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INVESTIGATION OF STOCHASTIC BEAM GROWTH WITH A NON-LINEAR LENS

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SUMMARY

The problem of the onset of stochastic motion in a non-linear system is of considerable importance for the understanding of the long-time stability of beams in hadron storage rings in the presence of strong high-order non-linear resonances. A non-linear lens has been installed in the SPS in order to simulate resonances driven by the beam-beam effect or by magnetic field imperfections in superconducting storage rings. In this paper, a criterion for the onset of stochastic motion is derived and compared with the results of computer simulations and of experiments performed on the machine.

Introduction

The stability of beams in storage rings in the presence of non-linear resonances has been the subject of considerable interest for many years. In 'classical' machines resonances higher than octupole are predominantly driven by the beam-beam effect. In contrast, in the new generation of storage rings using superconducting magnets the field quality is mainly governed by the accuracy of the positioning of the coil blocks. In addition, a larger fraction of the magnet aperture may be used to store the beam so magnetic field imperfection resonances are expected to play a more important rôle in these machines.

It is well known that under static conditions, one-dimensional resonances above octupole are intrinsically stable because the nonlinear detuning, always present in a real situation, prevents infinite growth of amplitude. In order to explain beam growth it is necessary that the tune is modulated in some way, for example, by synchrotron motion in the presence of non-zero chromaticity or by power supply ripple. Moreover, in order to have growth the betatron phase must be randomised between resonance crossings. A randomising process has been proposed[1),2)], which predicts that the motion will become stochastic when synchrobetatron resonances overlap.
In particular, according to this theory, the stochastic limit depends on the tune modulation frequency. In this paper, the dynamics of a stored proton beam in the present of a non-linear lens and tune modulation is discussed. Results of a computer simulation as well as an experiment performed on the SPS are given.

Theory

The nonlinear lens was first used in the ISR[3)] and later in the SPS for beam simulation studies. It consists of a pair of current-carrying copper bars which can be placed above and below the beam to produce a nonlinear field. The field components for current I are, with the nomenclature of figure 1,

$$B_{x,y}(x,y) = \frac{\mu_o I y,x}{\pi} \; \frac{h^2 \mp (x^2+y^2)}{\left[x^2+(h+y)^2\right]\left[x^2+(h-y)^2\right]} \qquad 1)$$

The quadrupole gradient is $\partial B_y/\partial x = -\partial B_x/\partial y = -\mu_o I/\pi h^2$, giving the linear tune shift $\Delta\nu_L = \beta_{x,y} L \, \mu_o I/(4\pi^2 B\rho h^2)$. where L is the length of the lens (1.5m).

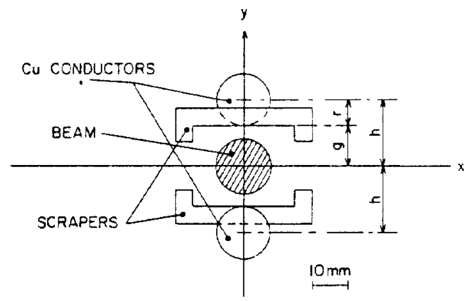

Figure 1

The Hamiltonian of the motion in the presence of the lens is [4)]

$$H(\eta,\eta';\theta) = \eta'^2/2 + \nu^2\eta^2/2 + \nu^2\beta^{3/2} f(\eta,\theta) \qquad 2)$$

where η, θ are the Courant and Snyder variables $\eta = x/\sqrt{\beta}$, $d\theta/ds = 1/\nu\beta$, and

$$f(\eta,\theta) = -\frac{2\Delta\nu_L}{\nu\beta}\eta_h^2\left[\frac{1}{2}+\sum_{p=1}^{\infty}\cos p\theta\right]\ln\left|\eta_h^2-(\eta-\eta_d)^2\right| \qquad 3)$$

where $\eta_h = h/\sqrt{\beta}$, $\eta_d = d/\sqrt{\beta}$ and d is the lens displacement with respect to the beam centre.

Transforming to a new Hamiltonian $\hat{H}$ via the action and angle variables ε, ϕ with $\eta = -\sqrt{2\varepsilon/\nu}\cos\phi$; $\eta' = \sqrt{2\nu\varepsilon}\sin\phi$ the phase equation can be obtained

$$\frac{\partial\phi}{\partial\theta} = \frac{\partial\hat{H}}{\partial\varepsilon} = \nu - \frac{4\Delta\nu_L\cos\phi\,(\cos\phi+\delta)}{1-\alpha^2(\cos\phi+\delta)^2}\left[\frac{1}{2}+\sum_{p=1}^{\infty}\cos p\theta\right] \qquad 4)$$

where $\alpha = \sqrt{\varepsilon\beta}/h$, $\delta = d/\sqrt{\varepsilon\beta}$

The nonlinear detuning is obtained by averaging the zeroth harmonic of 4)

$$\Delta\nu_{NL} = \frac{\Delta\nu_L}{\pi}\int_0^{2\pi}\frac{\cos\phi\,(\cos\phi+\delta)}{1-\alpha^2(\cos\phi+\delta)^2}\,d\phi \qquad 5)$$

The 'resonance width' $\Delta\nu_n$ is obtained by transforming to the 'slow' phase $\psi = \phi - p\theta/n$ and averaging the coefficient of the slowly varying term cos $n\phi$, giving

$$\Delta\nu_n = \frac{2\Delta\nu_L}{\pi}\int_0^{2\pi}\frac{\cos\phi\,(\cos\phi+\delta)\cos n\phi}{1-\alpha^2(\cos\phi+\delta)^2}\,d\phi \qquad 6)$$

The slowly varying part of the transformed Hamiltonian in the absence of tune modulation is of the form

$$\hat{H} = (\nu - p/n)\varepsilon + U(\varepsilon) + \nu\eta_h^2 V_n(\varepsilon)\cos n\psi \qquad 7)$$

In this form, $\nu\eta_h^2\, dV_n(\varepsilon)/d\varepsilon$ can be

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identified as the resonance width 6) and $dU(\varepsilon)/d\varepsilon$ is the nonlinear detuning function 5).

We now follow closely the paper by E.D. Courant[2] which predicts the threshold for the onset of stochastic motion in the presence of tune modulation in terms of these functions.

$$\nu_s \leq 4n \sqrt{U''(\alpha)V_n(\alpha)\; J_n(n\hat{\nu}/\nu_s)\,\nu\eta_h^2} \qquad 8)$$

where ν_s is the modulation tune and $\hat{\nu}$ its amplitude.

In the present study the lens and beam parameters were chosen so that the behaviour should become stochastic according to the above criteria in the region of 10 Hz. Most of the studies have been carried out on a 5th order resonance, generated by vertically displacing the lens with respect to the beam axis.

Simulation Results

The computer simulation of the non-linear lens in a storage ring tracks a particle in two transverse dimensions over 5 x 10^5 machine turns, or about 11 seconds in the SPS. Since the analytic theory above describes motion only in vertical phase space, the horizontal motion is usually turned off. Tune modulation, with a period of $1/\nu_s$, is introduced through longitudinal oscillations and linear chromaticities. A fixed vertical modulation amplitude of $\Delta\hat{\nu} = 2.8\times10^{-3}$ is used, corresponding to the SPS experiment described below.

Figure 2 shows the maximum vertical displacement of a simulation particle, as a function of its initial amplitude A_y. Here the lens strength is $\Delta\nu_L = 5\times10^{-3}$, the zero amplitude betatron tune is $\nu_y = .60$, and the modulation tune is $\nu_s = 2\times10^{-4}$. When lens and beam centres coincide, the lowest even resonance excited, 6/10, is negligibly weak, as shown by the dashed line. However, when the lens is displaced by d/h = 0.05, the 3/5 resonance is excited, and particles with initial amplitudes between about 0.40 and 0.55 reach a common value of maximum displacement, about $\hat{Y}$/h = 0.77 .

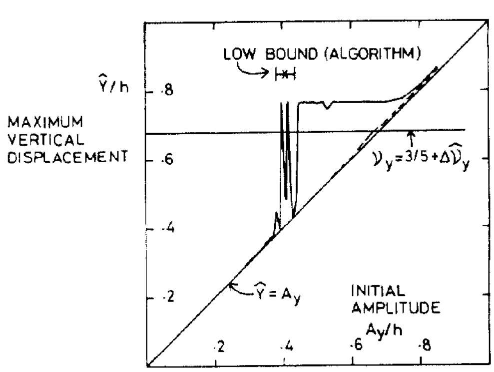


Figure 2

This common accessibility of a boundary in phase space, for a range of initial conditions, is characteristic of stochasticity (or chaos). Figure 3 confirms this interpretation, by showing that a pair of trajectories started very close together, with initial amplitudes near 0.50, diverge exponentially over many orders of magnitude.

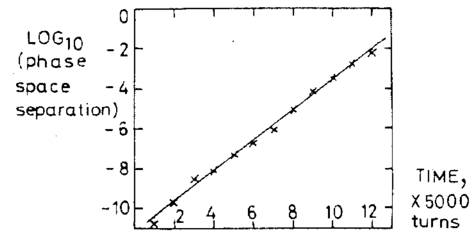


Figure 3

A stochastic band exists in vertical phase space, due to overlapping synchrobetatron sidebands of the nonlinear resonance $\nu_y = 3/5$. The amplitude at which $\nu_y = 3/5 + \Delta\hat{\nu}_y = .6028$ is quite successful as a simple estimate of the higher bound, giving A_y/h = 0.68 . Particles with larger amplitudes no longer cross and recross the resonance. A weaker lens leads to a more unstable situation.

Equation 8) predicts the location of the lower stochastic bound, and implies that the band is wider for small modulation frequencies. However, the simulation cannot necessarily find a clear cut lower bound, even granted infinite computer time, since stable and chaotic phase space regions may be intricately intertwined. The boundary algorithm used here searches down in amplitude steps of 0.01 for the first five 'stable' trajectories, which stay inside an aperture at Y/h = +/-0.65 for 5×10^5 turns. The mean amplitude of these five is taken to represent the location of the lower bound. If the transition from stability to stochasticity is not sharp, see for example figure 2, the variance of the five amplitudes increases, and is represented by large error bars.

Figure 4 shows how the low bound depends on the modulation frequency, confirming that small frequencies are the most dangerous. Stability is reached when the modulation frequency is roughly equal to the depth, when there are not 'many' sidebands to overlap. At the lowest frequency datum, $\nu_s = 10^{-5}$ (0.4 Hz in the SPS), particles are tracked for only five modulation oscillations.

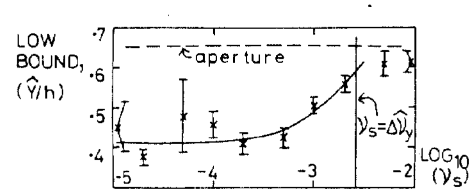


Figure 4

The reintroduction of horizontal motion shows that vertical motion, when $\nu_s = 2\times10^{-4}$, is usually stabilised by moderate amplitude oscillations. Little difference is seen when the horizontal tune modulation and tune shift are turned on, or, as here, turned off. Two dimensional resonances are excited at some horizontal tunes.

Figure 5 shows how the low bound depends on the horizontal amplitude A_x, for a zero amplitude tune of $\nu_x = 0.68$. Increasing amplitudes raise the low bound in a way which is well fitted by a quadratic, probably because the vertical tune drops as the square of small horizontal amplitudes, at a given vertical amplitude.

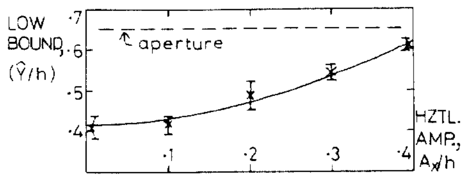
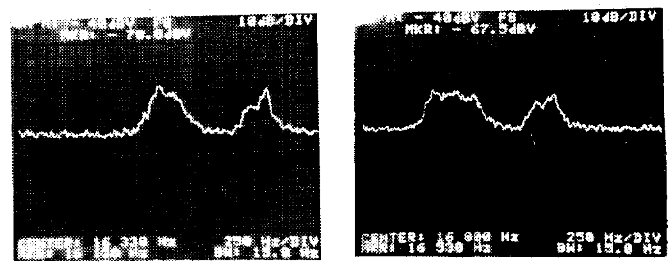


Figure 5



(a)

(b)

Figure 6

Experimental Results

The SPS was set up for acceleration from 26 GeV/c to 270 GeV/c where selected pulses could be stored. The chromaticities, were measured at 270 GeV/c by displacing the beam radially with the r.f. and measuring the tunes. The measured values were $\xi_H = (\Delta\nu_H/\nu_H)/(\Delta p/p) = \xi_V = 0.15$
The momentum spread measured from the voltage and bunch length was $\Delta p/p = \pm 7 \times 10^{-4}$, giving a maximum chromatic tune excursion of $\Delta\hat{\nu} = \pm 2.8 \times 10^{-3}$.

The machine was placed on the 5th order resonance $5\nu_y = 133$, which was excited by the displaced lens with the following characteristics

$h = 37.2$ mm	$\alpha = 0.25$
$I = 1500$ Amps	$\delta = 0.20$
$\Delta\nu_L = 0.005$	$\beta_V = 87.5$ m
$\Delta\nu_{NL} = 2.3 \times 10^{-4}$	$\beta_H = 25$ m
$\Delta\nu_5 = 2.9 \times 10^{-6}$	$\varepsilon_V = 1\pi \times 10^{-6}$ m. rad.

Under these circumstances we would expect the stochastic threshold to be around 10Hz.

The beam lifetime was measured against a scraper which was used to limit the beam emittance to 1π mm.mrad. The machine tune could be modulated in the range 1 - 40 Hz with an amplitude of $\pm 3 \times 10^{-3}$ using one of the machine quadrupoles on a separate power supply. In addition, with the RF on, the tune was modulated at the synchrotron frequency of 200Hz. No means was available to cover the interval 40 - 200 Hz.

Before each measurement, the beam was blown up to fill the aperture by successively exciting the first betatron line until a small beam loss was observed. In this way it was hoped that the initial conditions were re-established.

Figure 6a shows a Schottky scan of the horizontal and vertical betatron lines of the debunched beam after 1 hour of storage with the lens on. The resonance $5\nu_y = 133$ can easily be seen, together with the characteristic 'pile-up' of particles on one side of the resonance due to the nonlinear detuning. The lifetime is 20 hours, which corresponds well with the calculated lifetime of the aperture limited beam. Figure 6b shows the same Schottky scan with 1 Hz modulation at $\hat{\nu} = \pm 3 \times 10^{-3}$. The distribution clearly flattens as the hole fills up. The lifetime drops to 1 hour.

Table 1 shows the results of measurements made under different conditions.

Lens	Tune modulation	Amplitude	τ(h)
OFF	RF (200 Hz)	$\pm 2.8\times10^{-3}$	20
ON	RF (200 Hz)	$\pm 2.8\times10^{-3}$	20
ON	OFF	0	20
OFF	1 - 40 Hz	$\pm 3 \times 10^{-3}$	20
ON	1Hz	$\pm 3 \times 10^{-3}$	1
ON	40 Hz	$\pm 3 \times 10^{-3}$	4

The first four of these measurements showed that the beam lifetime was not affected by the lens with a tune modulation of 200 Hz or by tune modulation in the 1 - 40 Hz region without lens excitation. However, with the lens excited, low frequency modulation was found to have a dramatic effect on beam decay rate. The lifetime was found to increase slowly in the frequency range explored, but no clear threshold behaviour could be found. Unfortunately, due to hardware limitations, the interesting range from 40 Hz to 200 Hz could not be explored.

Conclusions

The catastrophic effect of low frequency tune modulation together with nonlinear resonance excitation has been clearly established both in computer simulations and in experiments on the SPS. The simulation verifies the threshold nature of the phenomenon. From the experimental results it would be tempting to conclude that a stochastic threshold exists somewhere between 40 and 200 Hz. However, the mechanism of the tune modulation in the two cases was different, so no such conclusion can be drawn. More experiments are needed if the means to cover the range 40-200 Hz would become available.

Acknowledgements

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