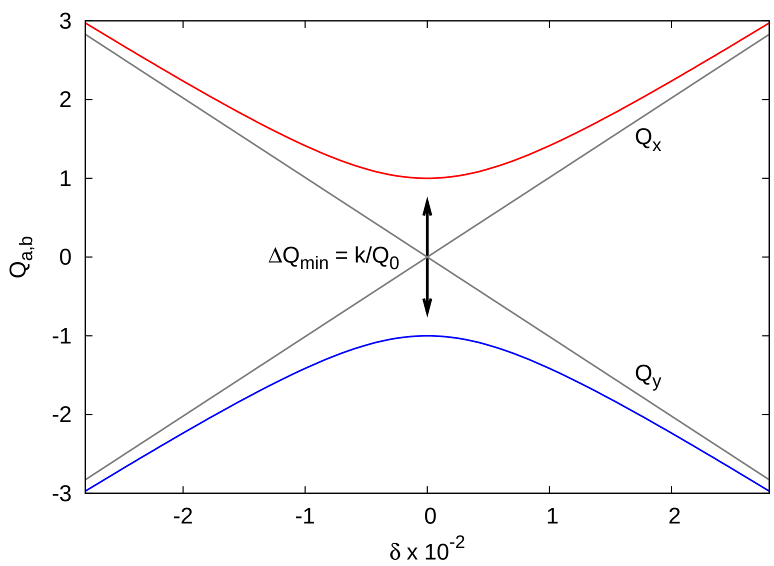
0-mode: $a = \frac{x+y}{2}$, π-mode: $b = \frac{x-y}{2}$

Resonance Condition:

$$Q_x \pm Q_y = n$$

Tune Split:

$$Q_a^2 - Q_b^2 = 2k$$



Why De-Couple:

- Accurate Tune Control
- $\epsilon_x \longrightarrow \epsilon_y$ exchange
- Reduction of dynamic aperture
- Machine ramping, polarization...

Coupling Formalisms & Equivalence

Hamiltonian Formalism

$$
\begin{aligned}
\hat{x} - i\hat{p}_x^h &= \sqrt{2I_x}e^{i\psi_x} - 2if_{1001}\sqrt{2I_y}e^{i\psi_y} \\
&\quad -2if_{1010}\sqrt{2I_y}e^{-i\psi_y}, \\
\hat{y} - i\hat{p}_y^h &= \sqrt{2I_y}e^{i\psi_y} - 2if_{1001}^*\sqrt{2I_x}e^{i\psi_x} \\
&\quad -2if_{1010}\sqrt{2I_x}e^{-i\psi_x}\ , \\
f(s)_{\substack{1001\\1010}} &= -\frac{1}{4(1-e^{2\pi i(Q_x \mp Q_y)})} \\
&\quad \times \sum_l k_l \sqrt{\beta_x^l \beta_y^l} e^{i(\Delta\phi_x^{sl} \mp \Delta\phi_y^{sl})}
\end{aligned}
$$

Matrix Formalism

$$
\mathbf{T} = \begin{pmatrix} \mathbf{M} & \mathbf{m} \\ \mathbf{n} & \mathbf{N} \end{pmatrix} = \mathbf{VUV^{-1}}
$$

$$
\mathbf{U} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} \gamma\mathbf{I} & \mathbf{C} \\ -\mathbf{C}^+ & \gamma\mathbf{I} \end{pmatrix}
$$

$$
|\mathbf{C}| + \gamma^2 = 1
$$

$$
\begin{pmatrix} \hat{x} \\ \hat{p}_x \\ \hat{y} \\ \hat{p}_y \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \overline{C}_{11} & \overline{C}_{12} \\ 0 & \gamma & \overline{C}_{21} & \overline{C}_{22} \\ -\overline{C}_{22} & \overline{C}_{12} & \gamma & 0 \\ \overline{C}_{21} & -\overline{C}_{11} & 0 & \gamma \end{pmatrix} \begin{pmatrix} A_x \cos\psi_x \\ A_x \sin\psi_x \\ A_y \cos\psi_y \\ A_y \sin\psi_y \end{pmatrix}
$$

$$
\begin{aligned}
f_{\substack{1001\\1010}} &= \frac{1}{4\gamma}(\pm\overline{C}_{12} - \overline{C}_{21} + i\overline{C}_{11} \pm i\overline{C}_{22}) \\
\frac{|\overline{C}|}{4\gamma^2} &= |f_{1001}|^2 - |f_{1010}|^2 \\
\Delta Q_{min} &\approx \frac{2\gamma}{\pi}\left(\frac{\cos\nu_x - \cos\nu_y}{\sin\nu_x + \sin\nu_y}\right)\sqrt{|\overline{C}|}
\end{aligned}
$$

SVD & $|\overline{C}|$ Matrix

$$x \;=\; \sqrt{2J_a\beta_a}\gamma\cos(\phi_a+\psi_a) \;+\; \sqrt{2J_b\beta_a}c_b\cos(\phi_b+\psi_b+\Delta\psi_b)$$

$$\gamma=\sqrt{1-|\overline{C}|}, \qquad c_b=\sqrt{\overline{C}_{11}^2+\overline{C}_{12}^2}, \qquad \Delta\psi_b=tan^{-1}(\overline{C}_{12}/\overline{C}_{11})$$

$$\mathbf{B}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{T}}$$ NOT EIGENMODES

Rotated SVD Matrix:

$$O^T\Sigma V^T=\begin{pmatrix} \cdots & \sqrt{\bar{J}_a\beta_a}\gamma\cos(\psi_a-\psi_a^0) & \cdots \\ \cdots & \sqrt{\bar{J}_a\beta_a}\gamma\sin(\psi_a-\psi_a^0) & \cdots \\ \cdots & \sqrt{\bar{J}_b\beta_a}c_b\cos(\psi_b+\Delta\psi_b-\tilde{\psi}_b^0) & \cdots \\ \cdots & \sqrt{\bar{J}_b\beta_a}c_b\sin(\psi_b+\Delta\psi_b-\tilde{\psi}_b^0) & \cdots \end{pmatrix}$$

$$\frac{\bar{C}_{12}}{\gamma}=\mathrm{sgn}(\sin\Delta\psi_a)\sqrt{\frac{\tilde{A}_a\tilde{A}_b}{A_aA_b}\sin\Delta\psi_a\sin\Delta\psi_b}$$

$$\frac{\overline{C}_{11}}{\gamma}=\frac{\overline{C}_{12}}{\gamma}\cot\Delta\psi_b \quad ; \quad \frac{\overline{C}_{22}}{\gamma}=-\frac{\overline{C}_{12}}{\gamma}\cot\Delta\psi_a$$

$\overline{C}$ Matrix

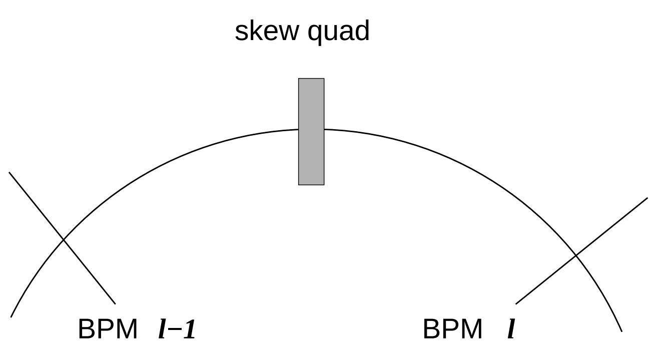
Propagation of $\overline{C}$ Matrix **w/o** skew quads:

$$\overline{C}_{\mathbf{2}} = \mathbf{R}_x(\phi_x)\overline{C}_{\mathbf{1}}\mathbf{R}_y^{-1}(\phi_y)$$

$$\overline{C}_{21}^{(1)} = \frac{-\overline{C}_{11}^{(1)} \cos\phi_a \sin\phi_b + \overline{C}_{12}^{(1)} \cos\phi_a \sin\phi_b + \overline{C}_{22}^{(1)} \sin\phi_a \cos\phi_b - \overline{C}_{12}^{(2)}}{(\sin\phi_a \sin\phi_b)}$$

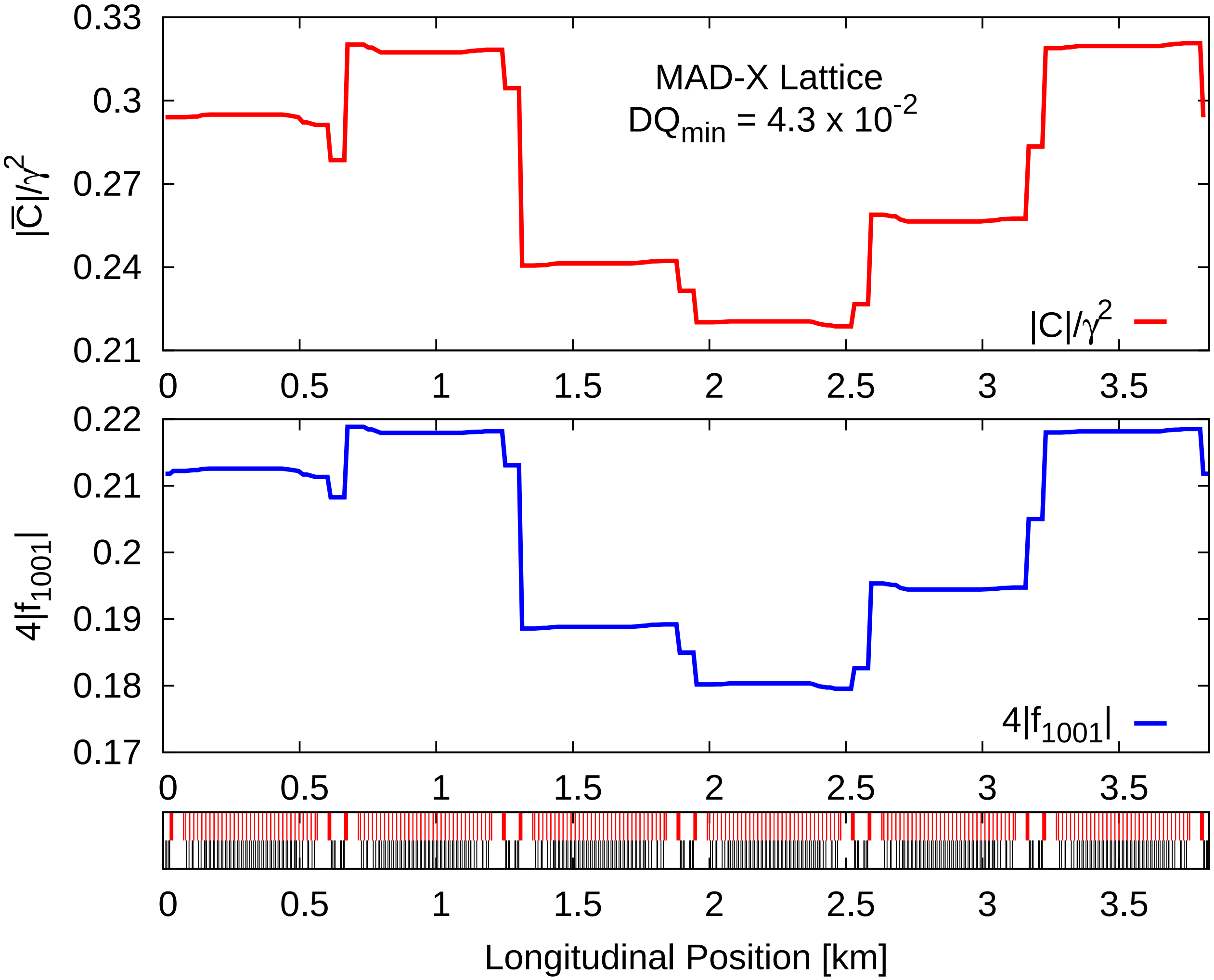
The $|\overline{C}|$ is discontinuous thro' Skew Quad

$$\overline{C}_{\mathbf{2}} = \overline{C}_{\mathbf{1}} - \overline{k}$$



$$\bar{k} = -\frac{|C^{(2)}| - |C^{(1)}|}{C_{12}^{skew}}$$

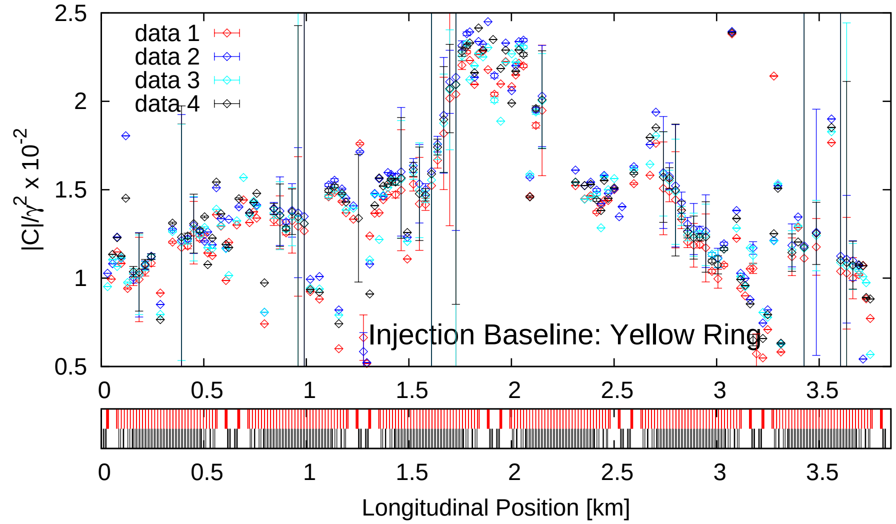
RHIC Lattice With Skew Errors

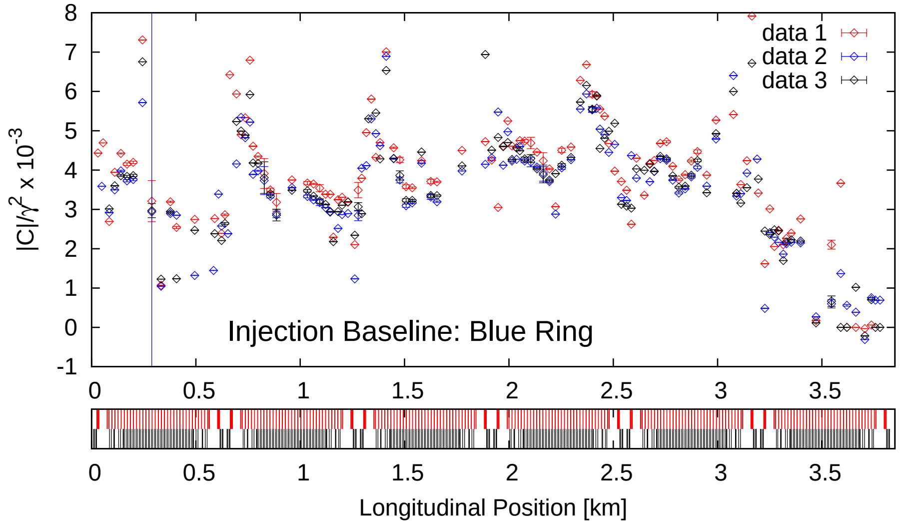


*** Coupling terms derived from single particle tracking

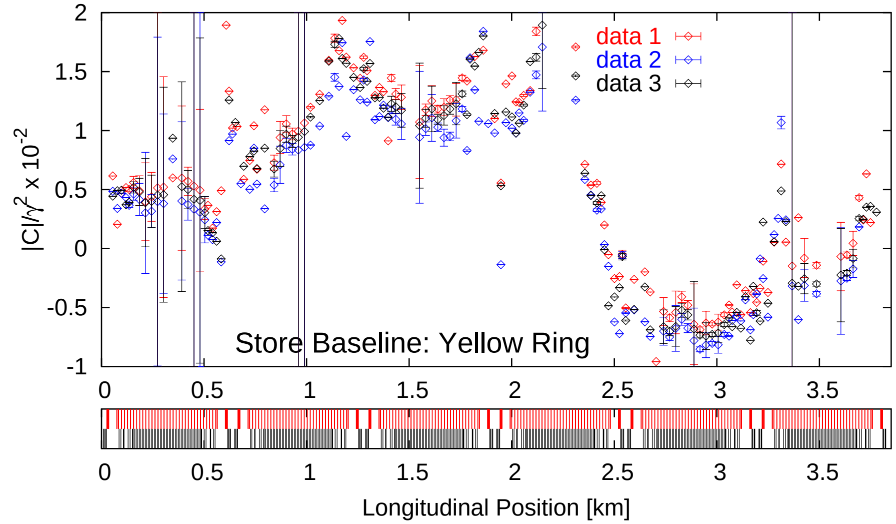
Run 2005: Cu-Cu (AC Dipole Data)

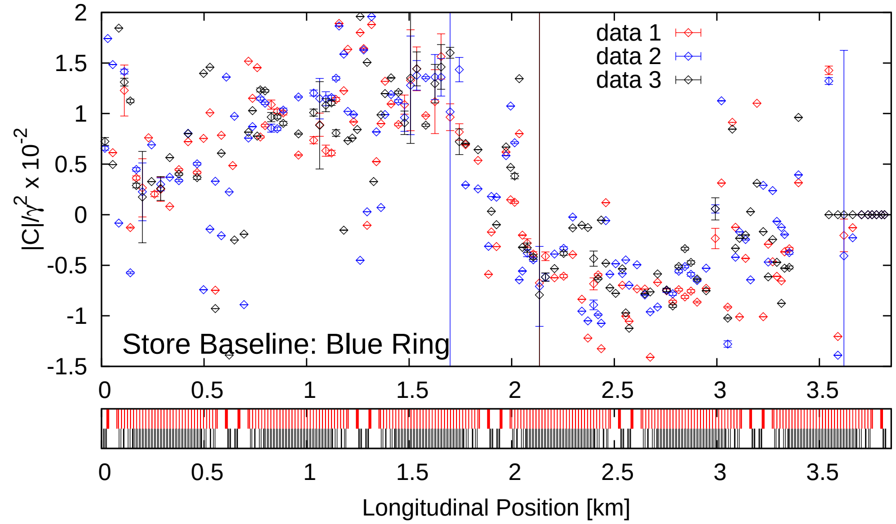
Injection



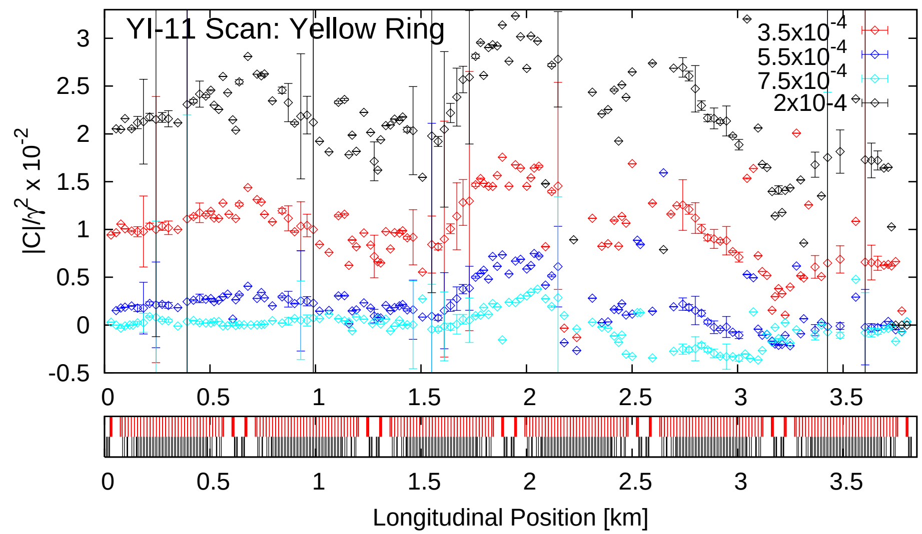


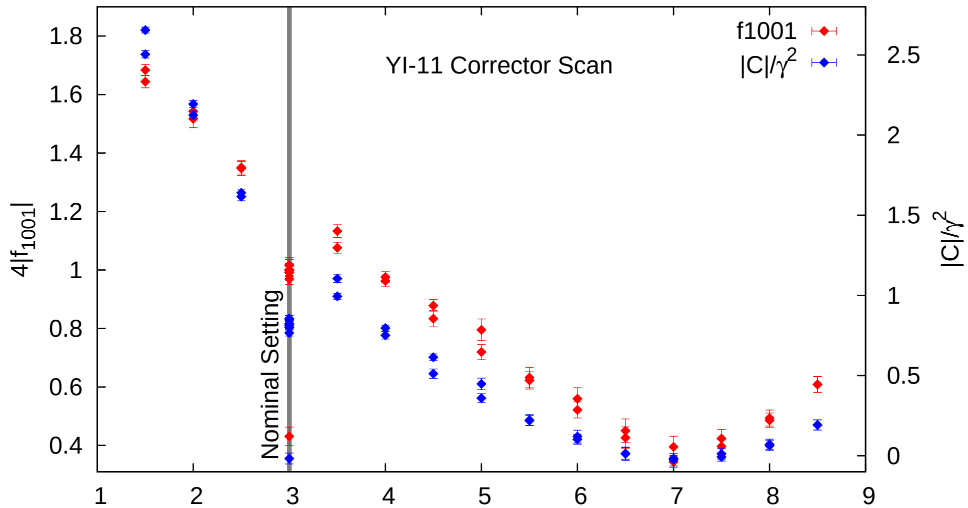
Top Energy





IR Corrector Scan: Injection

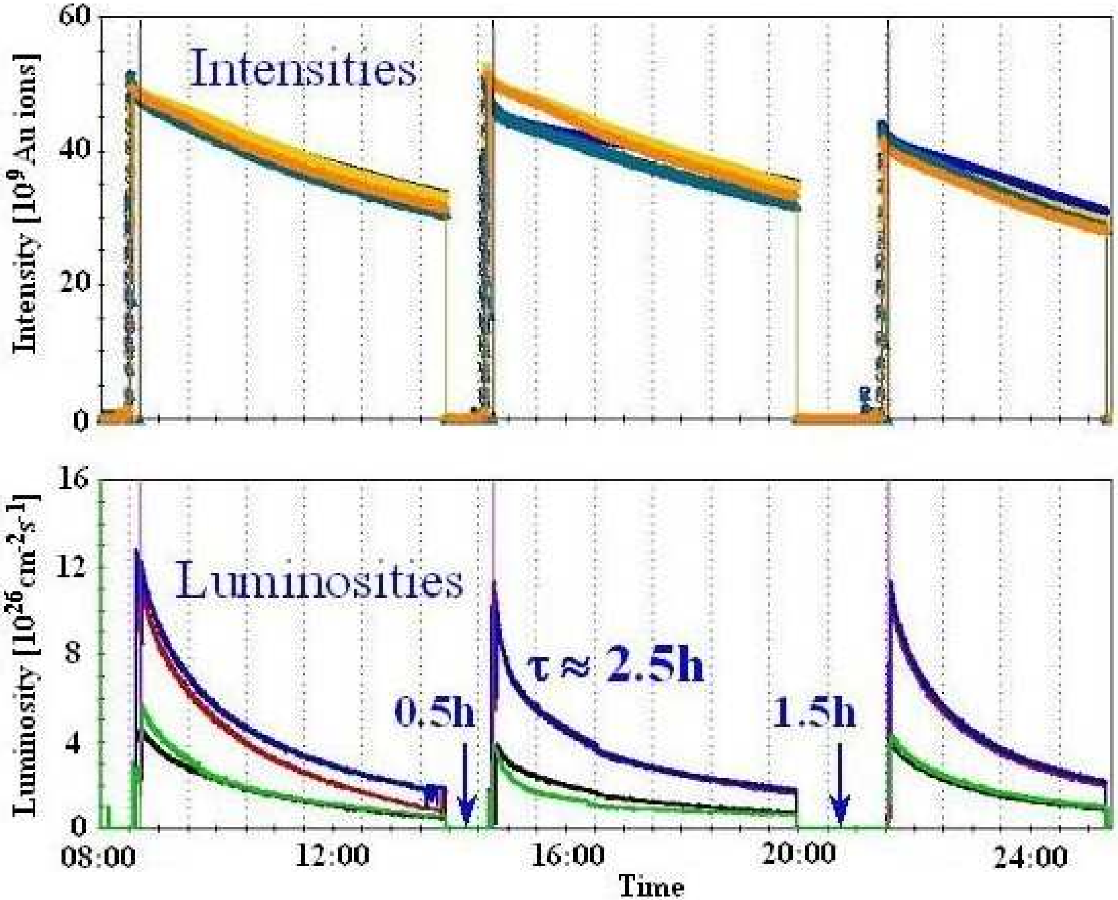




- Possible correction strategy by scanning IR skew correctors
- Minimize both local excursions and average value
- Identify slopes
  - Quadrupole tilts
  - Vertical offsets in sextupoles
  - BPM offsets ??

Superconducting RF Cavities @RHIC

Intra-Beam Scattering (IBS)

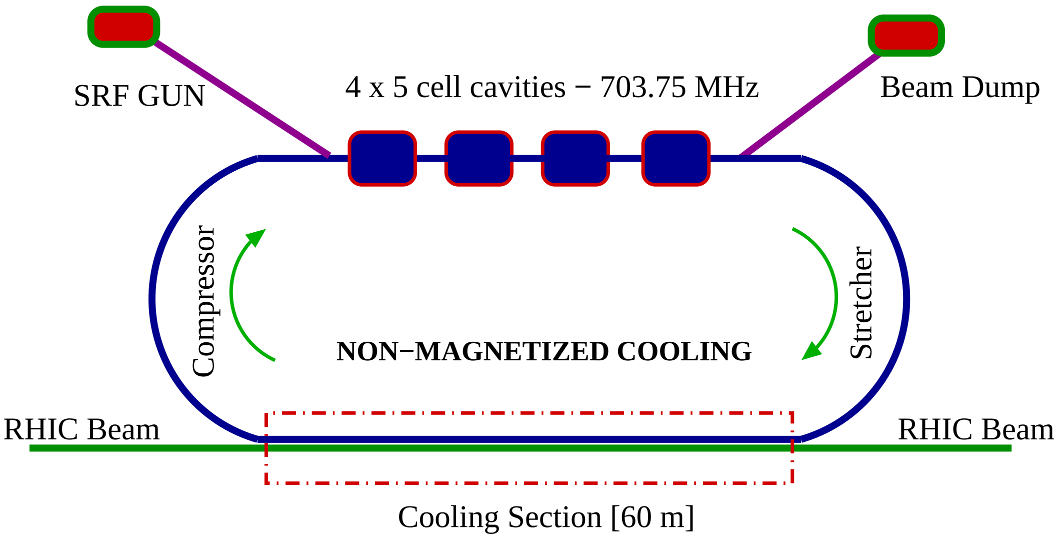


(Courtesy W. Fischer)

Main limitation for luminosity and beam lifetime due to multiple small angle Coulomb scattering

ecooling@RHIC (ca. 2008)

- Cooling Au beams at 100 GeV requires $\sim$ 54 MeV e^-
- Low $\epsilon_{x/y}$, High Current, and High Bunch Charge
- CW e^- beam for continuous cooling
- Replenish e^- every cycle - superconducting energy recovery linac

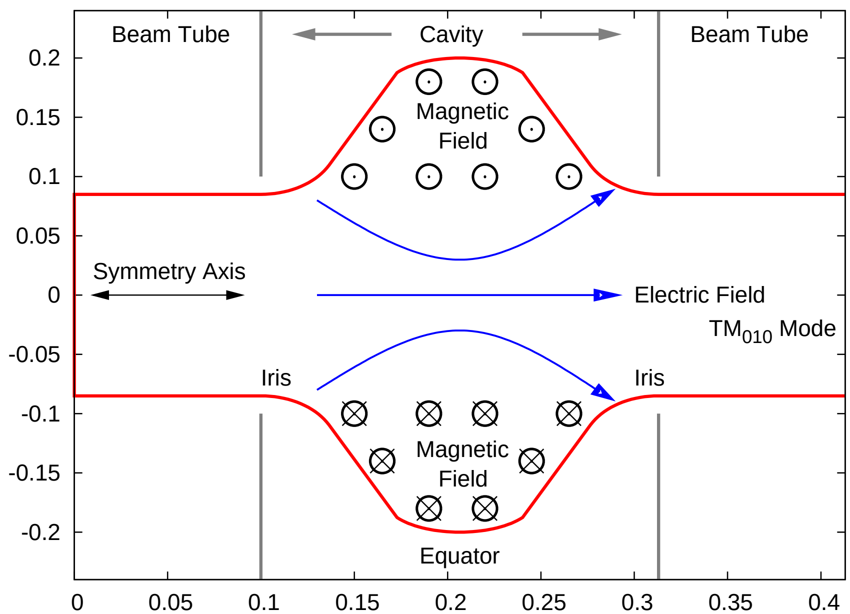


Inj. energy [MeV]	5.2
Max energy [MeV]	20-40
Avg. I [mA]	50-200
Rep. rate [MHz]	9.4-28
Charge/Bunch [nC]	5-20
ϵ_y [mm.mrad]	3
Bunch length [cm]	1.0
Energy recovery	$>$ 99.95 %

RF - 101

Cylindrical Cavity (TM_{010}):

$$E_z = E_0 J_0(\omega_0 r/c) \cos(\omega_0 t)$$
$$H_\phi = -\frac{1}{\mu_0 c} E_0 J_0(\omega_0 r/c) \sin(\omega_0 t)$$
$$\omega = \frac{2.405\ c}{R}$$



** Elliptical shape due to multipacting

Accelerating Voltage:

$$V_{acc} = \left| \int_{z=0}^{z=l} E_z e^{i\omega_0 z/c} dz \right|$$

Power Dissipated:

$$P_d = \frac{1}{2} R_s \int_S |\vec{H}|^2\ ds, \qquad R_s = R_{BCS} + R_{res}$$

Quality Factor & Shunt Imp.:

$$Q_0 = \frac{\omega_0 U(t)}{P_d(t)}, \qquad R_a = \frac{V^2}{P_d}$$

$$\frac{R_a}{Q} = \frac{V^2}{\omega U}$$

Why Superconducting ??

- High average accelerating field - $E_{acc} \sim$ 20-50 MV/m
- High quality factor ($\sim 10^{11}$)

$$Q = \frac{\omega U}{P}$$

- Low power dissipation on surface - CW operation

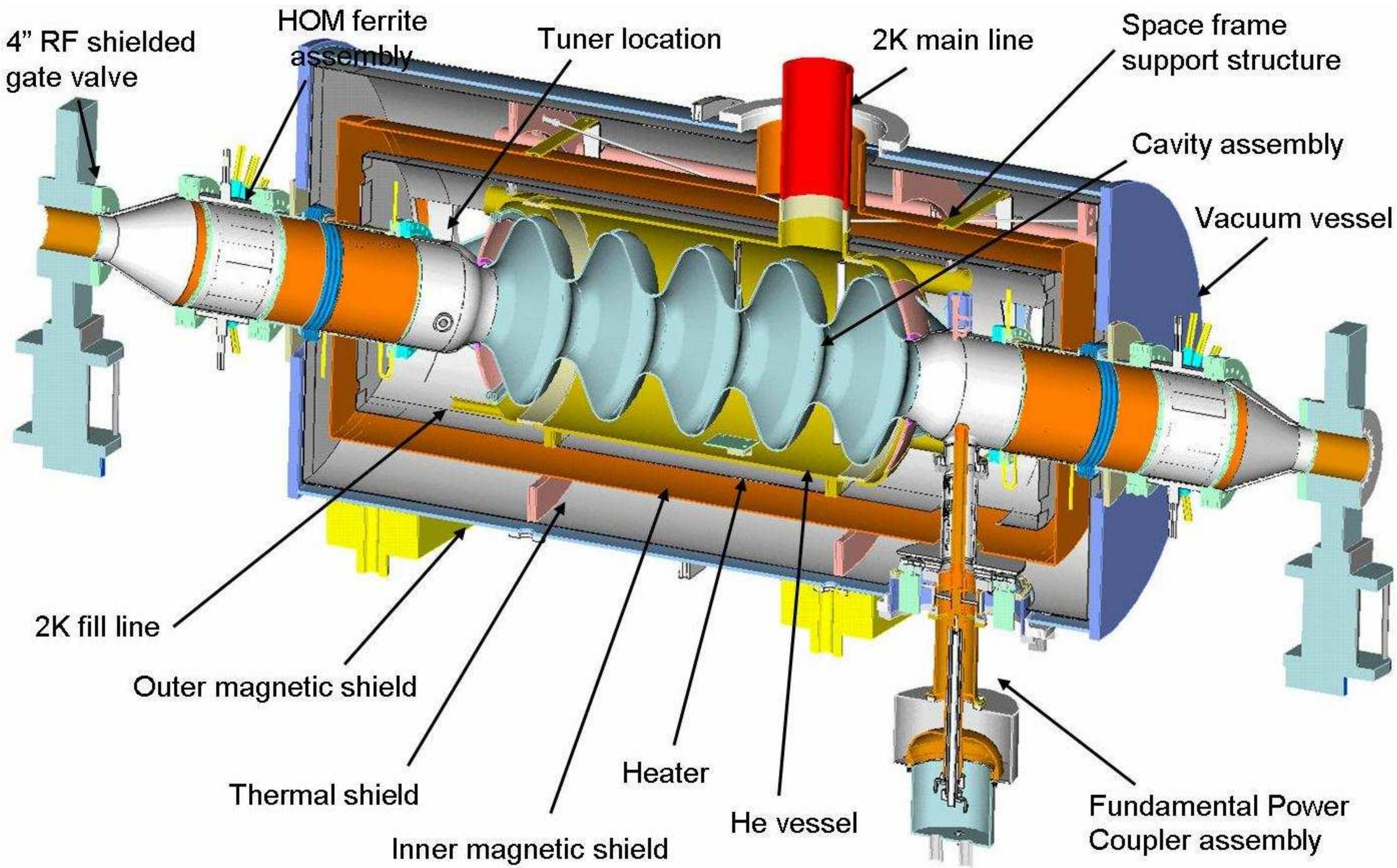
$$\frac{P}{L} = \frac{E_{acc}^2}{\frac{R_a}{Q} Q}$$

- Larger irises $\rightarrow$ smaller $k_{||}$ & $k_{\perp}$

Power comparison for $E_{acc} = 1$MV/m

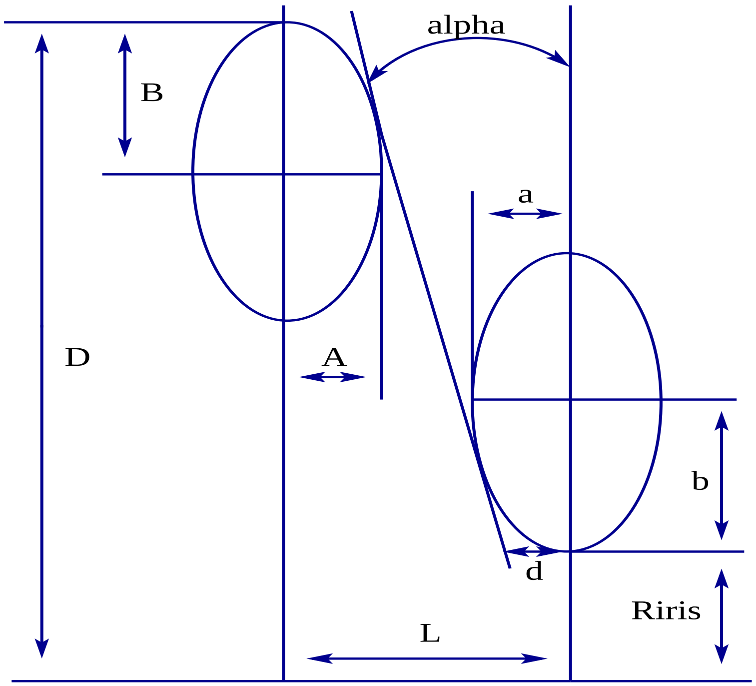
Option	SRF	Normal
Q	2×10^9	2×10^4
$\frac{R}{Q}$	330	900
$\frac{P}{L}$	1.5 W/m	5.6×10^4
AC Power	0.54 kW/m	112 kW/m

Five-Cell SRF Cavity

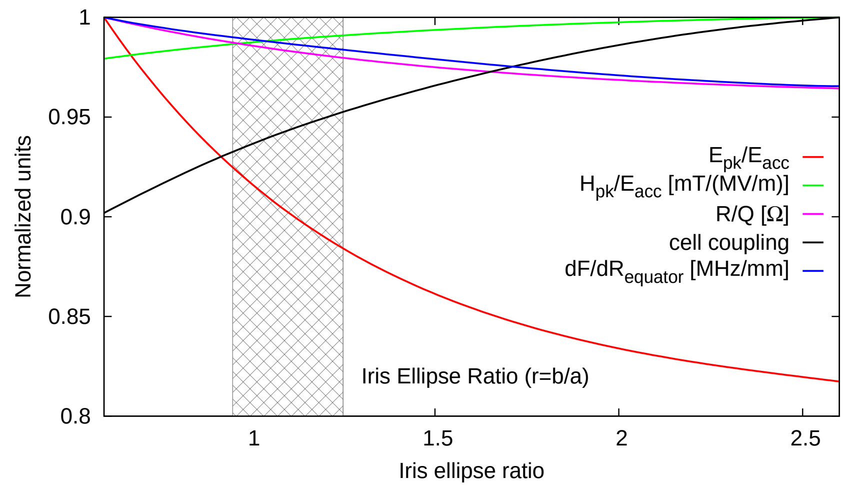


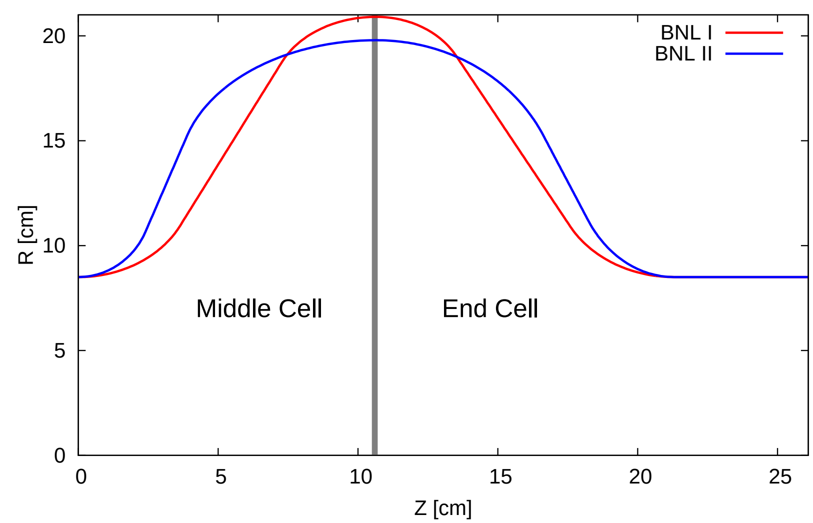
Courtesy AES

Elliptical Cavity Parametrization



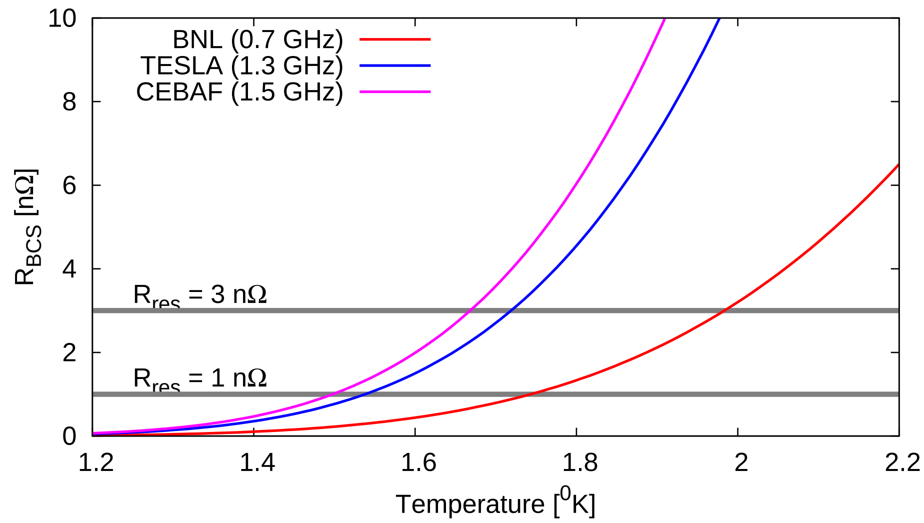
Iris Radius, R_{iris}	8.5 [cm]
Wall Angle, α	25 [deg]
Equatorial Ellipse, $R = \frac{B}{A}$	1.0
Iris Ellipse, $r = \frac{b}{a}$	1.1
Cav. wall to iris plane, d	2.5 [cm]
Half Cell Length, $L = \frac{\lambda\beta}{4}$	10.65 [cm]
$H = D - (R_{iris} + b + B)$	4.195 [cm]
Cavity Beta, $\beta = \frac{v}{c}$	1.0



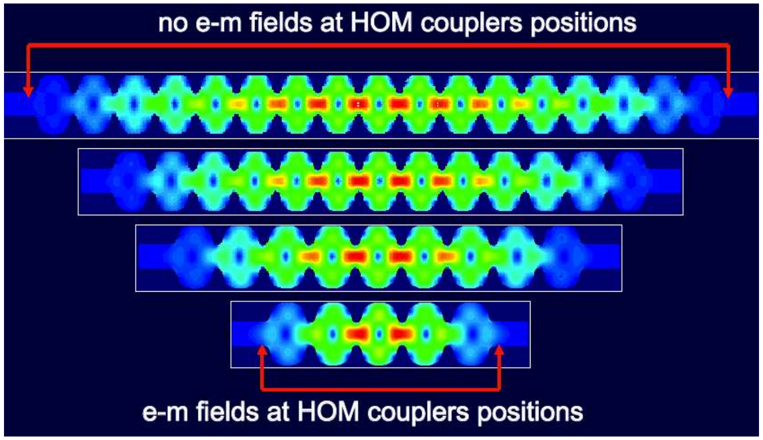


Design Criteria

- Freq: 703.75 MHz
  - 25^{th} harmonic of RHIC
  - Lower $k_{||}$, $k_{\perp}$
  - Power sources
  - Chemical treatment

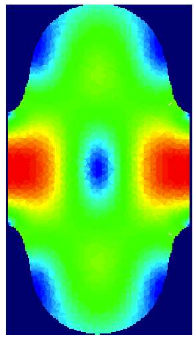


- $P_{cav} \propto \frac{Rs}{(R/Q)G}$ $(\downarrow)$
  - $R_s \propto \omega^2$ $(R_s = R_{BCS} + R_{res})$
  - $\frac{R}{Q}G \propto const.$ $(dim. \propto \omega)$



- Five Cells
  - Field sensitivity factor: $\frac{N^2}{k_{cc}}$
  - Trapped HOM modes

Trapped Modes

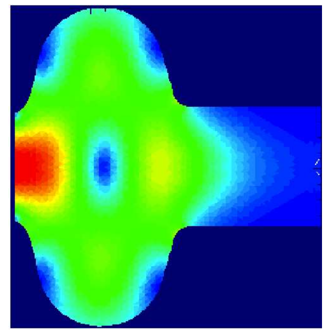


TESLA

$f = 2.4 GHz$

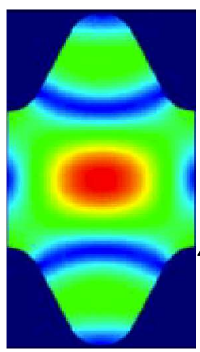
$\Longleftrightarrow$

$\Delta f = 30 MHz$



Middle Cell

End Cell

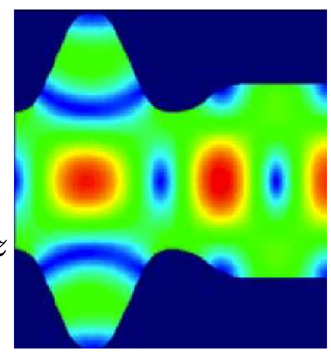


BNL I

$f = 2.4 GHz$

$\Longleftrightarrow$

$\Delta f = 13 MHz$



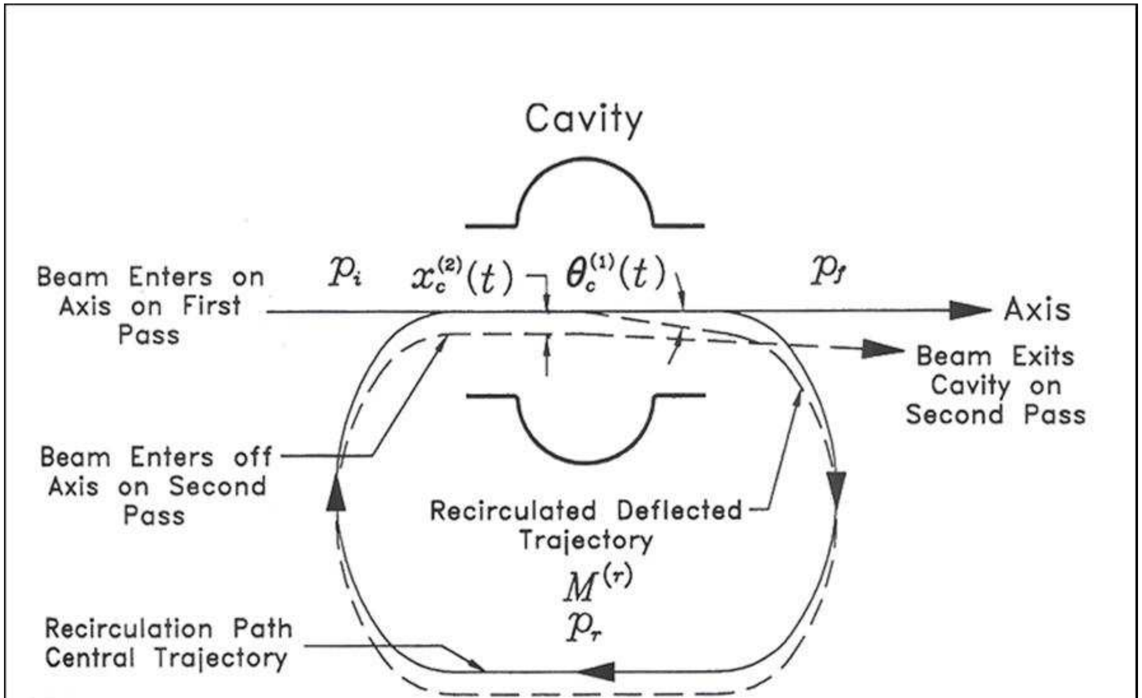
HOM Power & Instabilties

Longitudinal Modes:

$$P_{HOM} = 2k_{||}IQ$$

Transverse Modes:

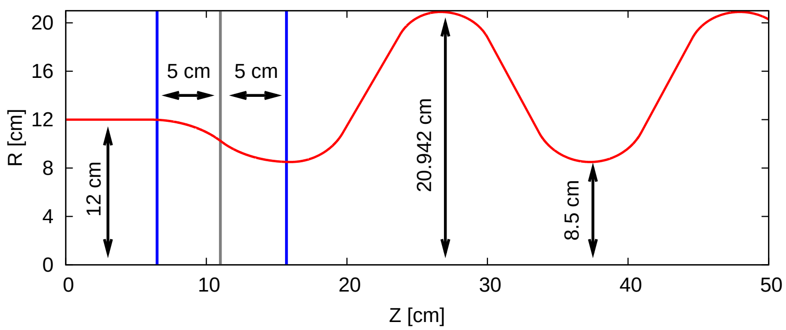
$$I_{thr} = \frac{-2p_r c}{e\left(\frac{R}{Q}\right)Q_e k_m M_{12} sin(\omega_m t_r)}$$

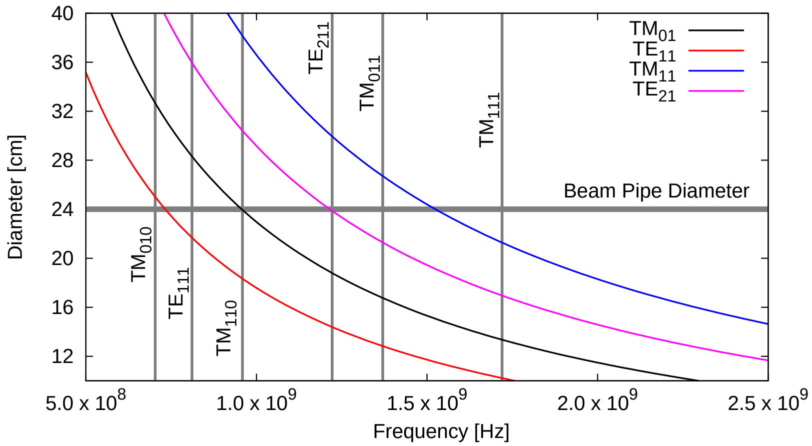


Courtesy G. Krafft

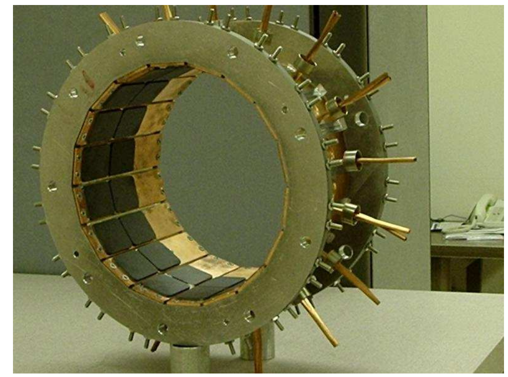
HOM Extraction & Damping

- Enlarged Beam
  - Propagate HOMs to warm section
  - Minimize power coupler kick
- BP Flutes, Loop couplers

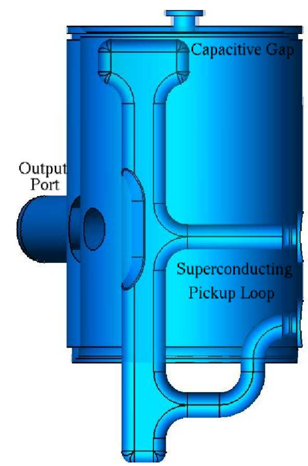




Ferrite Absorbers
Broadband (300 K)



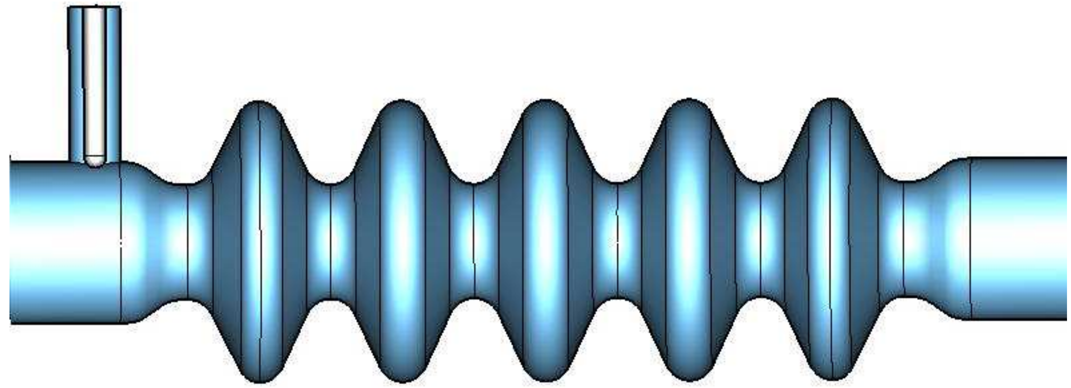
Loop Couplers
Resonant Ciruit (2 K)

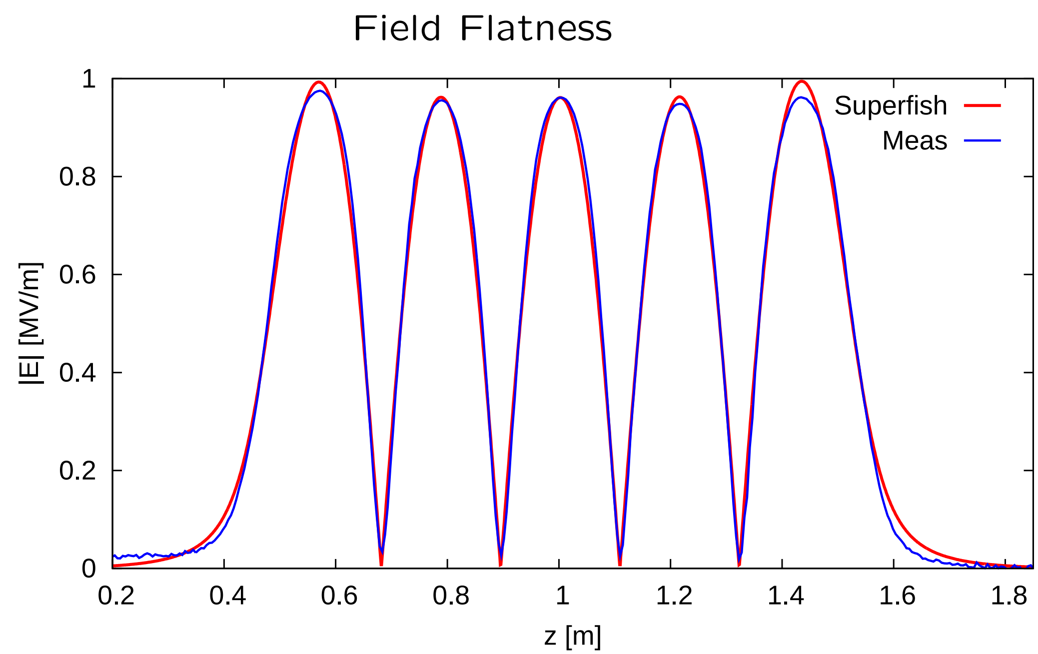


BNL I Cavity

Main Parameters:

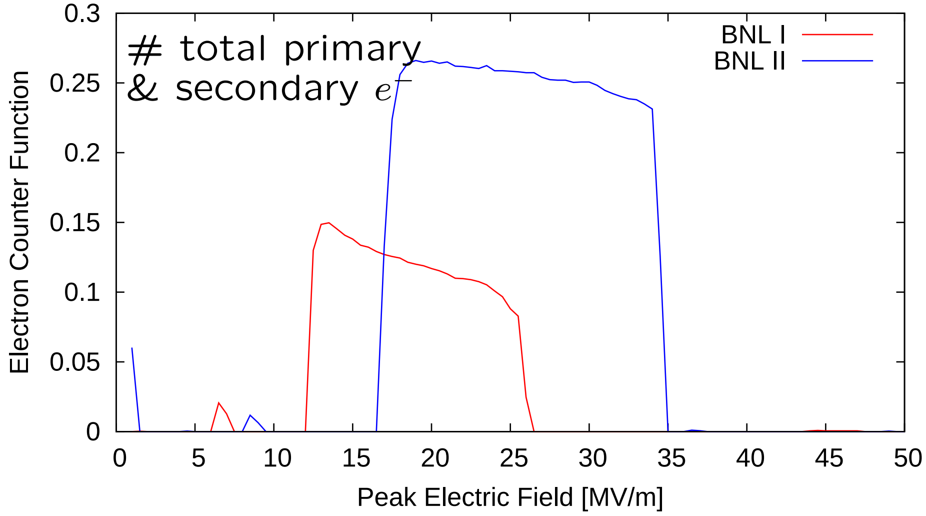
Frequency [MHz]	703.75
RHIC Harmonic	25
Number of cells	5
Active cavity length [m]	1.52
Iris Diameter [cm]	17
Beam Pipe Diameter [cm]	24 [cm]
G (Ω)	225
R/Q [Ω]	403.5
Q BCS @ 2K	4.5×10^{10}
Q_{ext}	3×10^{7}
E_p/E_a	1.97
H_p/E_a $[mT/MV/m]$	5.78
cell to cell coupling	3%
Sensitivity Factor ($\frac{N^2}{\beta}$)	833
Field Flatness	96.5 %
Det. Coeff [Hz/MV/m]	1.2
Low. Mech. Reson. [MHz]	96
$k_{\|\|}$ $(\sigma_z - 1cm)$ [V/pC]	1.1
$k_{\perp}$ $(\sigma_z - 1cm)$ [V/pC/m]	3.1
Power$_{HOM}$ (10-20 nC) [kW]	0.5-2.3

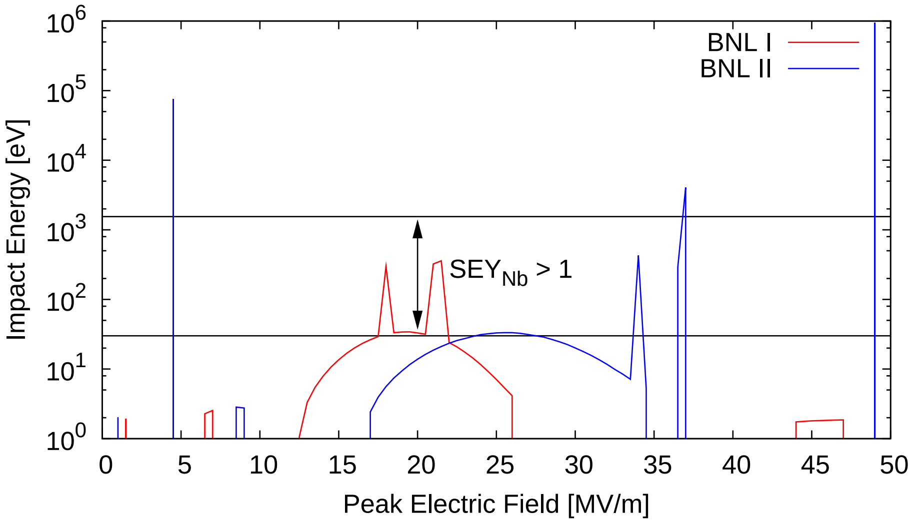


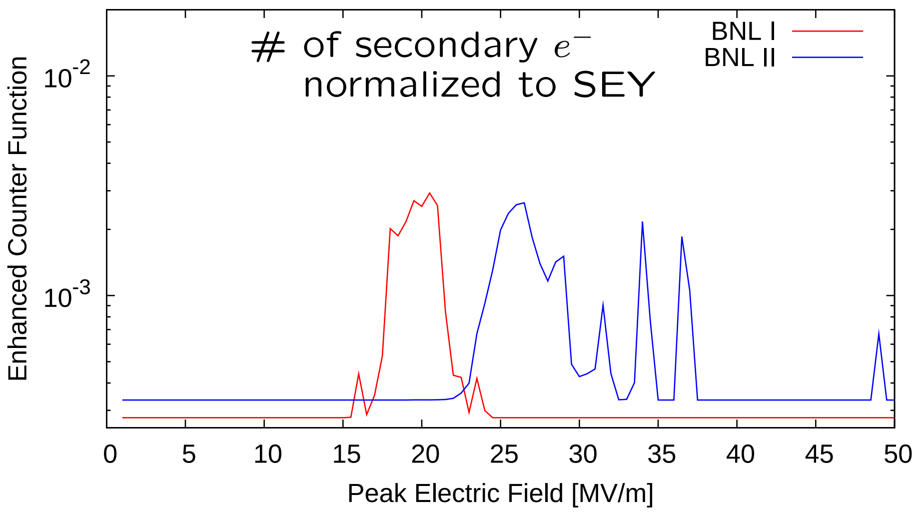


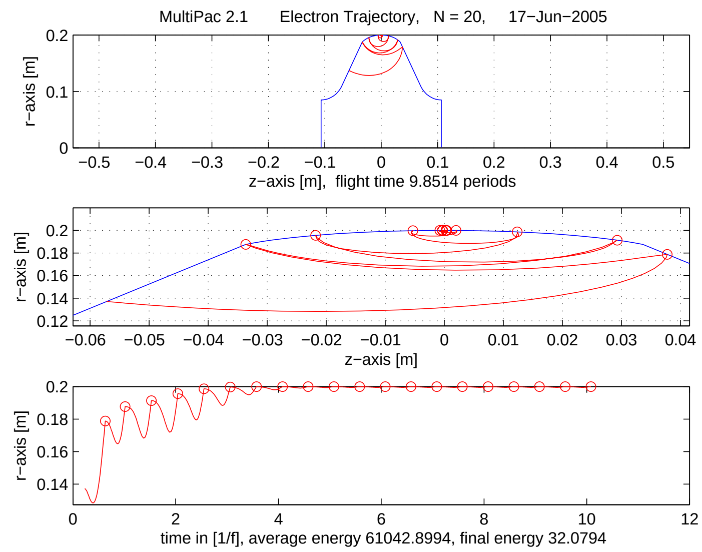
** Field measured using bead-pull technique

Multipacting

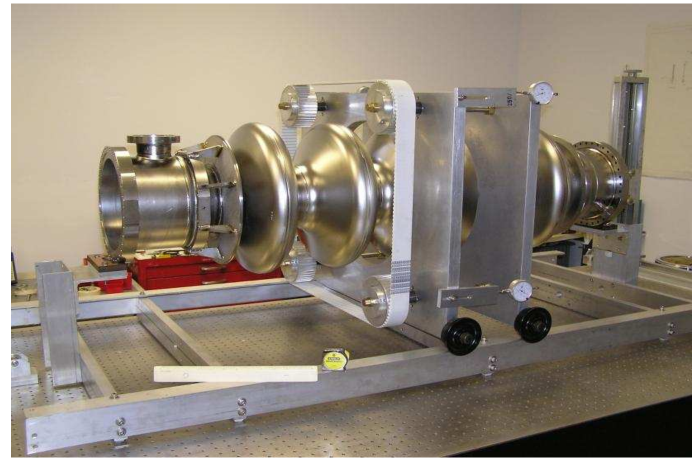






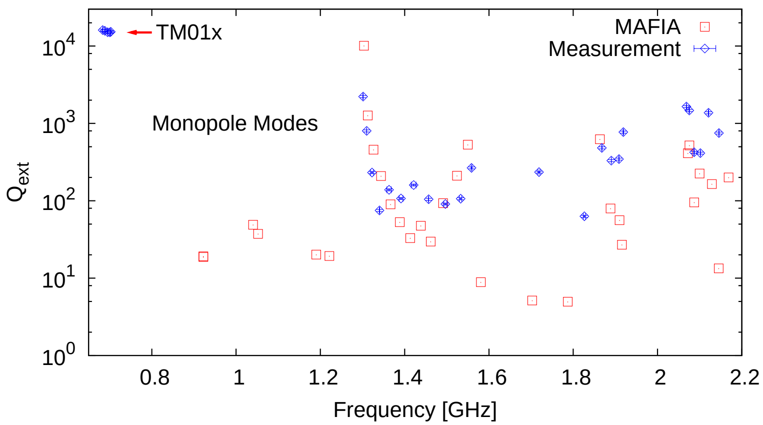


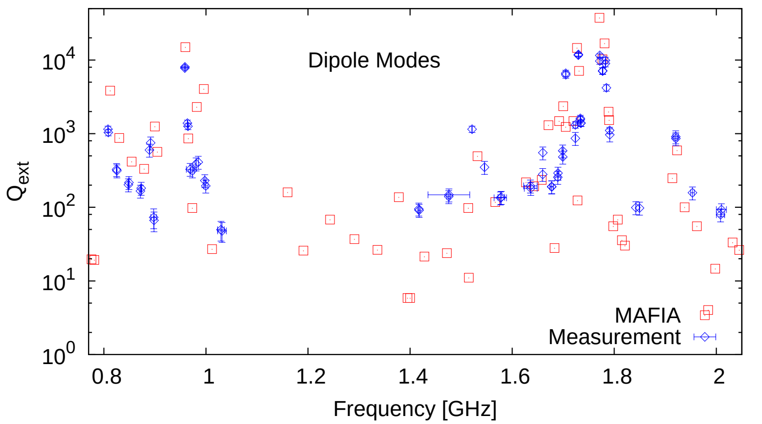
Fabrication, Tuning & Testing



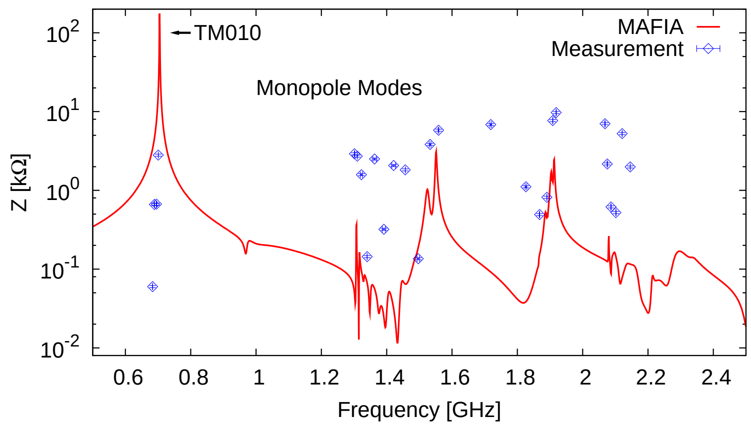
HOMs: Simulation & Measurements

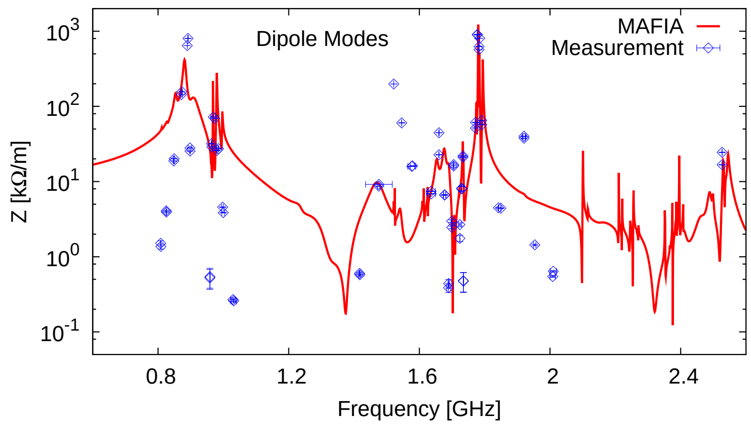
Frequency Domain





Time Domain

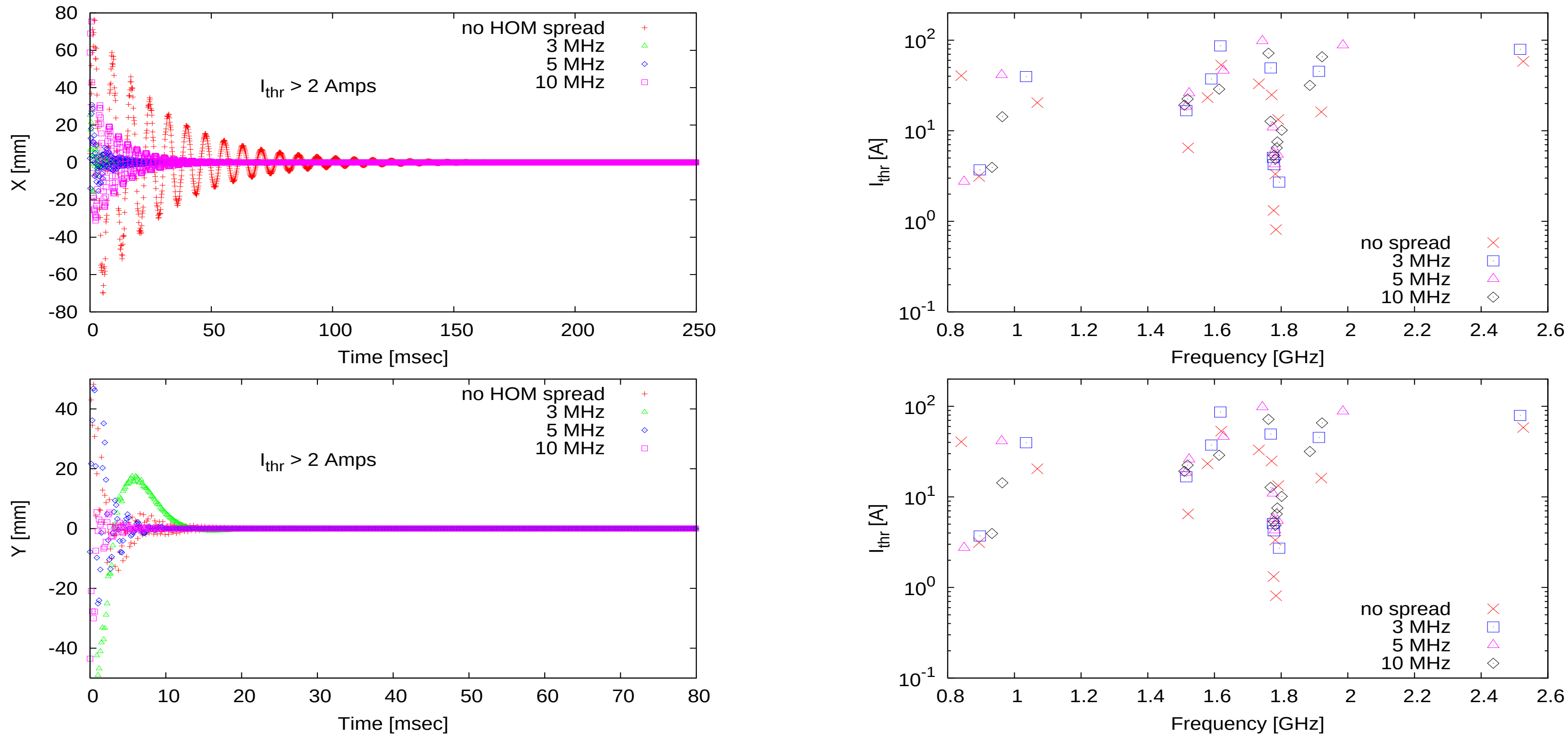




Multibunch Beam BreakUp

Time-Domain

Frequency Domain



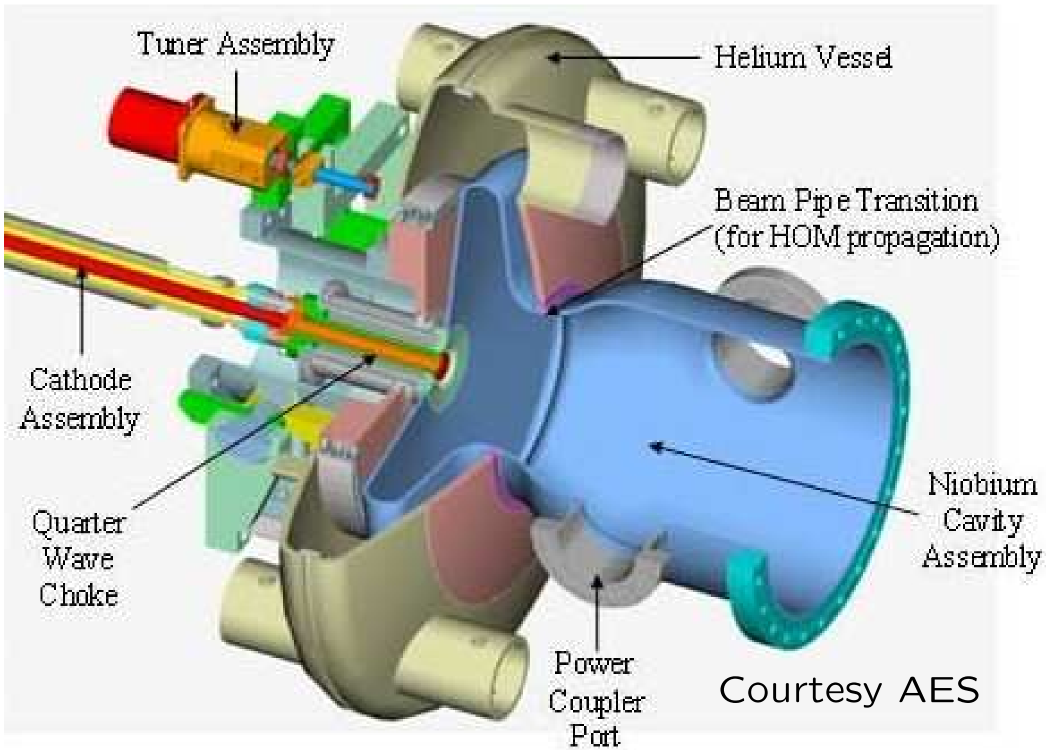
Threshold Current > 2 Amps

BNL eCooling Configuration - 4 Cavities - 54 MeV

(Numerical Codes from JLAB)

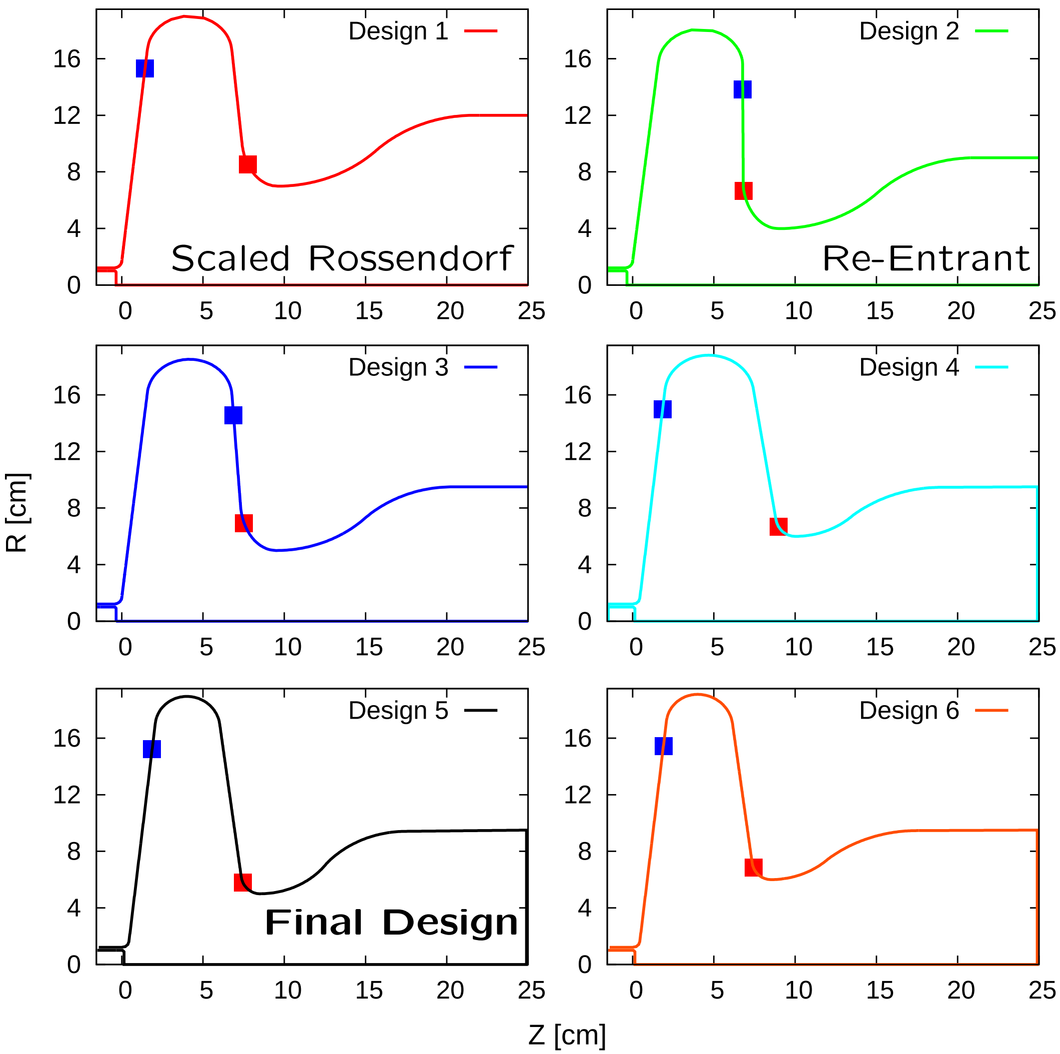
$\frac{1}{2}$ Cell SRF Gun
ERL Prototype

$\frac{1}{2}$ Cell SRF Gun



- Generation of ampere class CW beam
- $\beta < 1$ and varying
- Low $\epsilon_{x/y}$ & $\delta E/E$
- E_z at the cathode
- Strong Coupling $Q_{ext} \sim 10^4$
- Coupler Kicks
- Multipacting
- HOMs & Stability Criteria
- Cathode Issues and Isolation

SRF Gun Design



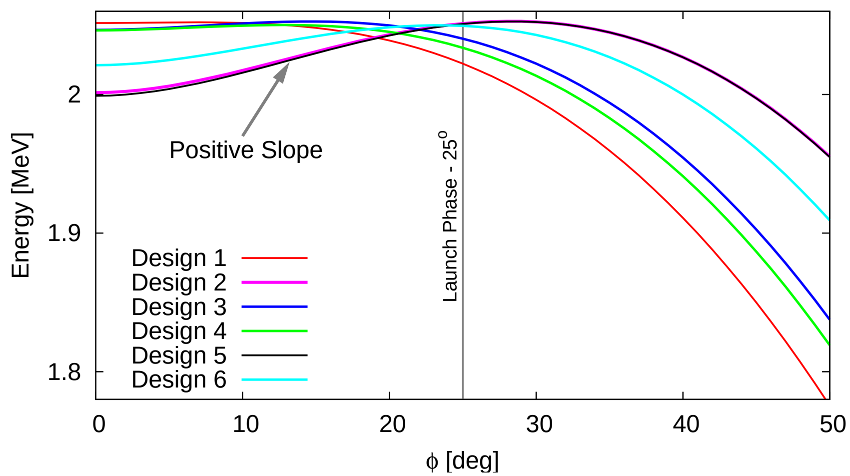
Some Comparisons

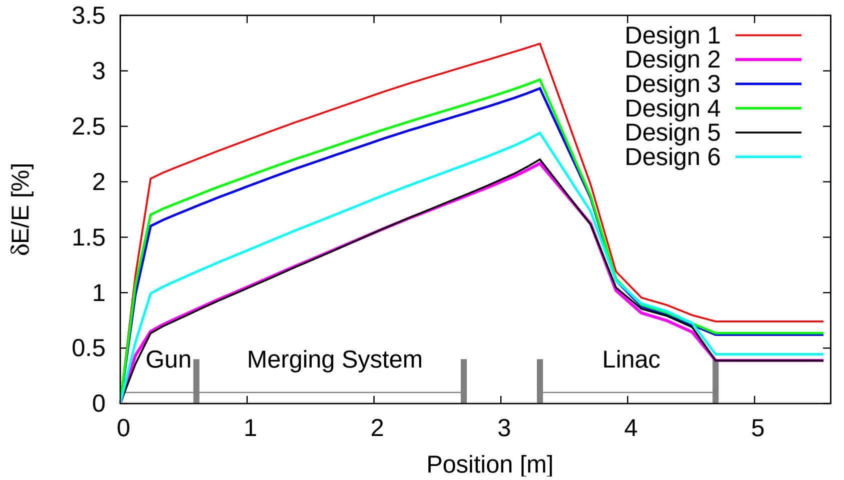
Shape	r/Q [Ω]	E_p/E_a	B_p/E_a $[\frac{mT}{(MV/m)}]$
Design 1	101	1.14	2.73
Design 2	105	1.39	2.97
Design 3	103	1.20	2.81
Design 4	112	1.33	2.69
Design 5	**95**	**1.42**	**2.96**
Design 6	92	1.42	2.87

Design 5	**Right Cell**
Frequency	703.75 MHz
Iris Radius, R_{iris}	5.0 cm
Wall Angle, α	6.5°
Equatorial Ellipse, $R = \frac{B}{A}$	1.1
Iris Ellipse, $r = \frac{b}{a}$	1.2
Cav. wall to iris plane,	1.0 cm
Active cavity Length, L	8.5 cm
Center to equator end	18.95 cm
Avg. Beta, $< \beta = \frac{v}{c} >$	0.587

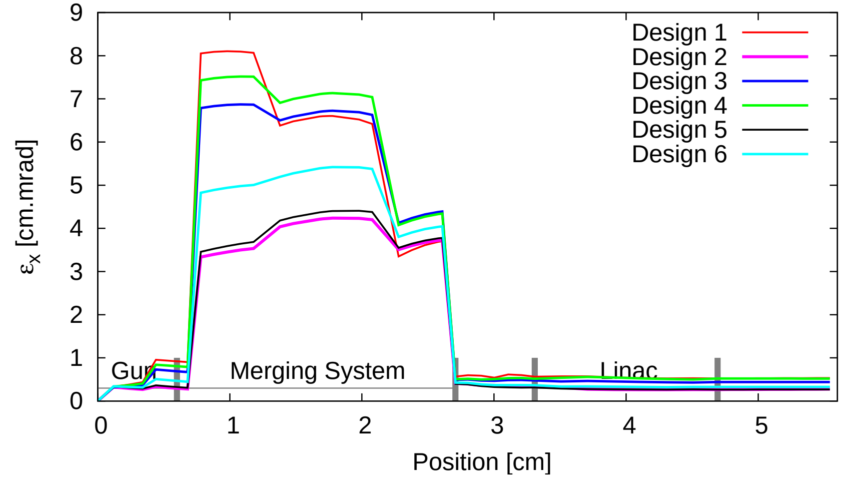
Energy Spread & Emittance (D. Kayran)

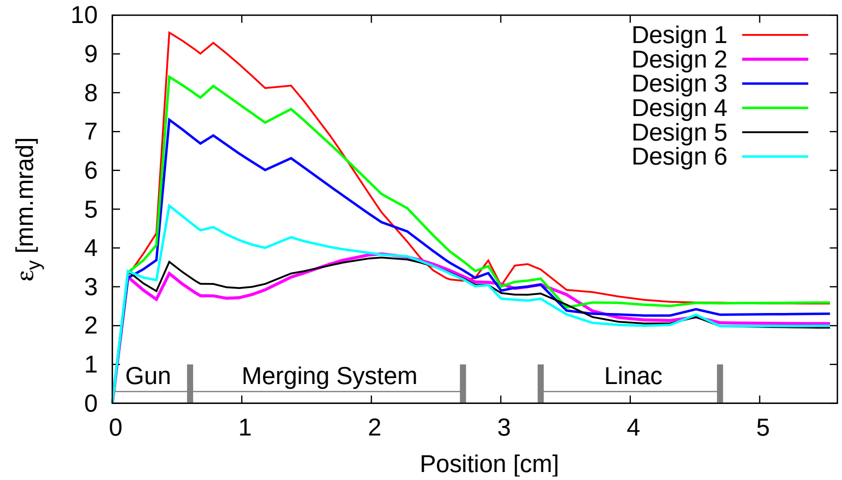
Longitudinal



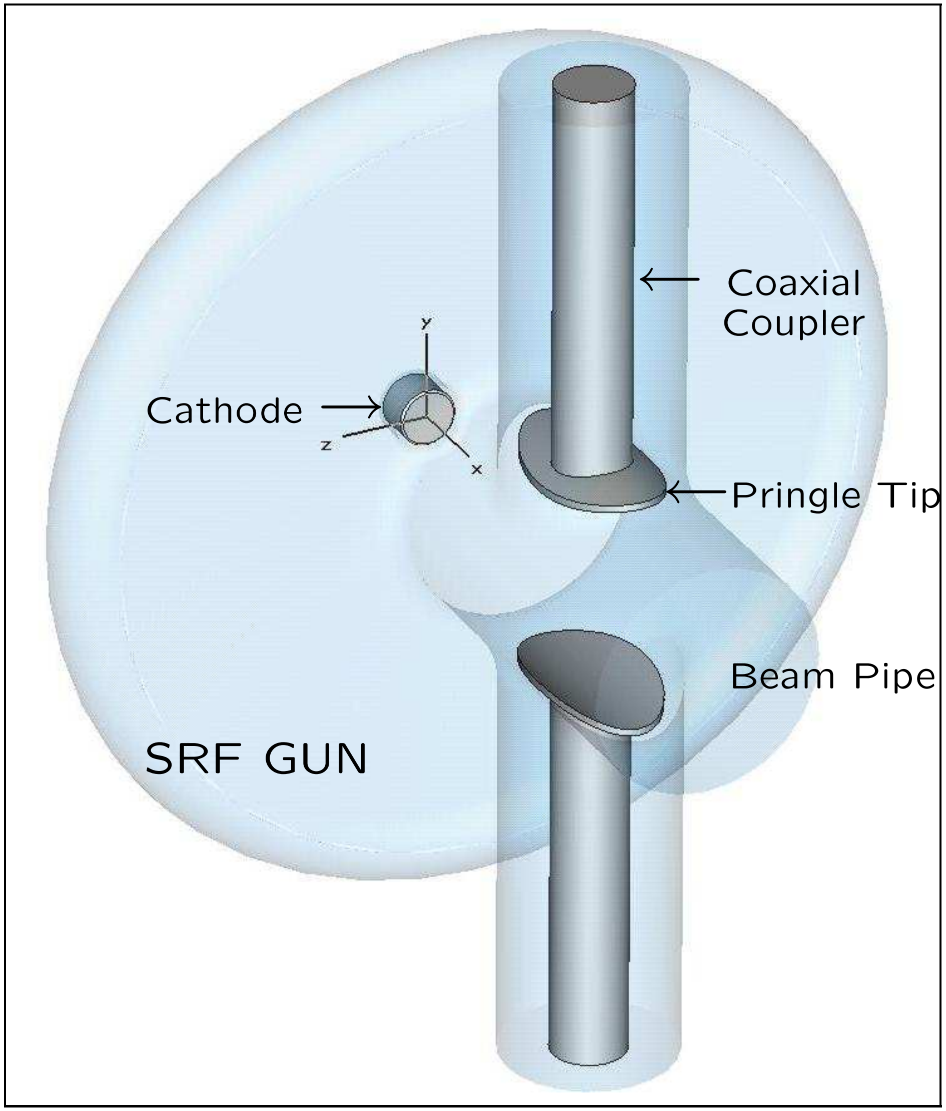


Transverse





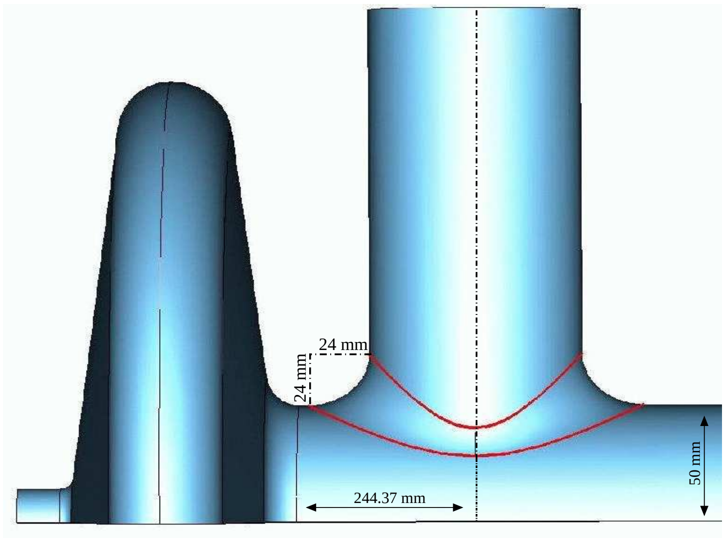
Coupling Fundamental Power



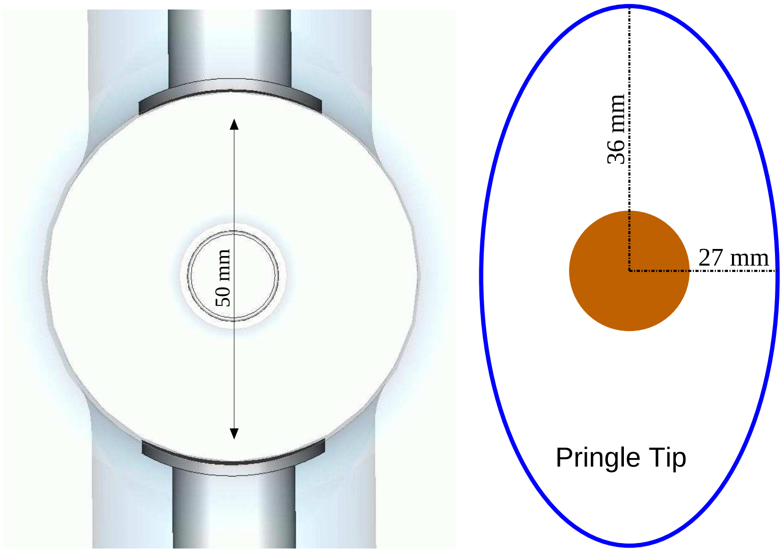
- Couple strongly: $Q_{ext} \sim 3 - 5 \times 10^4$
- Coupler kicks
- Reduce wakefields
- Engineering, alignments, etc..

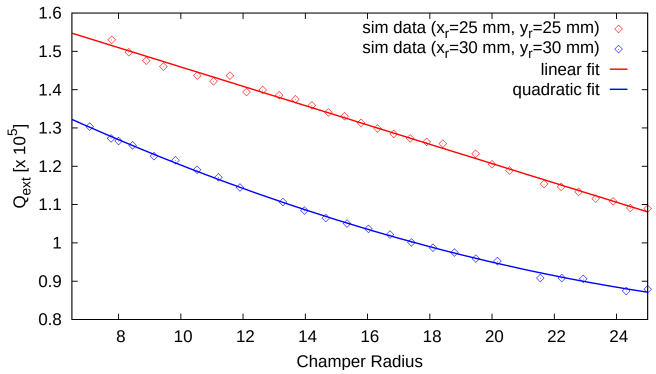
FPC Optimization

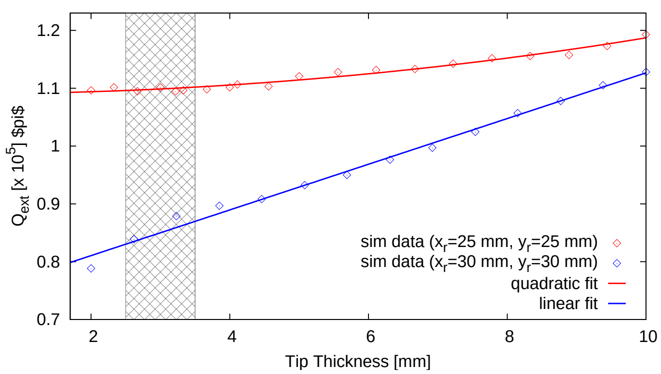
FPC - Beam Pipe Edge

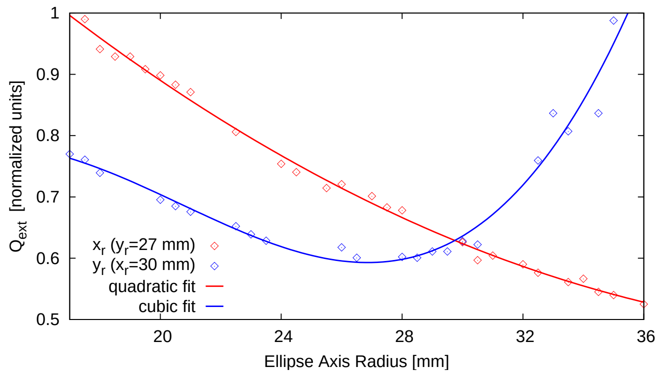


Pringle Tip Thickness

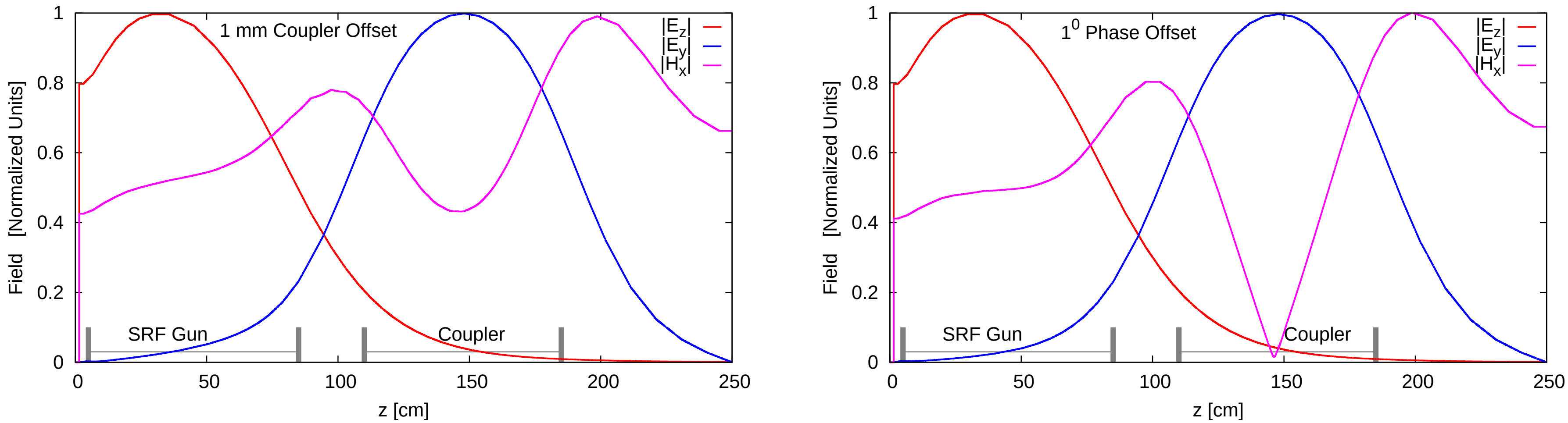








Coupler Kicks



$$\delta_t = \frac{\int (E_y + v_z B_x) dz}{\int E_z dz}$$

$$d\epsilon_n = \sigma_t \frac{2\pi\sigma_z}{\lambda_{RF}} \frac{eV_{acc}}{E_0} \left| \mathrm{R}e(\delta_t) \sin\phi_0 + \mathrm{I}m(\delta_t) \cos\phi_0 \right|$$

Asymmetry	Kick	$d\epsilon_n/\epsilon_n$
Tip Penetration	(-6.1 - 5.0i)$\times 10^{-5}$ mm^{-1}	$<$ 3%
Phase Offset	(8.4 - 5.9i)$\times 10^{-5}$ deg^{-1}	$<$ 3%

Conclusions and Outlook

- Linear optics & coupling using SVD formalism
- Measurements of optics and local coupling sources
- Investigation of slopes in coupling terms

- Design of Five-Cell SRF cavity for electron cooling
- Several RF issues and HOM measurements on prototype
- Beam breakup thresholds

- $\frac{1}{2}$ cell SRF gun design
- RF & beam dynamics issues, coupler optimization

Back Up Plan

Date: Wed, 15 Mar 2006 11:11:04 +0200
From: "Raj Lippi"
X-Priority: 3
Subject: BACHELORS,MBA,**DOCTORATE,PHD**
Status: RO

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Acknowledgements

Advisors: Ilan Ben-Zvi, Steve Peggs

Collaborators: Rogelio, Andrea, Jacek

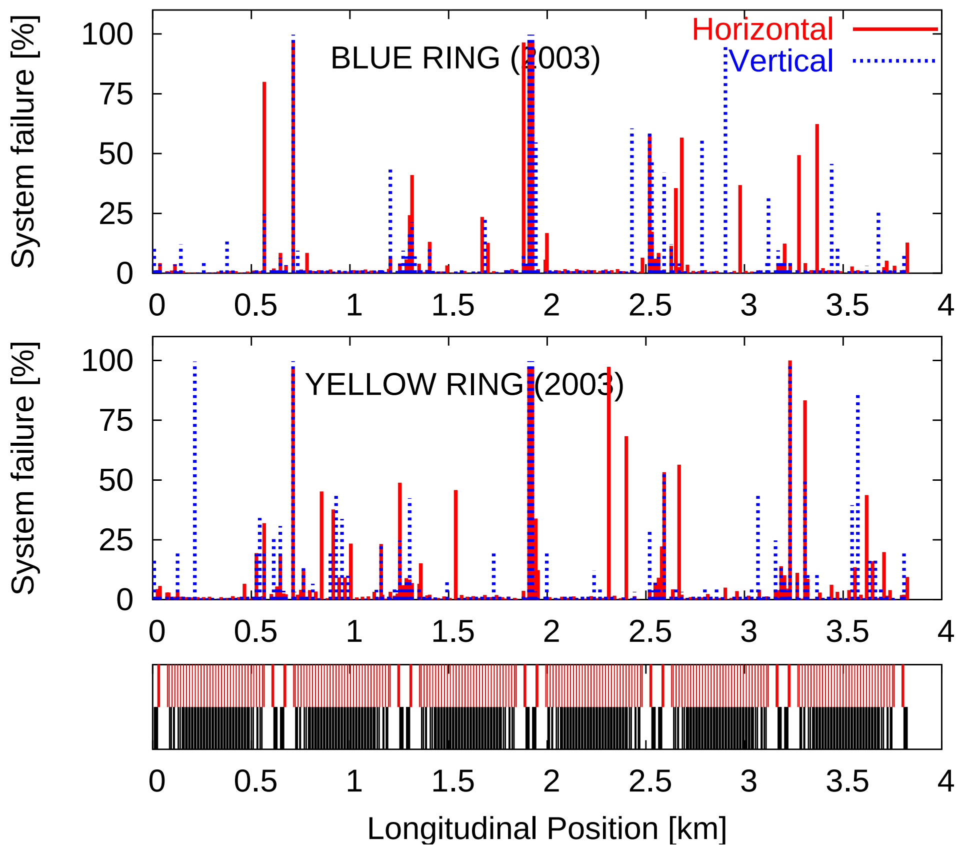
RHIC Accelerator Physics & Operations Group (Wolfram, Todd, Thomas, Yun, Mei...)

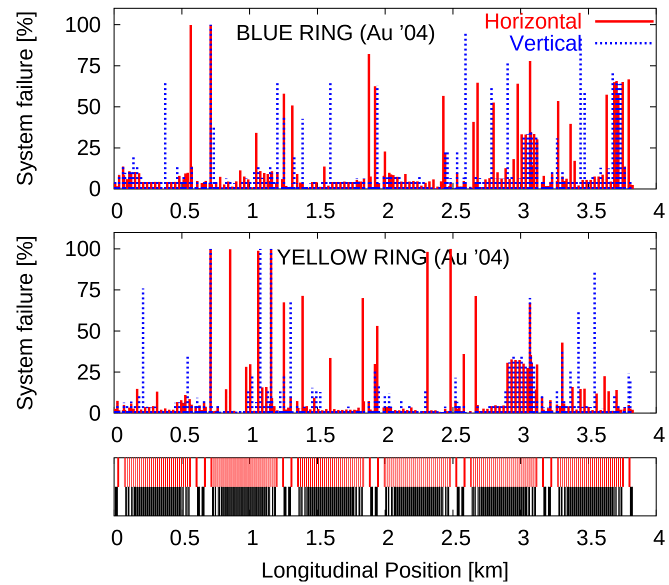
Electron Cooling Group & AES (Vladimir, Andrew, Dimitri, Gary, Dave ...)

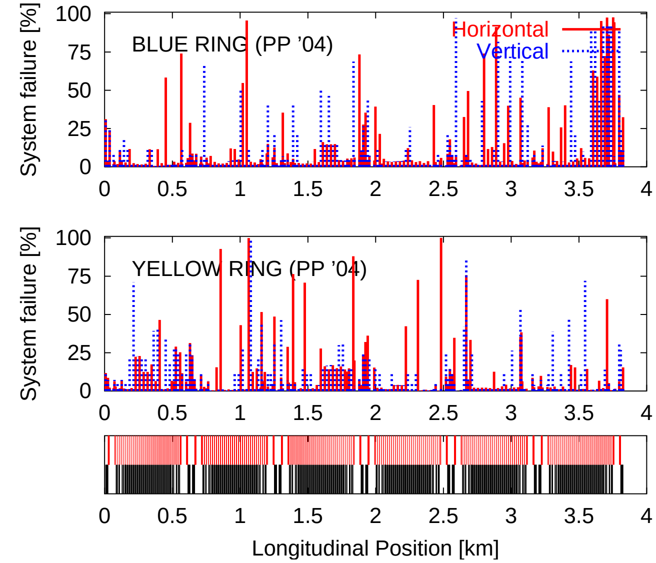
Extra Slides

Hardware Cut

BPM is tagged faulty due to obvious electronic failures by some hardware thresholds

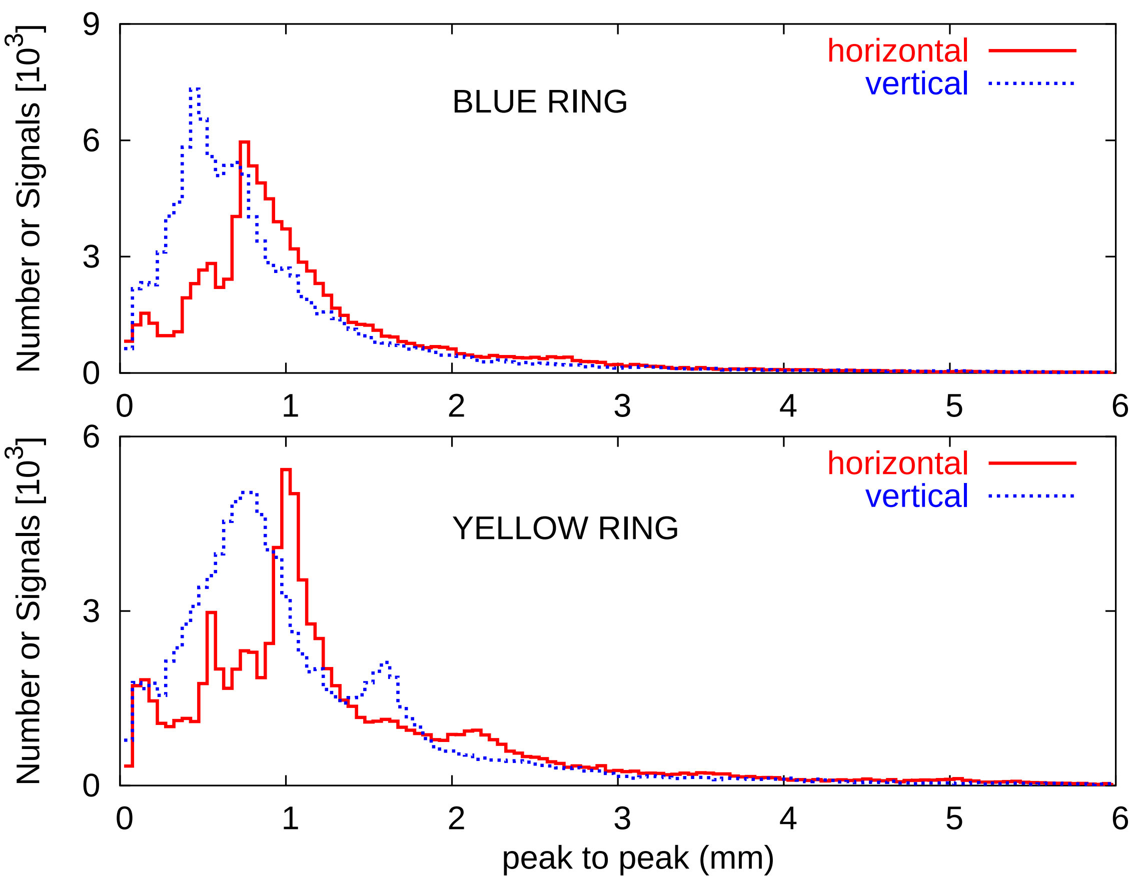




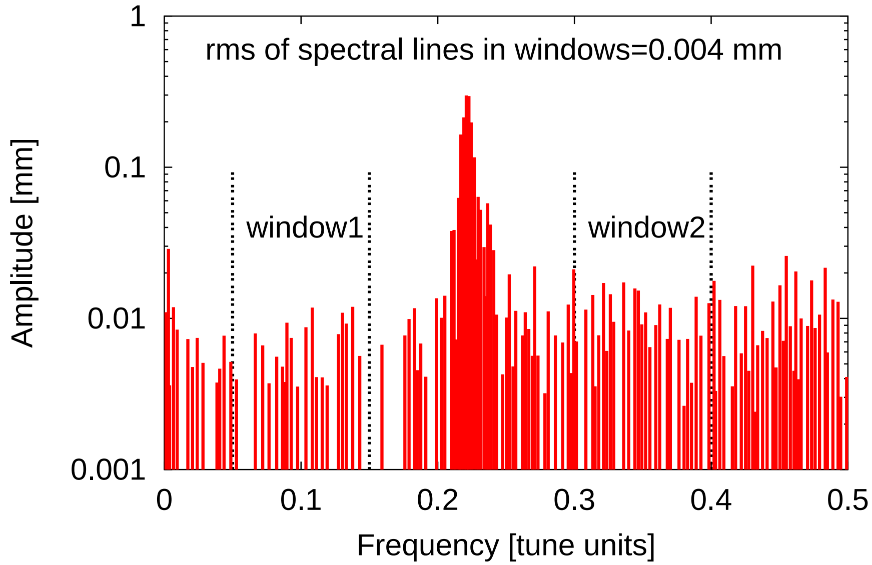


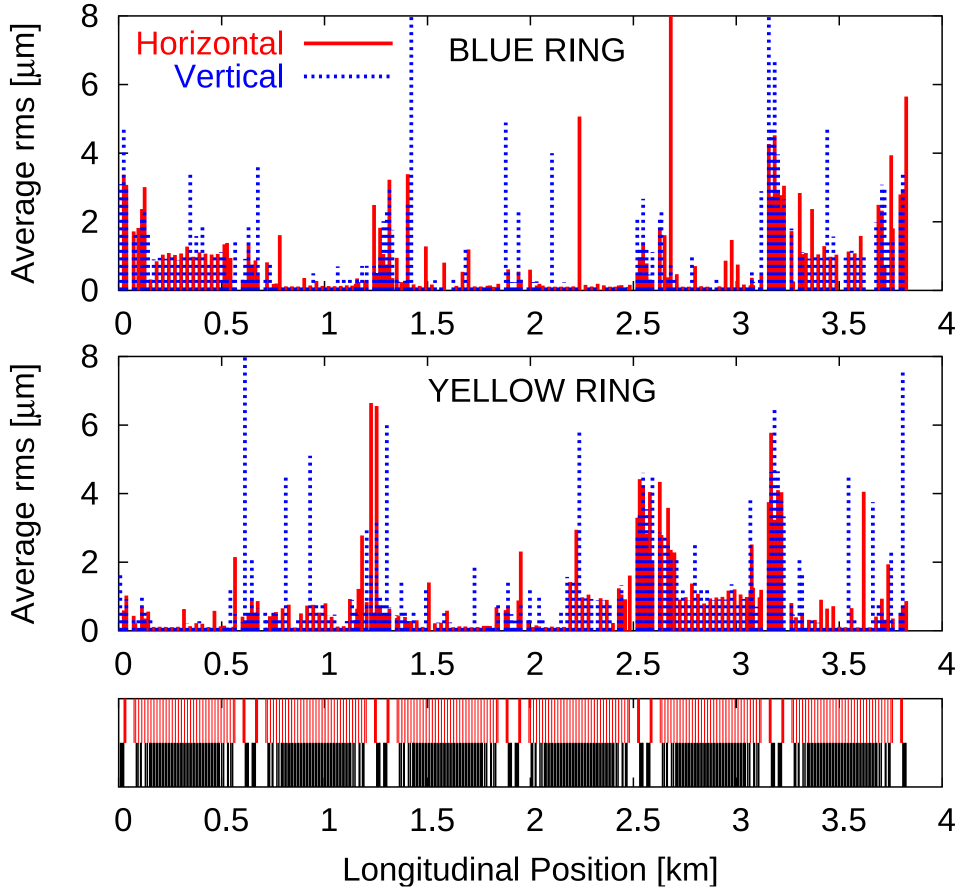
Peak to Peak Cut

Numerical techniques used to identify faulty bpms become less sensitive when signal to noise to ratio is small



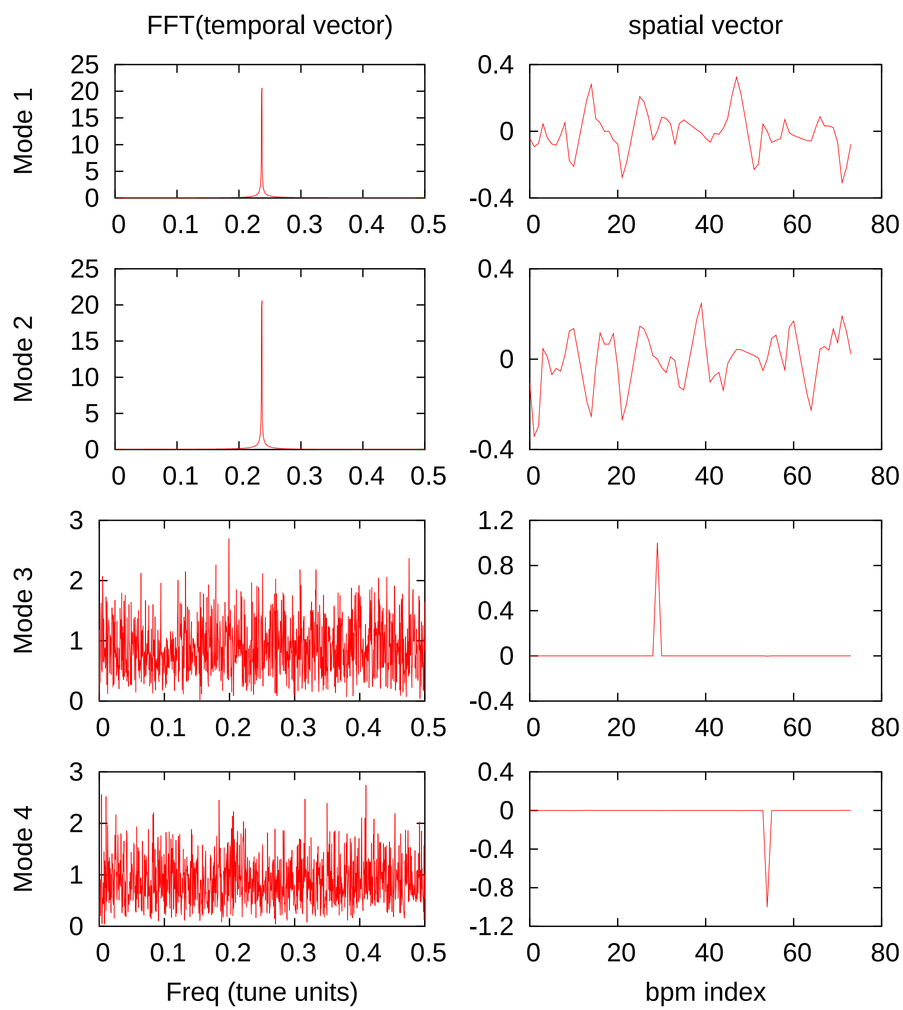
FFT Technique

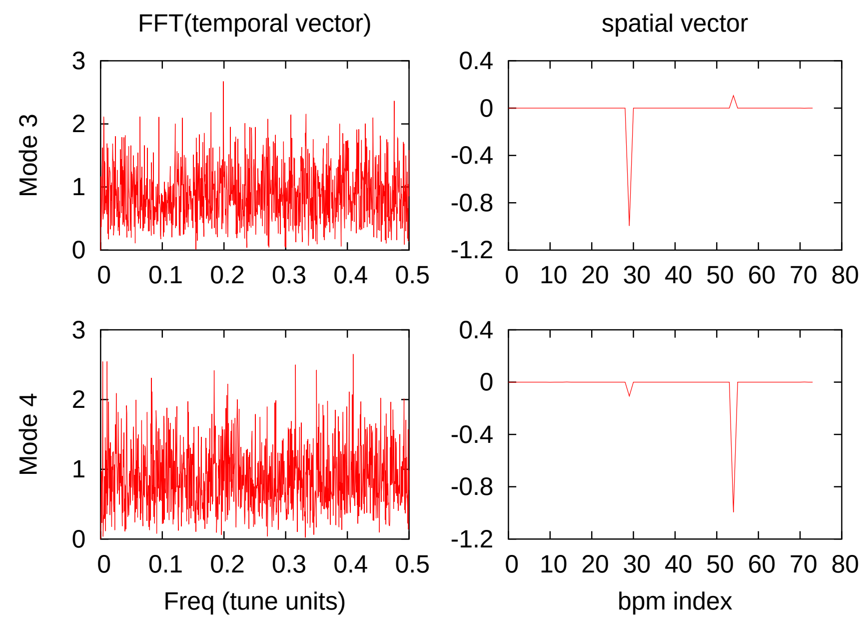


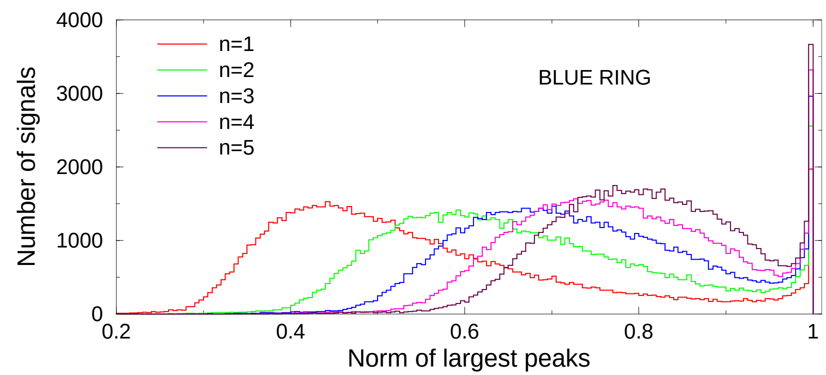


The rms of the background is use as the observable to determine the threshold for a faulty BPM

SVD Technique

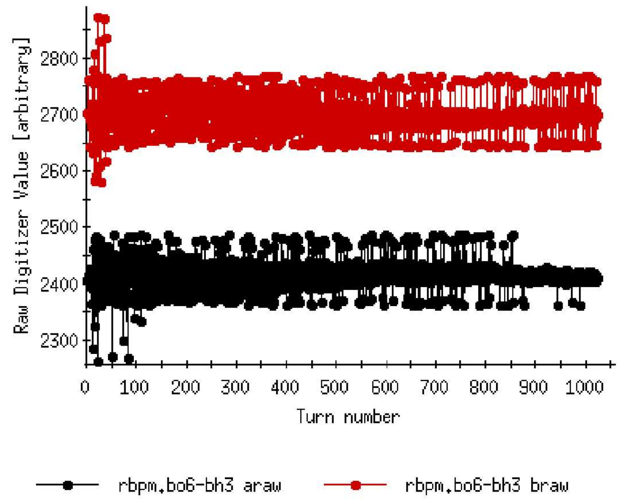


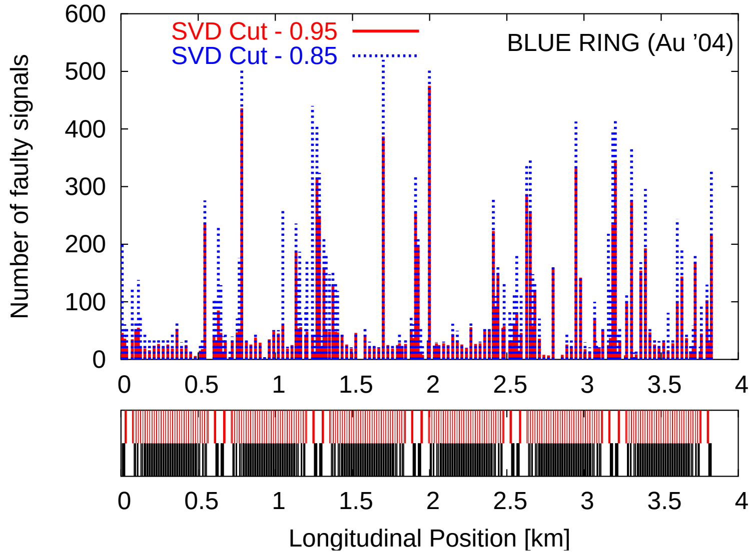


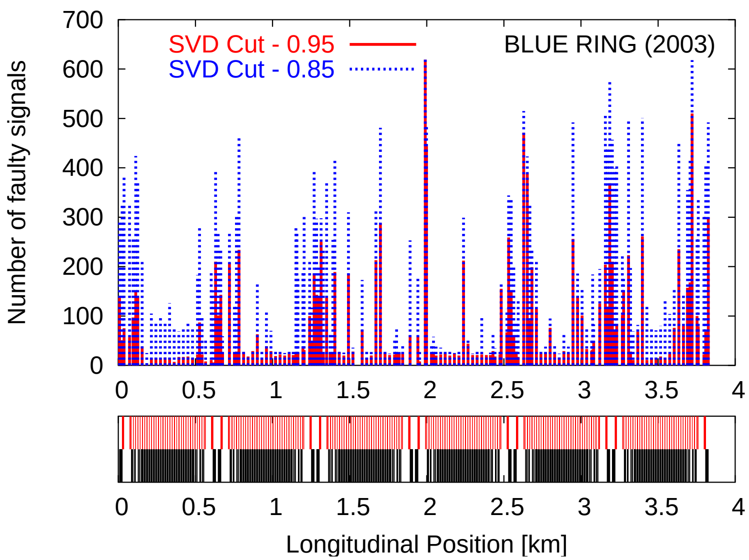


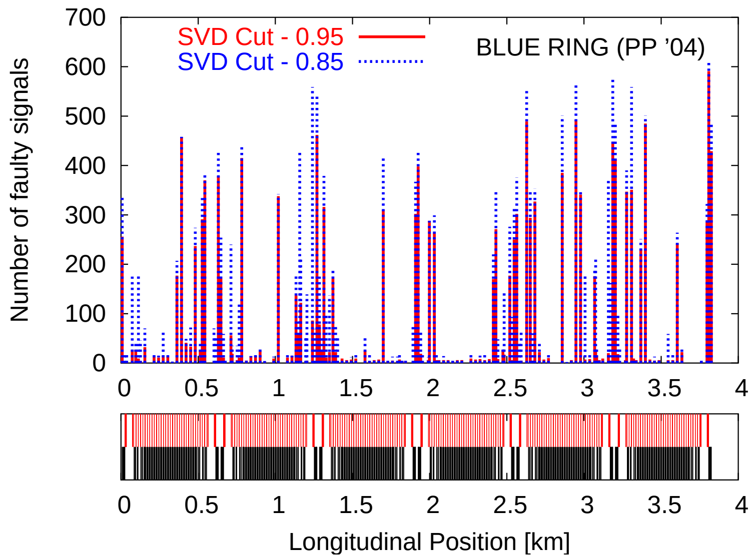
Sum of the largest peaks of the spatial vectors are used as observables

“Hairs” & Improvement









Betatron Eigenmodes

The m^{th} BPM reading for the t^{th} turn can be expressed as:

$$b_t^m = \sqrt{2J_t\beta_m}\cos\left(\phi_t + \psi_m\right)$$

For convenience, normalize the BPM matrix by $B = \frac{B}{\sqrt{T}}$ so that covariance matrix $C_B = B^T B$ is simply:

$$\begin{aligned} C_B^{mn} &= \frac{1}{T}\sum_{t=1}^{T} b_t^m b_t^n \\ &= \sum_{t=1}^{T}\frac{J}{T}\sqrt{\beta_m\beta_n}[\cos\left(\psi_m - \psi_n\right) + \cos\left(2\phi_t + \psi_m + \psi_n\right)] \\ &= \langle J\rangle\sqrt{\beta_m\beta_n}\cos\left(\psi_m - \psi_n\right) \end{aligned}$$

Eigenmodes Contd..

From SVD of a matrix B we can see:

$$C_B V = (USV^T)^T (USV^T) V = VS^2$$

To find eigenvalues & eigenvectors, we need to solve

$$C_B v = \lambda v$$

where $v = \sqrt{2J\beta_m} \cos(\phi_0 + \psi_m)$
From the m^{th} component of the secular equation

$$\begin{aligned}
\lambda \cos(\phi_0 + \psi_m) &= \langle J \rangle \sum_{n=1}^{M} \beta_n \cos(\psi_m - \psi_n) \cos(\phi_0 + \psi_n) \\
&= \langle J \rangle \Big[\cos(\phi_0 + \psi_m) \sum_{n=1}^{M} \beta_n \cos^2(\phi_0 + \psi_n) \\
&\quad + \frac{1}{2} \sin(\phi_0 + \psi_m) \sum_{n=1}^{M} \beta_n \sin 2(\phi_0 + \psi_n) \Big]
\end{aligned}$$

Eigenmodes Contd..

Therefore, we have the condition

$$\sum_{n=1}^{M} \beta_n \sin 2(\phi_0 + \psi_n) = 0$$

The two solutions are

$$\phi_0 = -\frac{1}{2}\tan^{-1}\left(\frac{\sum_n \beta_n \sin 2\psi_n}{\sum_n \beta_n \cos 2\psi_n}\right), \qquad \phi_0 + \frac{\pi}{2}$$

The two eigenvalues are

$$\lambda_{\pm} = \frac{1}{2}\langle J\rangle \left[\sum_{n=1}^{M} \beta_n \pm \sum_{n=1}^{M} \beta_n \cos 2(\phi_0 + \psi_n)\right]$$

Eigenvectors

The normalized eigenvectors (spatial) are

$$v_+ = \frac{1}{\sqrt{\lambda_+}} \left[\sqrt{\langle J \rangle \beta_m} \cos(\phi_0 + \psi_m) \right]$$

$$v_- = \frac{1}{\sqrt{\lambda_-}} \left[\sqrt{\langle J \rangle \beta_m} \sin(\phi_0 + \psi_m) \right]$$

Corresponding, normalized temporal vectors

$$u_+ = \sqrt{\frac{2J_t}{T\langle J \rangle}} \cos(\phi_t - \phi_0)$$

$$u_- = -\sqrt{\frac{2J_t}{T\langle J \rangle}} \sin(\phi_t - \phi_0)$$

Phase Advance and Beta Functions

Therefore, BPM matrix of betatron oscillation

$$\boxed{\mathbf{B} = \sigma_+ \mathbf{u}_+ \mathbf{v}_+^{\mathbf{T}} + \sigma_- \mathbf{u}_- \mathbf{v}_-^{\mathbf{T}}}$$

Twiss functions can be derived from the betatron vectors

$$\psi = \tan^{-1}\left(\frac{\sigma_- v_-}{\sigma_+ v_+}\right)$$

$$\beta = \langle J \rangle^{-1}(\lambda_+ v_+^2 + \lambda_- v_-^2)$$

Error in bounds for measurements is given by

$$\sigma_\psi \approx \frac{1}{\sqrt{T}}\frac{\sigma_r}{\sigma_s}$$

$$\sigma_{\frac{\Delta\beta}{\beta}} \approx 2\sigma_\psi$$

Quadrupole Errors & Beta Beating

$$\frac{\Delta\beta}{\beta} \approx -\Delta q\,\beta_0 \sin(2(\phi-\phi_0))$$

Global Correction:

$$\mathcal{A}\Delta\vec{q} = \left[\frac{\Delta\vec{\beta}}{\vec{\beta}}\right]_{1...m}$$

$$\mathcal{A}_{mn} = \frac{\beta_n}{2\sin(2\pi Q)} \times \cos(2|\psi_m - \psi_n| - 2\pi Q).$$

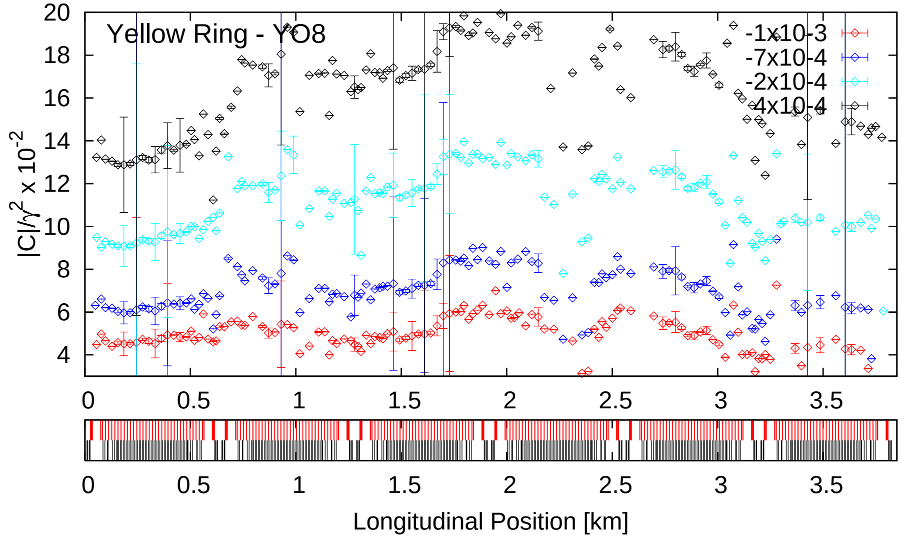
$$\left\|\mathcal{A}\Delta\vec{q} - \frac{\Delta\vec{\beta}}{\vec{\beta}}\right\|^2 = min.$$

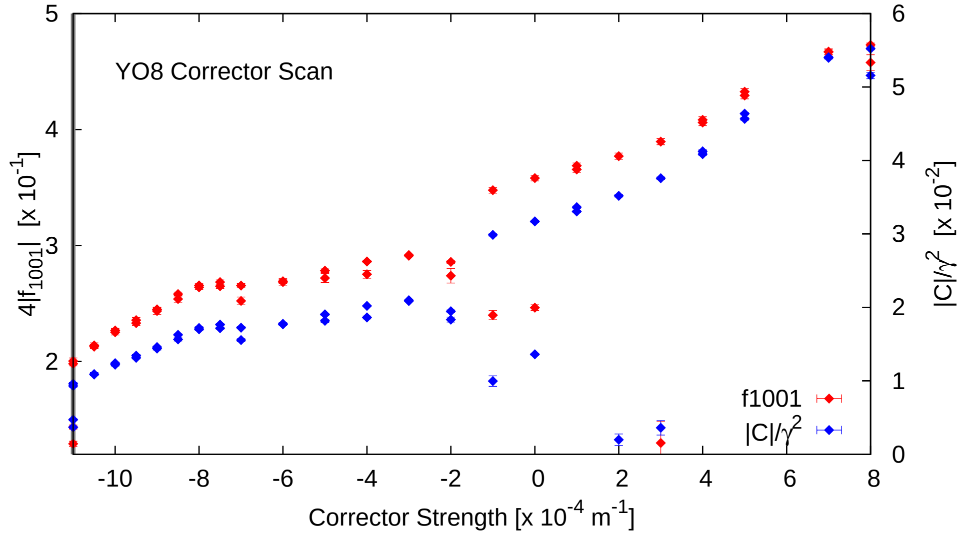
$$\Delta\vec{q} = (\mathcal{A}^T\mathcal{A})^{-1}\mathcal{A}^T\left[\frac{\Delta\vec{\beta}}{\vec{\beta}}\right]$$

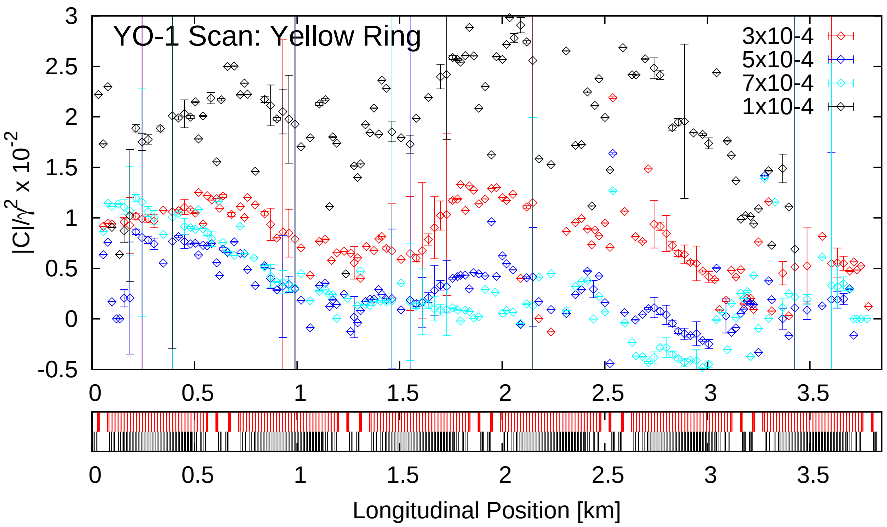
Local Correction:

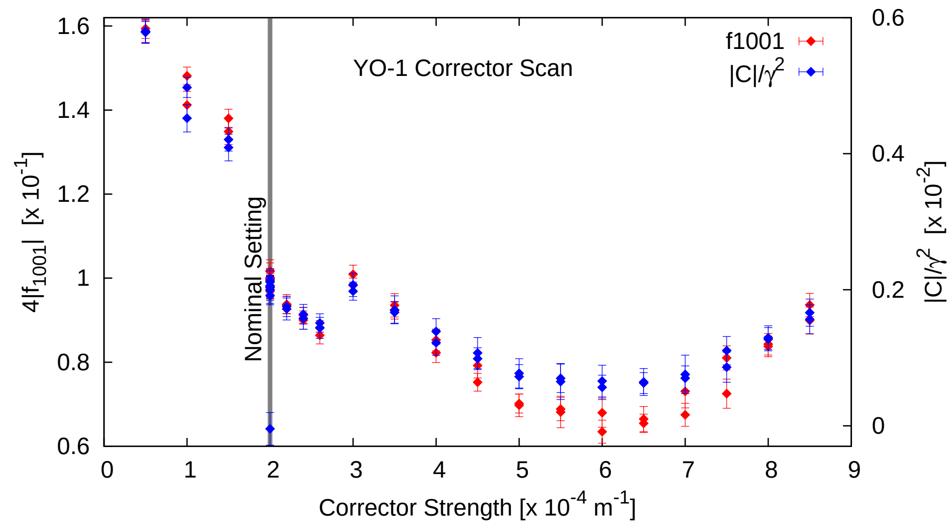
$$\begin{aligned}
\Delta q_1 &= -\frac{\Delta\beta_2}{\beta_2}\,\frac{1}{\beta_1}\,\frac{1}{\sin(2\psi_{21})}\\
\Delta q_2 &= +\frac{\Delta\beta_2}{\beta_2}\,\frac{1}{\beta_2}\,\frac{\sin(2\psi_{31})}{\sin(2\psi_{32})\ \sin(2\psi_{21})}\\
\Delta q_3 &= -\frac{\Delta\beta_2}{\beta_2}\,\frac{1}{\beta_3}\,\frac{1}{\sin(2\psi_{32})}
\end{aligned}$$

IR Corrector Scan: IR-8 & IR-2

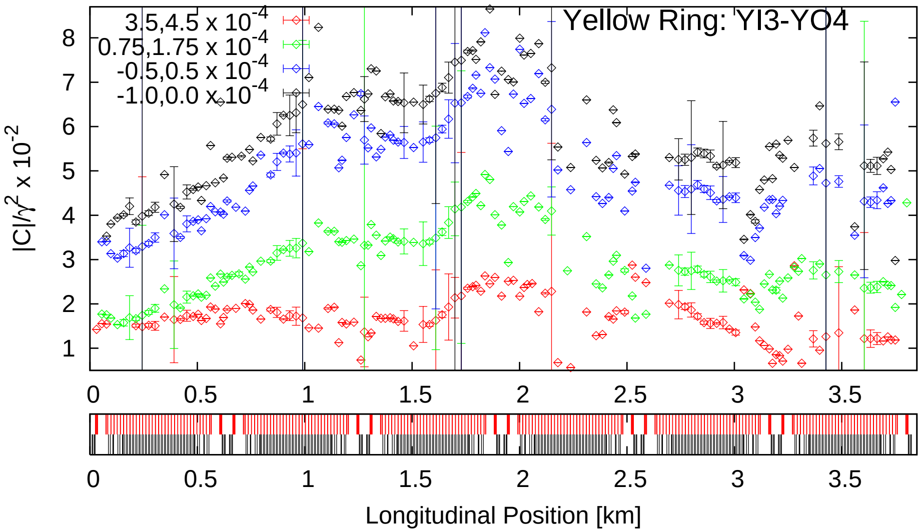


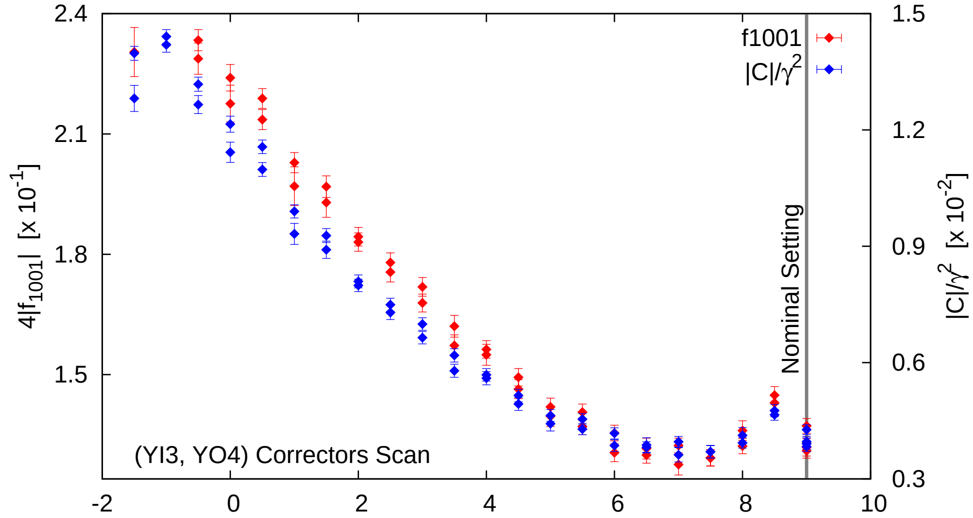


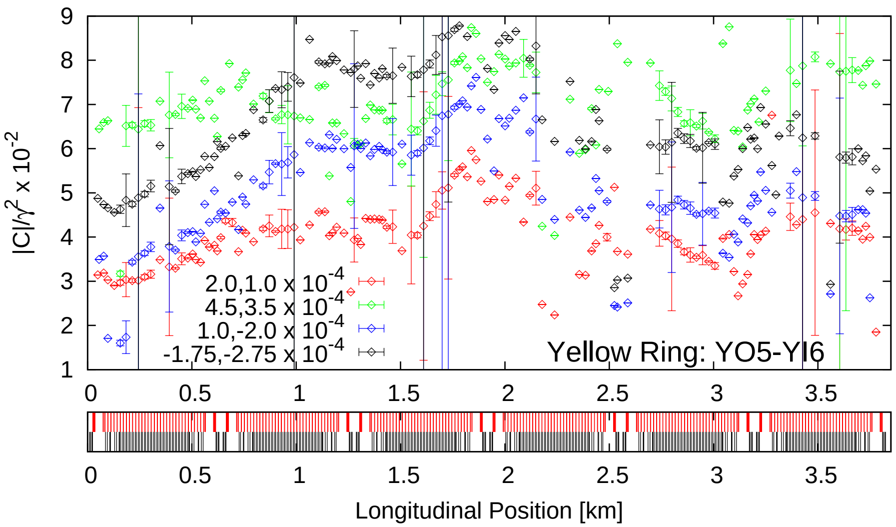


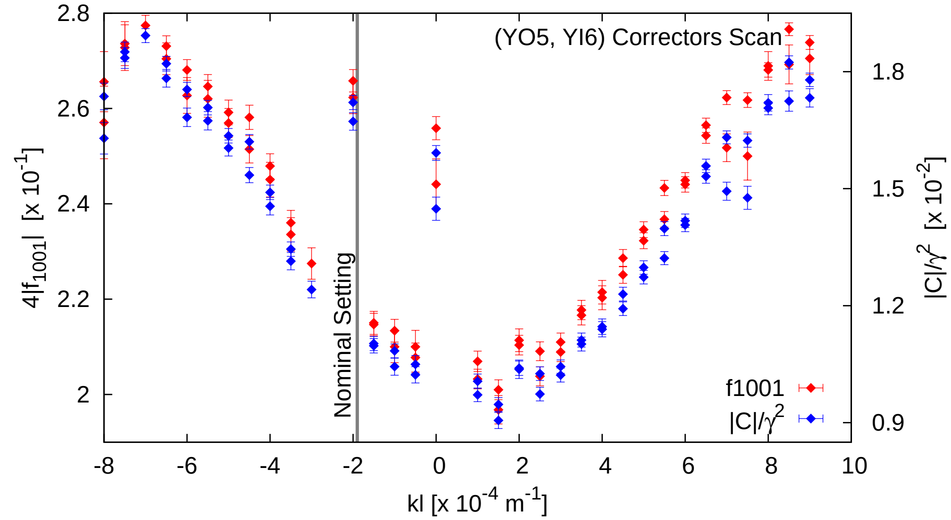


IR Corrector Scan: IR-4 & IR-6

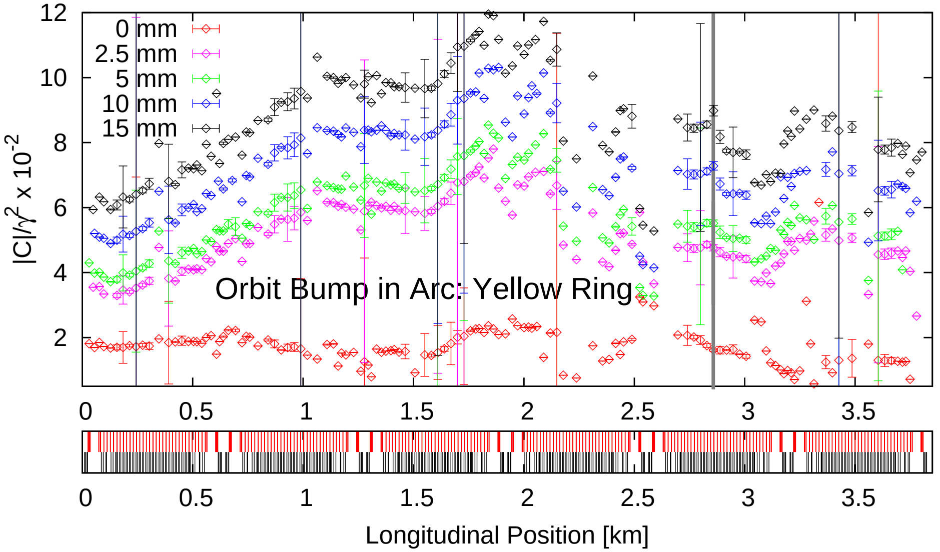


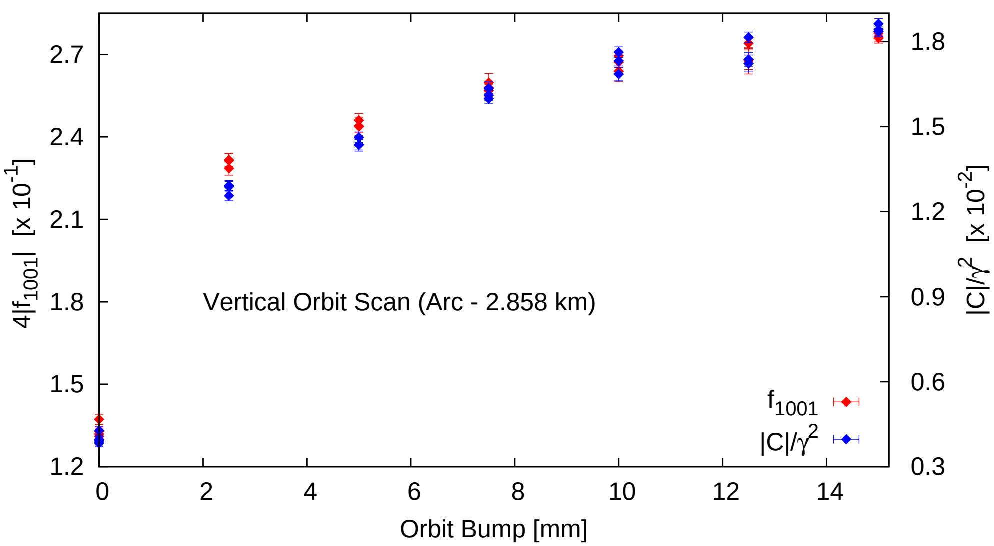




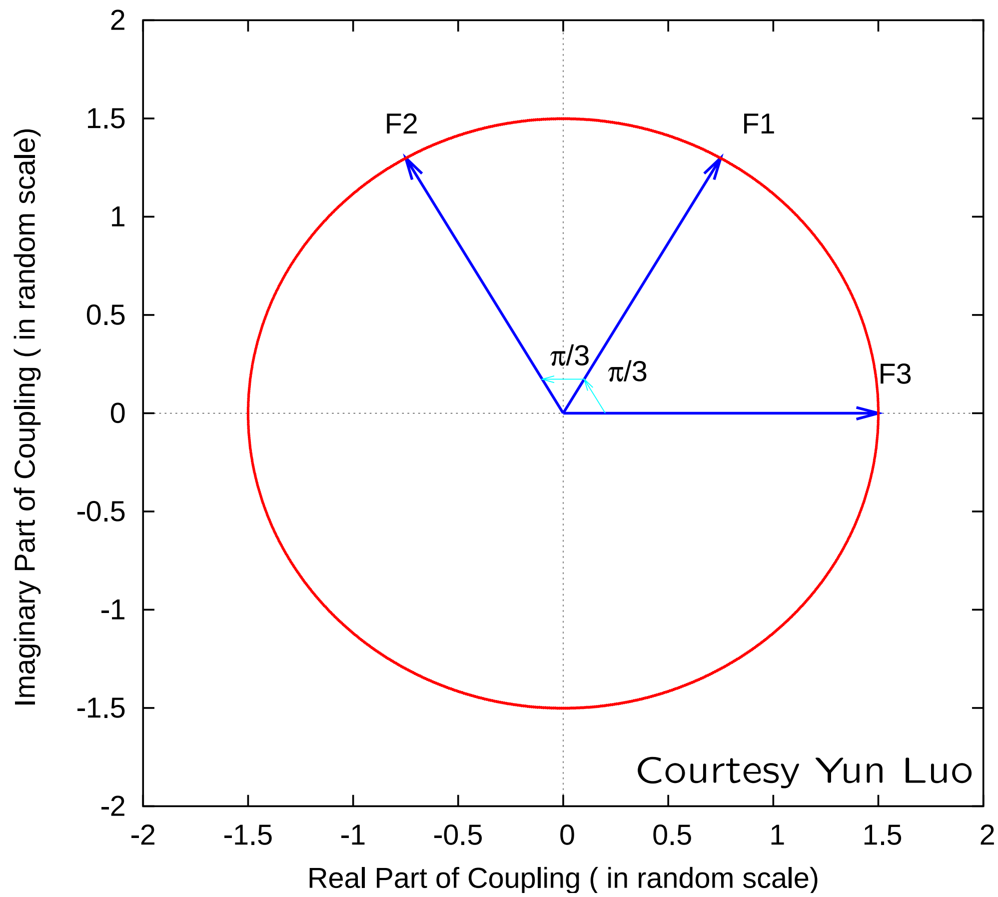


IR Corrector Scan: Vertical Orbit Bump





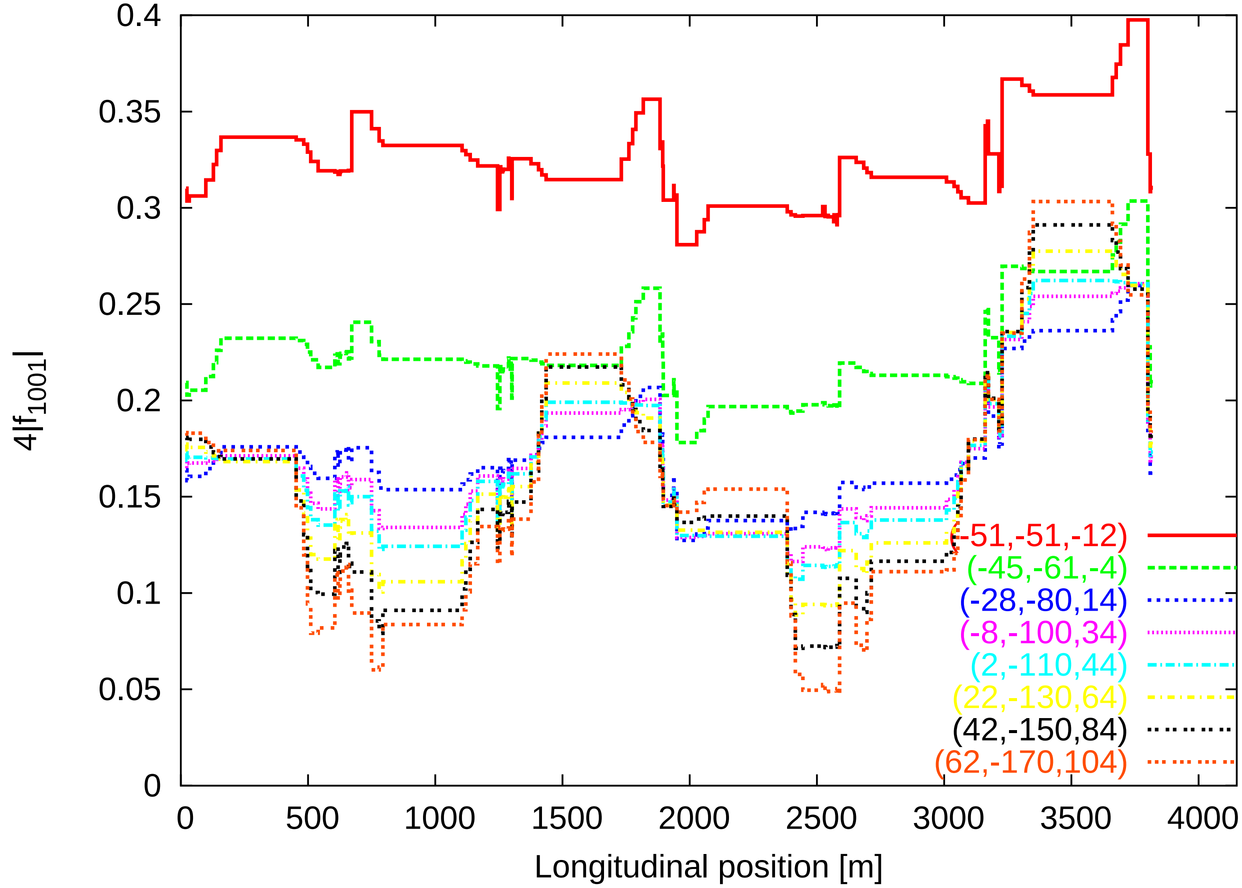
Global Coupling - RHIC Three Family Scheme



All settings of the form (F1$-\Delta$,F2$+\Delta$, F3$+\Delta$) have the same ΔQ_{min}

MAD-X RHIC Model

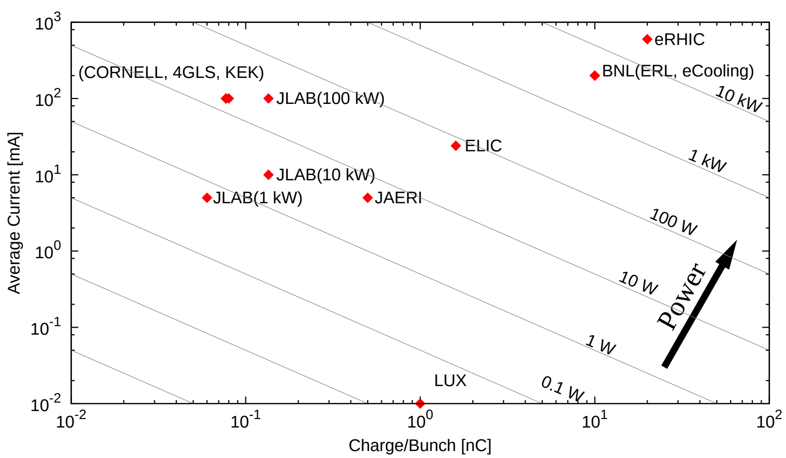
Global coupling compensated ($\Delta Q_{min} = 1 \times 10^{-3}$)



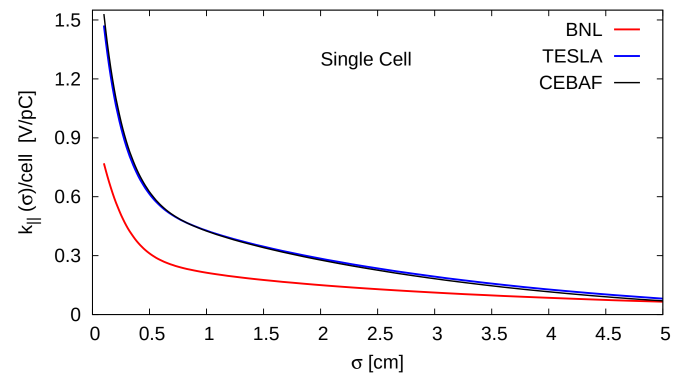
The numbers in brackets represent the strength of the families in units of $10^{-5}m^{-1}$.

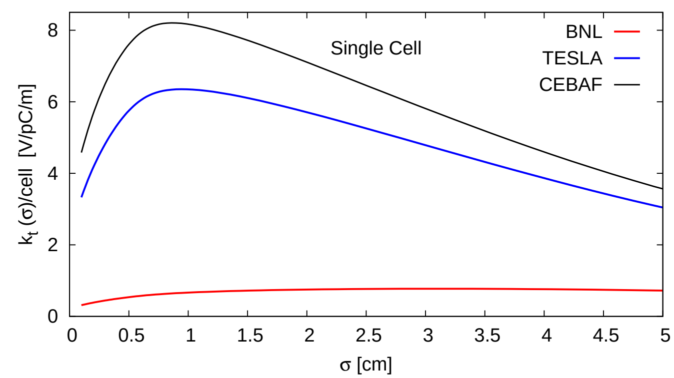
Design Criteria

- $\frac{E_{peak}}{E_{acc}}$ $(\downarrow)$, $\frac{H_{peak}}{E_{acc}}$ $(\downarrow)$
- $P_{cav} \propto \frac{Rs}{(R/Q)G}$ $(\downarrow)$
  - $R_s \propto \omega^2$ $(R_s = R_{BCS} + R_{res})$
  - $\frac{R}{Q}G \propto const.$ $(dim. \propto \omega)$
- Field sensitivity: $a \propto \frac{N^2}{k_{cc}}$ $(\downarrow)$



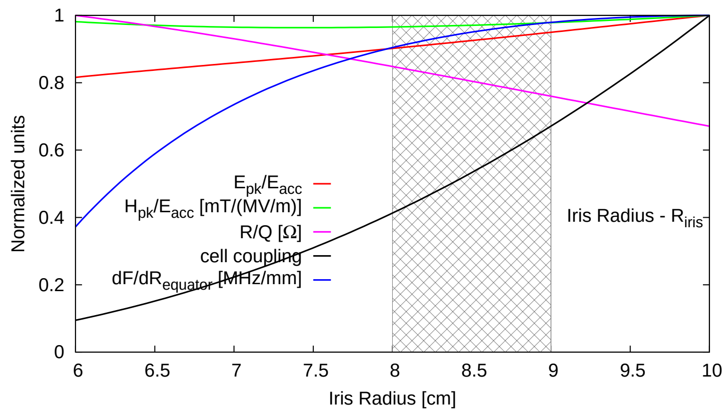
- $P_{avg} = 2k_{||}IQ$
- $k_{||} \propto \frac{1}{\mathbf{R_{iris}}}\sqrt{\frac{d}{\sigma_z}}\sqrt{N_c}$
- $k_{\perp} \propto \frac{1}{R^3_{iris}}\sqrt{d\sigma_z N_c}$



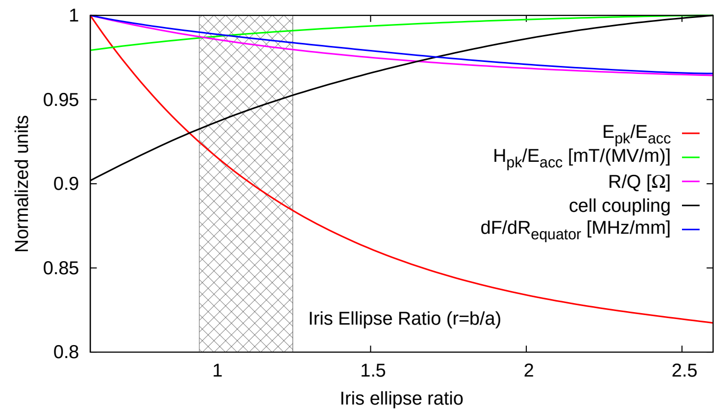


Cavity Design

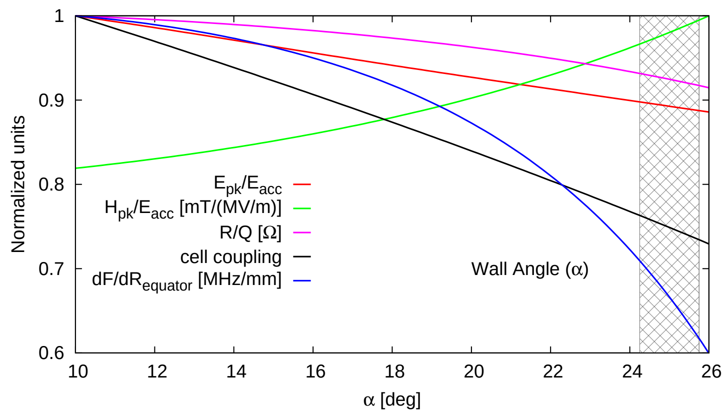
Aperture:



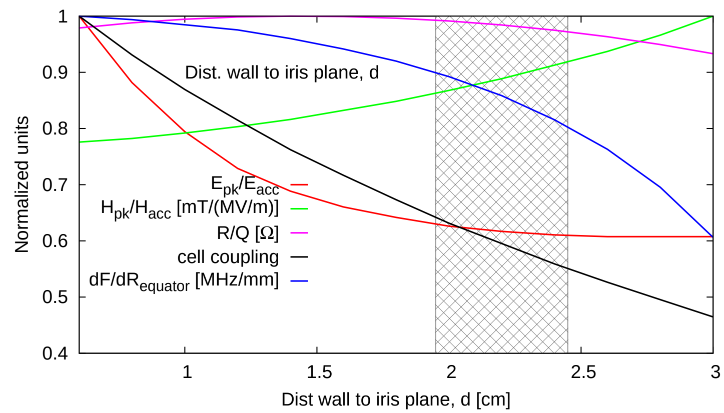
Iris Ellipse Ratio:



Wall Angle:



Dist. from wall to iris plane:



Cavity Comparisons

Par	BNL(HC)	CEBAF(HG)	TESLA(HG)
Freq. [MHz]	703.75	1497	1300
$\frac{R}{Q} * G$ [Ω^2]	9×10^4	2.1×10^5	2.8×10^5
E_p/E_a	1.97	1.96	1.98
H_p/E_a $[mT/MV/m]$	5.78	4.15	4.15
k_{cc}	3%	1.89%	1.87%
N_{cells}	5	7	9
$\frac{N^2}{\beta k_{cc}}$	8.3×10^2	2.6×10^3	4.1×10^3
Lorentz. Det. Coeff $[Hz/(MV/m)^2]$	1.2 (UnStiff)	2	1
$k_{\|\|}$ $(\sigma_z - 1mm)$ [V/pC]	4.25	10.71	13.14
$k_\perp$ $(\sigma_z - 1mm)$ [V/pC/m]	0.1	2.24	2.07
Q_{ext} (Dipole)	$10^2 - 10^4$	$10^3 - 10^6$	$10^3 - 10^7$

Simulations Techniques (MAFIA, ABCI)

Frequency Domain:

$$Conventional: \quad k = 2\frac{|f_{mag} - f_{ele}|}{f_{mag} + f_{ele}}$$

$$log(\frac{1}{k}) \approx \begin{cases} 0 & : \quad untrapped \\ \infty & : \quad trapped \end{cases}$$

$$Complex: \quad Q_{ext} = \frac{1}{2}\frac{Re(f)}{Im(f)}$$

$$Dispersion: \;\; \widehat{E}(r, z + L) = \widehat{E}(r, z)e^{i\phi}$$

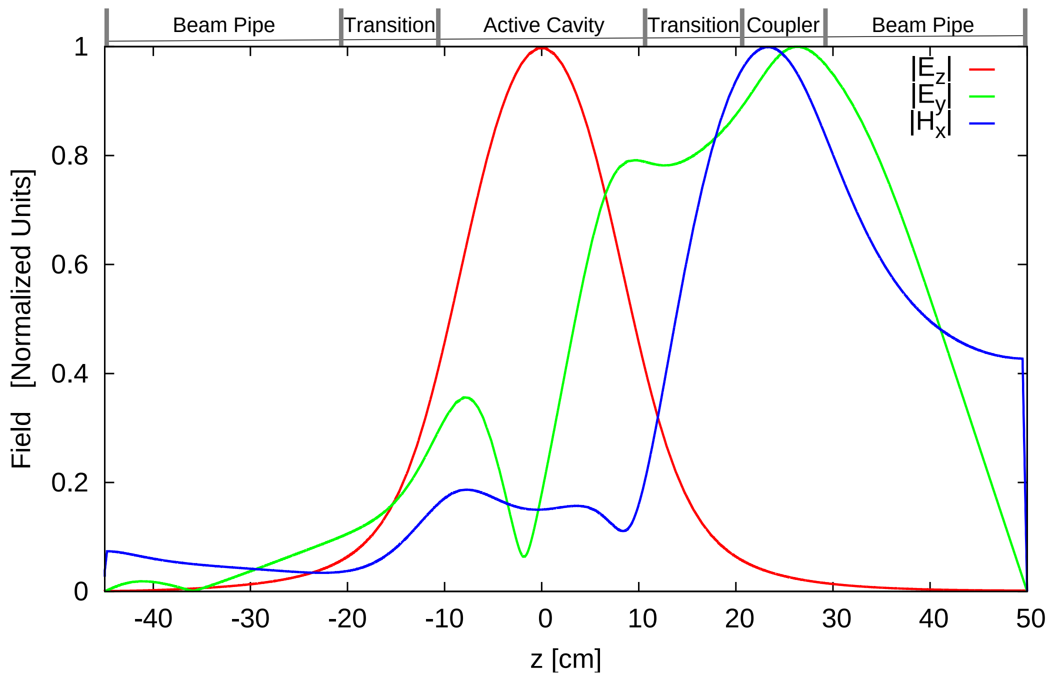
$$\phi(z) = \cos^{-1}\left(\frac{E_z(r, z + L_{cell}) + E_z(r, z - L_{cell})}{2E_z(r, z)}\right)$$

Time Domain:

$$\begin{aligned} W_{||} &= \frac{1}{q}\int_{-\infty}^{\infty} E_z(x, y, (s + z)/c)\, dz \\ I(\omega) &= qe^{-\frac{1}{2}\frac{(\sigma_s\omega)^2}{c^2}} \quad (Gaussian\ Bunch) \end{aligned}$$

$$\begin{aligned} Z_{||} &= \frac{1}{cI(\omega)}\int_{-\infty}^{\infty} W_{||}(x, y, s)e^{-i\frac{\omega}{c}s} ds \\ Z_{\perp} &= \frac{Z_{||}(x, y, s)}{kr^2} \end{aligned}$$

Coupler Kick



$$\delta_t = \frac{\int (E_y + cB_x)dz}{\int E_z dz}$$

	δ_t	Kick
Single Coupler	$(0.3 - 1.2i)\times10^{-3}$	$\approx$ 0.27 mrad
Symmetric Couplers	$(5.3 - 8.7i)\times10^{-5}\ mm^{-1}$	$\approx$ 48 μrad

Bead-Pull Technique & R/Q

$$S_{21} = \frac{2\sqrt{\beta_1\beta_2}}{(1+\beta_1+\beta_2) + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$$\frac{\delta\omega}{\omega_0} = \begin{cases} -\frac{\pi r^3}{U}(\epsilon_0 \frac{\epsilon_r+2}{\epsilon_r-1} E_0^2) & : \quad dielectric \\ -\frac{\pi r^3}{U}(\epsilon_0 E_0^2 - \frac{\mu_0}{2} H^2) & : \quad metal \end{cases}$$

$$\frac{\delta\omega}{\omega_0} \approx -\frac{1}{2Q_L}\tan(\phi)$$

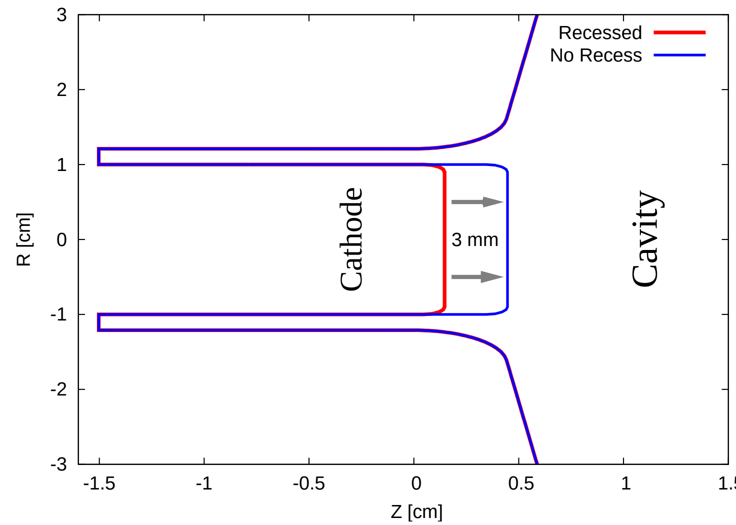
$$\frac{R}{Q}^{sphere} = -\frac{1}{2\pi\omega_0 r^3 \epsilon_0}[\int\sqrt{\frac{\delta\omega}{\omega_0}}\cos(kz)dz]^2$$

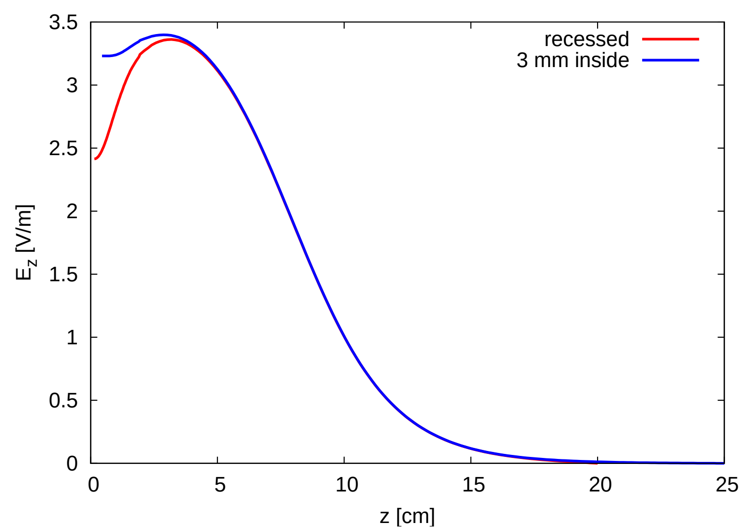
Material	R (mm)	R/Q Ω
Teflon	4.77 ± 0.02	474 ± 10
Steel	3.96 ± 0.01	532 ± 10
Aluminum	2.5 ± 0.01	370 ± 10

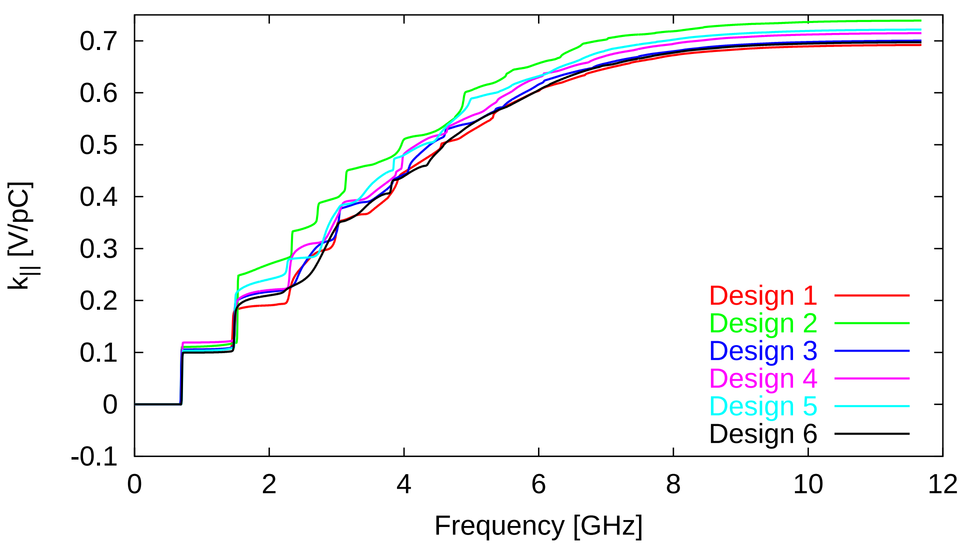
Average HOM Losses

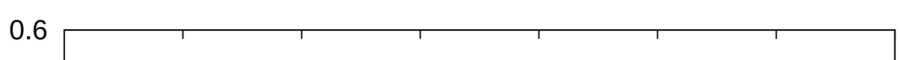
$$\boxed{P_{HOM} = k_{||} Q_b I_b}$$

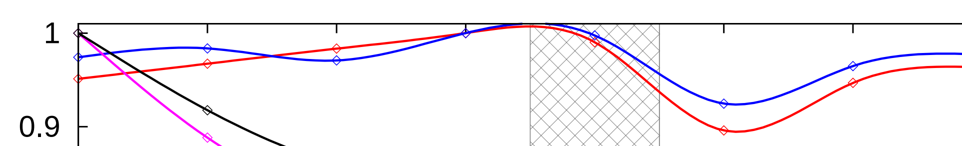
Cathode Recess



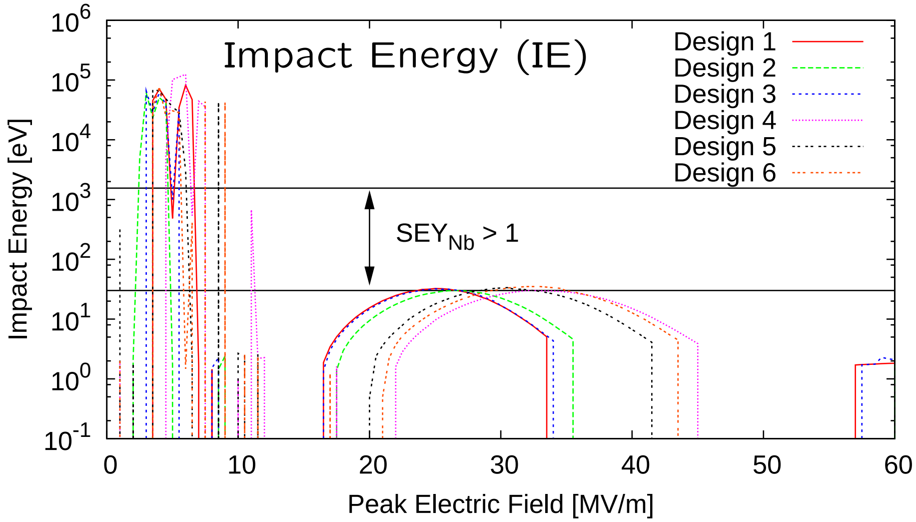


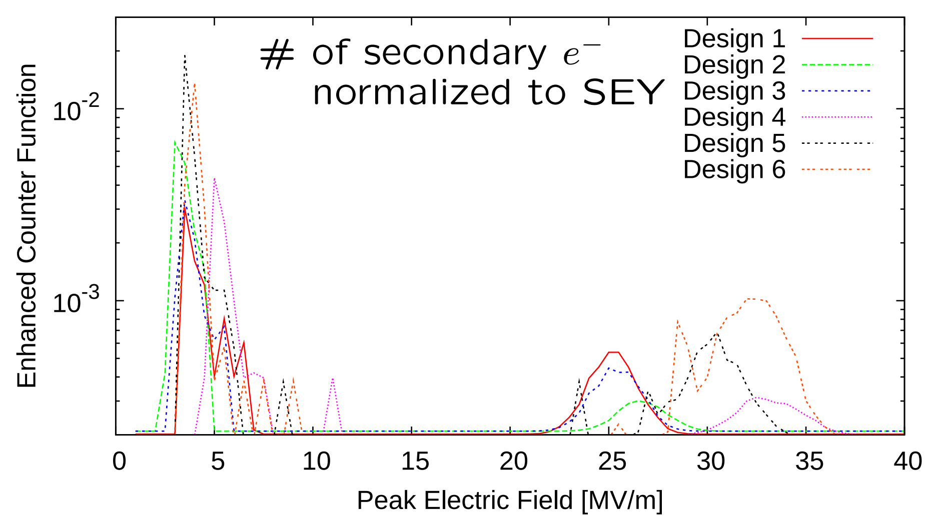


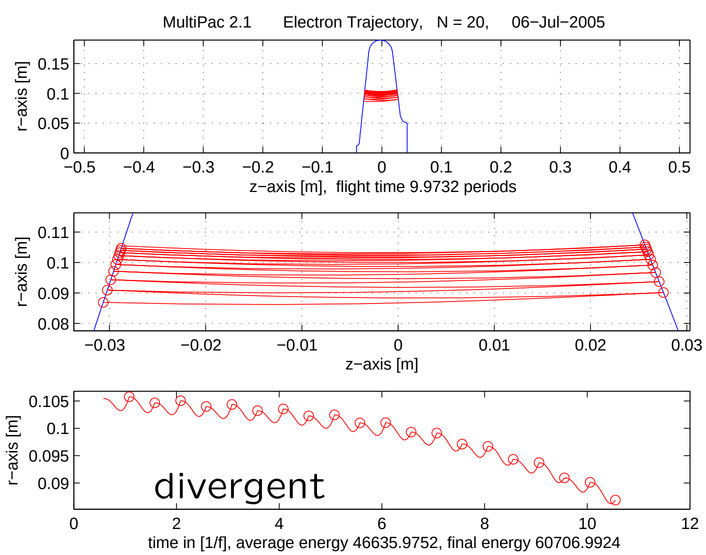


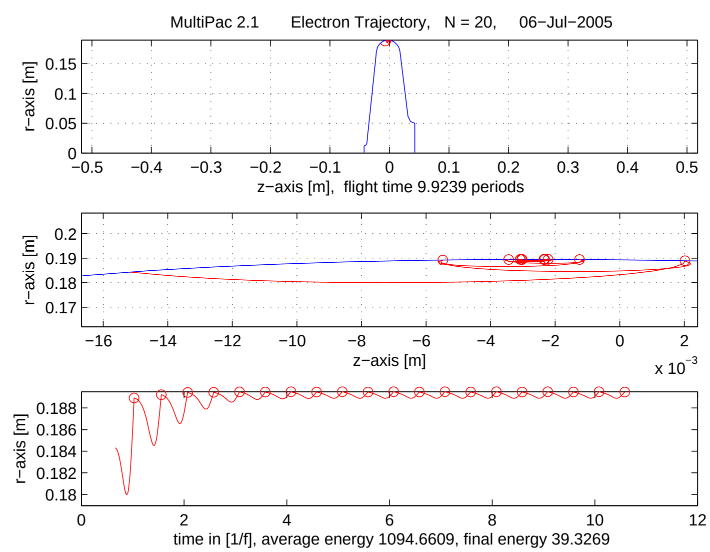


Multipacting

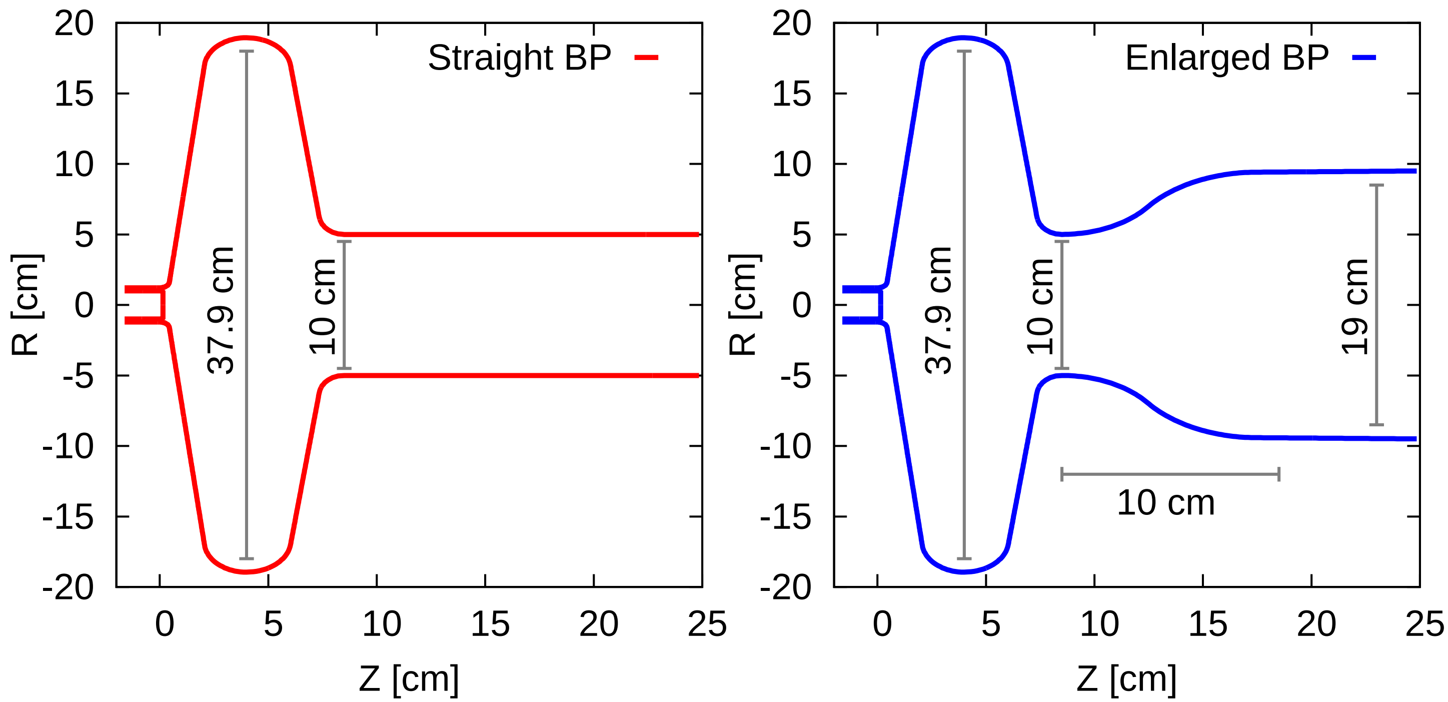






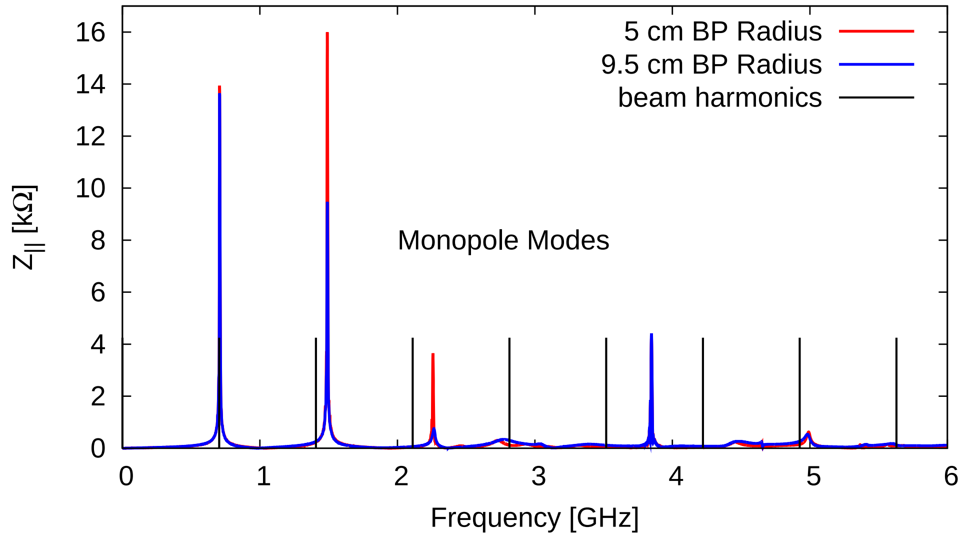


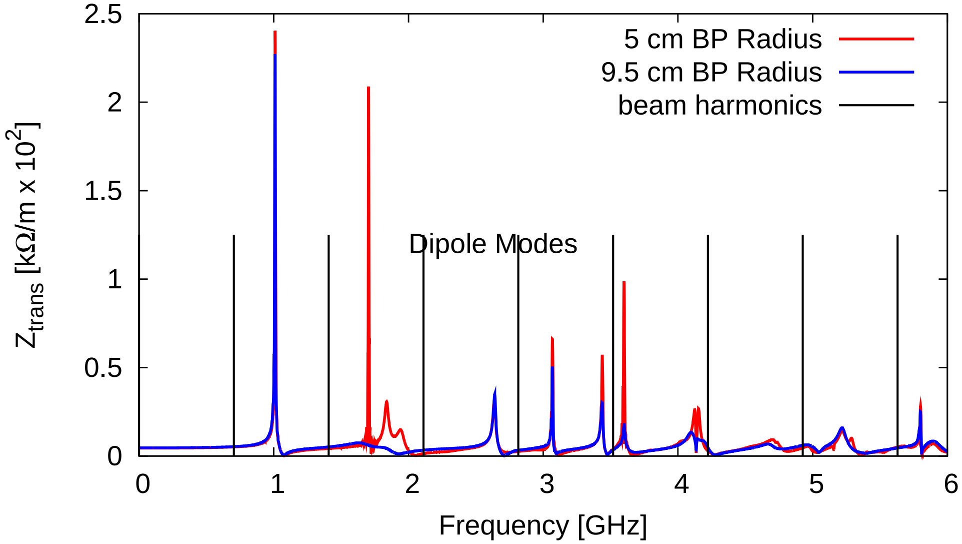
Beam Pipe Transition



- HOM Damping ☺
- FPC Coupling (field level $< 10^2 \rightarrow$ 10 cm away) ☹
- Mechanical Design (manufacturing, valves etc..) ☹

Impedance Spectrum & Laser Stability





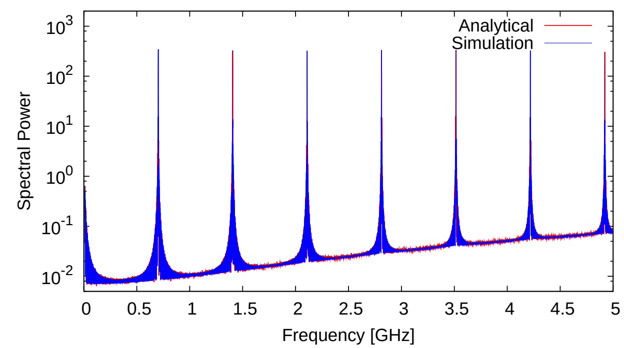
Phase Modulation:

$$I(t) = \sum_{n=-\infty}^{\infty} a_n \delta(t - nT_0 - \epsilon_n)$$

Spectral Power Density:

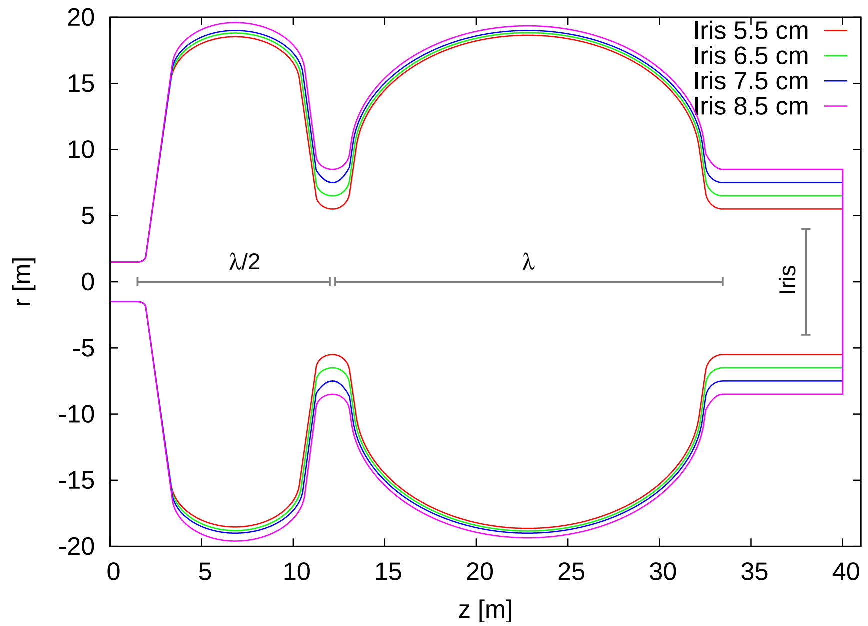
$$P(\omega) = \underbrace{\frac{2\pi}{T_0^2}\left[\frac{\sin(\sqrt{3}\omega\sigma_\epsilon)}{(\sqrt{3}\omega\sigma_\epsilon)}\right]^2}_{envelope} \sum_{m=-\infty}^{\infty} \underbrace{\delta\left(\omega - \frac{2\pi m}{T_0}\right)}_{harmonics}$$

$$+\frac{1}{T_0}\underbrace{\left[\left(1 - \left[\frac{\sin(\sqrt{3}\omega\sigma_\epsilon)}{(\sqrt{3}\omega\sigma_\epsilon)}\right]^2\right) + \sigma_a^2\right]}_{baseline}$$



$$\frac{V_{HOM}}{V_{acc}} \approx 9 \times 10^{-3} \;\; (\sigma_a = 1\%, \;\sigma_\epsilon = 1 \text{ ps})$$

eCooling 1.5 Cell Gun



- Optimize Iris Radius
  - f_{HOMs} & $f_{cut-off}$
  - Trapped Modes
- Beam pipe transition
  - HOM damping
  - FPC Coupling
- Optimize L_1 & L_2
  - Energy Vs. Phase Slope
  - Longitudinal Emittance
  - Transverse Emittance
- Optimize cavity ellipses
  - Peak fields, R/Q, etc...