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THREE PRETZEL EFFECTS

CBN-84-14

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SUMMARY

1) Synchrotron radiation in the pretzel quadrupoles, due to the displaced central orbit, causes the synchrotron and horizontal partition numbers to change, if the pretzels are east-west symmetric. This changes the damping times of electrons and positrons in opposite directions, by a total amount which is statistically expected to be about 10% per centimetre of pretzel. Quadrupole radiation also differentially splits the horizontal betatron emittances of electrons and positrons. The resultant splitting of the horizontal collision size modifies the beam-beam tune shift parameters, but is partially cancelled by the accompanying changes in the energy spreads. Partition number variation with pretzel amplitude is eliminated by using antisymmetric pretzels, or by the design control of a linear lattice sum over pretzel quadrupoles. No direct evidence of any of these effects has been seen, other than the marginally better performance of antisymmetric pretzels.

2) A pretzeled orbit passing through a sextupole gives a horizontal dipole kick of the same magnitude and sign to both beams, causing a common mode closed orbit distortion. This magnetic distortion is usually canceled for one beam outside the pretzel by the adjustment of the electrostatic separation, during the usual pretzel tune-up. However, this makes the exterior orbit of the other beam twice as bad, since for it the magnetic and the electrostatic distortions add together. A pretzel of one centimetre amplitude is statistically expected to cause about a half millimetre orbit split, comparable to the orbit separation due to the energy difference between the two beams[1]. Attempts to measure such orbit splittings in an unexpectedly 'clean' luminosity lattice in CESR have proved inconclusive. The common mode orbit split has an amplitude which is proportional to the square of the pretzel amplitude, and has a phase which is independent of the pretzel symmetry.

3) About half of the natural chromaticity in a CESR luminosity lattice comes from outside the pretzels, while essentially all of the correction chromaticity comes from the 36 out of 38 sextupoles in each arc which are inside a pretzel. The phase advance across the pretzel therefore oscillates appreciably with the off energy parameter, $\delta = \delta E/E$, spoiling the horizontal closure of each pretzel. This is a convenient heuristic description of a contribution to the horizontal dispersion which changes η^* at the collision points by up to about ten times the pretzel amplitude, if the pretzels are symmetric. Electrons and positrons can have their collision dispersions split by as much as ±10%, with 1.5 centimetre pretzels. The additional dispersion wave is present at about the same level with antisymmetric pretzels, but with nodes at both collision points. Measurements made on CESR agree very well with theory[2].

Two typical lattices, B999XX.9AD and H99328.9AE, are used below to represent typical injection and luminosity conditions. In addition to calculating the particular strengths of the three effects for the two lattices, generalised statistical estimates for 1) and 2) are made under the assumption, where necessary, that the betatron phases of quadrupoles and sextupoles are randomly distributed. This is a convenient assumption, but one that is clearly not true in a storage ring with regular cells, and one which is at best arguable for CESR. While the two chosen lattices are theoretically cleaner than expected, a sign that non-random phases lead to significant cancellations in the relevant lattice sums, it is wise to be conservative and take the statistical estimates seriously.

None of these three effects are critically important, while the first two are difficult to measure or observe, even at their statistically predicted levels. Nonetheless, if a particular lattice happened to be two or three standard deviations out in the tail of the ensemble of lattices, either of the first two effects could become unexpectedly important.

The first and the third effects support the general (but not well substantiated) feeling that CESR works better with east-west symmetric pretzels, while the second effect, which is also the most trivial, does not take sides in the arguement.

PRETZEL NOTATION

The betatron phase origin $\phi=0$ is at the south collision point, with increasing phase going westward, and with $p(\phi)$ the west pretzel displacement of the positron beam from the nominal central orbit

$$p(\phi) = p_0\,[\beta_x(\phi)/\beta_{mx}]^{1/2}\,[\sin(\phi_x-\phi_{x1}) / |\sin(\phi_{mx}-\phi_{x1}|]$$
$$= H\,p_0\,\beta_x(\phi)^{1/2}\sin(\phi_x-\phi_{x1}) \qquad (1)$$

while the total separation of electrons and positrons is twice this. Here ϕ_{x1} is the horizontal phase at the beginning of the pretzel, and p_0 is the absolute value of the peak displacement, which occurs at β_{mx} and ϕ_{mx}, with a typical value p_0=1.3 centimetres which corresponds to 1300 scalar units.

All of the sums which will be quoted are sums over the west arc only unless otherwise stated, while the east-west pretzel symmetry is to be specified in each particular case.

PARTITION NUMBER MODIFICATION

DAMPING TIMES

The damping time associated with synchrotron radiation for vertical betatron (z), horizontal betatron (x) or synchrotron (s) oscillations is given by[3,4]

$$\tau_i = (2\,TE/U) / J_i \qquad i = x, z \text{ or } s \qquad (2)$$

where the J_i are the partition numbers and where T is the revolution period, E is the design energy, and U is the nominal energy loss per turn of the storage ring. Assuming from here on that the vertical orbit is flat, then J_z=1, while the other two partition numbers are related to each other through

$$J_x = 1 - D, \qquad J_s = 2 + D \qquad (3)$$

This introduces the key parameter D

$$D = \int \eta\, G(G^2+2K).ds \;/\; \int G^2.ds \qquad (4)$$

where η is the dispersion, G is the horizontal dipole strength 1/R, and K is the quadrupole strength in (metres)$^{-2}$. The integrals are taken azimuthally

around the entire machine, using quantities as actually encountered on the real closed orbit.

In an ideal separated function machine with no orbit errors, and at the correct synchronous energy,

$$GK = 0 \qquad (5)$$

except for the fringe fields at the ends of the dipoles. This means[4,5] that D has a nominal value of

$$D \approx 1/Q^2 \approx 10^{-2} \qquad \text{sector magnets}$$
$$= 0 \qquad \text{rectangular magnets} \qquad (6)$$

so that at CESR, with the pretzels off, D is between these two values.

When the pretzel is turned on there are extra dipole field components

$$G_Q = p.K \qquad (7)$$

due to the offset passage of the closed orbit through the quadrupoles. This causes D to depend on the pretzel amplitude p_0,

$$D(p_0) = \frac{\int_D \eta G^3.ds + \int_Q \eta pK.((pK)^2 + 2K).ds}{\int_D G^2.ds + \int_Q (pK)^2.ds} \qquad (8)$$

where the subscripts D and Q indicate integration over dipoles or quadrupoles. Ignoring the dipole fringe fields, and recognising that second and higher order terms are negligible, (8) is well approximated by

$$D(p_0) = 0.009 + \frac{2.\int_Q \eta pK^2.ds}{\int_D G^2.ds} \qquad (9)$$

which shows that D only varies to first order in pretzel amplitude if the pretzels are east-west symmetric. If this is the case, (9) is simplified by replacing the upper integral with a sum, calculating the lower integral as a geometric constant for CESR, and by dropping the constant term, to give

$$D(p_0) = 25.6\ \Sigma\ p\eta K^2 L \qquad (10)$$

where L=0.6 metres is the length of a regular quadrupole.

There are N=37 independently powered quadrupoles in each pretzel. If their phases really are well randomised, then

$$\langle (D(p_0)/p_0)^2 \rangle^{1/2} = 25.6\ N^{1/2} \langle (p\eta K^2/p_0)^2 \rangle^{1/2} \qquad (11)$$

where the angle brackets represent expectation values over an ensemble of lattices (on the left hand side) or over a typical injection or luminosity lattice (on the right hand side). Table 1 shows the value of $D(p_0)$ as 'expected' by (11), and as 'measured' by (10), for one centimetre pretzels in the lattices B999XX.9AD and H99328.9AE.

Table 1. $D(p_0)$ for p_0 = 1.0 centimetre

	EXPECTED (standard deviation)	MEASURED
INJECTION	0.126	-0.011
LUMINOSITY	0.124	-0.068

The horizontal betatron damping time of one beam in a lattice with east-west symmetric pretzels is expected to decrease by almost 20% when the pretzel amplitude is 1.5 centimetres, while the damping time of the other beam is expected to increase by the same amount. The synchrotron oscillation damping times also change, by half as much, in the opposite direction to the betatron damping times for each beam.

HORIZONTAL COLLISION SIZE

The total horizontal collision size is found by adding the energy and betatron contributions in quadrature

$$\sigma^*_H = (\epsilon_x \beta^*_x + [(\sigma_E/E)\ \eta^*]^2)^{1/2} \qquad (12)$$

where in the chosen CESR luminosity lattice the ratio of the components is

$$r = (\sigma_E/E)\ \eta^* / (\epsilon_x \beta^*_x)^{1/2} = 0.38 \qquad (13)$$

In general the horizontal emittance and the relative energy width depend on the partition numbers J_x and J_s, and through them on D and p_0,

$$\epsilon_x = \epsilon_{x0} / J_x$$
$$(\sigma_E/E)^2 = 2.(\sigma_{E0}/E)^2 / J_s \qquad (14)$$

Combining (3), (12), (13) and (14) the horizontal collision size depends to first order on the pretzel amplitude, for symmetric pretzels, as

$$\begin{aligned} &[(\sigma^*_H(p) - \sigma^*_H(0)) / \sigma^*_H(0)] / p_0 \\ &= [(1-r^2/2) / (2.(1+r^2))] \ .\ D(p_0)/p_0 \\ &= 0.37\ .\ D(p_0)/p_0 \end{aligned} \qquad (15)$$

where real numbers have been inserted for the luminosity lattice.

While one beam grows larger, the other gets smaller, leading to no net first order change in luminosity with p_0. The danger of this effect lies rather in the resultant splitting of the beam-beam tune shift parameters for each beam, causing one beam to be more susceptible than the other at equal beam currents. Note that the larger beam, which sees the stronger beam-beam effect, also has the longer damping time.

According to Table 1, the horizontal sizes and the horizontal tune shifts are expected to change in opposite senses for the two beams by about 7% for 1.5 centimetre symmetric pretzels. In the particular luminosity

lattice used the change is 'measured', in theory, to be only 3.8%, below the level of systematic measurement errors.

CLOSED ORBIT SPLITTING

If the strength of a thin sextupole is m, so that the horizontal kick due to a horizontal displacement x is

$$\Delta x' = -m.x^2 \tag{16}$$

then the dipole kick seen by a pretzeled orbit is

$$\Delta x' = -m.p^2 \tag{17}$$

which has the same sign for electrons and positrons. Adding together the magnetic and electrostatic closed orbit shifts X_M and X_E, the net closed orbit as a function of pretzel amplitude is symbolically written

$$X^{\pm}(p_0) = X^{\pm}(0) \pm p_0.X_E + p_0^2.X_M \tag{18}$$

where the + or − sign is for positrons or electrons.

Now, suppose that the horizontal positron orbit outside the pretzels has been flattened for positrons in the usual way using X_E, by adjusting the phase advance across each pretzel, and by adjusting the separator strengths, so that

$$X^{+}(p_0) - X^{+}(0) = p_0.X_E + p_0^2.X_M = 0 \tag{19}$$

There is a residual horizontal orbit split between the two beams, which outside both pretzels is

$$X^{+}(p_0) - X^{-}(p_0) = [\, X^{+}(0) - X^{-}(0) \,] - 2p_0^2.X_M \tag{20}$$

where the term in square brackets is predicted[1] to be typically less than half a millimetre, due to energy offsets between the two beams. The term $2p_0^2.X_M$ will be compared to this to assess the importance of the effect.

At a point of arbitrary phase ϕ, the magnetic displacement is

given by

$$p_0^2.X_M = (\beta^{1/2} / 2.\sin(\pi Q)).\ \Sigma_{W+E}[-m_S p_S^2 \beta_S^{1/2}.\cos(|\phi - \phi_S| - \pi Q)] \tag{21}$$

which shows that the symmetry of the pretzels is irrelevant. Concentrating now on the peak orbit separations at half integer wavelengths from the north and south collision points, $\phi = 0$ and πQ,

$$\begin{pmatrix} 2(p_0^2.X_M)_N \\ 2(p_0^2.X_M)_S \end{pmatrix} = \frac{2H^2\beta^{1/2}}{\sin(\pi Q)} \begin{pmatrix} -\cos(\phi_1) & \sin(\phi_1) \\ -\cos(\pi Q - \phi_1) & \sin(\pi Q - \phi_1) \end{pmatrix} \begin{pmatrix} S_C \\ S_S \end{pmatrix} \tag{22}$$

where the component sums over the sextupoles in the west pretzel are

$$\begin{pmatrix} S_C \\ S_S \end{pmatrix} = \Sigma\, m\beta^{3/2}.\sin^2(\theta) \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \tag{23}$$

with

$$\theta = \phi - \phi_1 \tag{24}$$

Before making statistical estimates of the sizes of the component sums, it is necessary to invoke the trigonometric identities

$$\begin{aligned} 4.\sin^2(\theta)\cos(\theta) &= \cos(\theta) - \cos(3\theta) \\ 4.\sin^3(\theta) &= 3.\sin(\theta) - \sin(3\theta) \end{aligned} \tag{25}$$

and to note that the 'Sakazakisation' process[6] to eliminate the betatron tune dependence on p_0 makes

$$\Sigma\, m\beta^{3/2}\sin(\theta) = \pi.\Delta Q / Hp_0 = 0 \tag{26}$$

The component sums in (23) can now be replaced with the Fourier sums

$$\begin{pmatrix} S_c \\ S_s \end{pmatrix} = 1/4\,\Sigma\, m\beta^{3/2} \begin{pmatrix} \cos(\theta) - \cos(3\theta) \\ -\sin(3\theta) \end{pmatrix} \tag{27}$$

Under the naive assumption that the 36 sextupoles in each pretzel are uncorrelated except for (26), the expected standard deviations of the component sums in an ensemble of lattices are

$$\begin{pmatrix} \langle S_c^2\rangle^{1/2} \\ \langle S_s^2\rangle^{1/2} \end{pmatrix} = \langle m^2\beta^3\rangle^{1/2} \begin{pmatrix} 3/2 \\ 3/2\sqrt{2} \end{pmatrix} \tag{28}$$

The actual components in the luminosity lattice H99328.9AE, where the r.m.s. normalised strength of the Sakazaki sextupole distribution is

$$\langle m^2\beta^3\rangle^{1/2} = 10.0 \text{ metres}^{-1/2} \tag{29}$$

are less than the expected ensemble standard deviations,

$$\begin{pmatrix} S_c / \langle S_c^2\rangle^{1/2} \\ S_s / \langle S_s^2\rangle^{1/2} \end{pmatrix} = \begin{pmatrix} 0.27 \\ 0.34 \end{pmatrix} \tag{30}$$

suggesting that the sextupoles are not in fact well randomised. Using these 'measured' values, and the miscellaneous luminosity lattice conditions

$$\begin{aligned} H^{-2} &= 36.0 \quad \text{metres} \\ Q &= 9.385 \\ \Phi_1 &= 4.507 \\ p_0 &= 0.015 \quad \text{metres} \end{aligned} \tag{31}$$

then the estimated size of the orbit splitting amplitude, at a point with a typical β = 36.0 metres, is

$$\begin{pmatrix} 2(p_0^2 x_M)_N \\ 2(p_0^2 x_M)_S \end{pmatrix} = \begin{pmatrix} 2.72 \\ 2.16 \end{pmatrix} \cdot 10^{-4} \text{ metres} \tag{32}$$

These separations are somewhat smaller than the energy difference effects already mentioned, and are below the common levels of pretzel tuning. Horizontal orbit measurements which were made on CESR to look for the splitting had systematic errors of about this size, and were therefore inconclusive.

DISPERSION PERTURBATION

A substantial proportion, 1-a, of the natural horizontal chromaticity, X_N, comes from the interaction region quadrupoles and from other regular quadrupoles outside the pretzels. On the other hand a fraction b≈1 of the correction chromaticity, X_S, comes from the sextupoles inside the pretzels, so that the net horizontal chromaticity is

$$X_T = -X_N + X_S \tag{33}$$

The horizontal phase advance across a pretzel differs from 6π, to first order in the off energy parameter δ, by

$$\begin{aligned} \Delta\Phi_x &= [\,(b-a).\,X_N + b.\,X_T\,].\pi\delta \\ &= X_p.\pi\delta \end{aligned} \tag{34}$$

where in the luminosity lattice, save set #5761, the quantities of interest are

$$\begin{aligned} a &= 0.42 \\ b &= 1.00 \\ X_N &= 20.3 \\ X_T &= 0.3 \\ X_p &= 12.1 \end{aligned} \tag{35}$$

It should be noted that b values larger than 1.0 are not unphysical.

Since a pretzel is therefore not perfectly closed for off energy particles, there is a piece of the horizontal dispersion which is proportional to p_0, the pretzel amplitude. If the pretzels are east-west symmetric the extra contribution will have peaks at half-integer wavelengths away from either of the collision points, of opposite sign for opposite kinds of particle. At a peak of the dispersion wave

$$\begin{pmatrix} \Delta\eta_N \\ \\ \Delta\eta_S \end{pmatrix} = X_p \cdot \frac{\pi p_0 \cdot (\beta/\beta_{mx})^{1/2}}{\sin(\pi Q)} \begin{pmatrix} \cos(\phi_1) \\ \\ \cos(\phi_1 - \pi Q) \end{pmatrix} = \begin{pmatrix} -0.124 \\ \\ 0.603 \end{pmatrix} \text{metres} \quad (36)$$

where values from (31) and (35), and a typical β_x of 36.0 metres have been used to get the numerical values. When scaled down to the south collision β^*_x of 0.914 metres, the expected contribution is $\Delta\eta^*_S = \pm\, 0.096$ metres, which begins to be significant when compared to the design $\eta^*_S = 0.88$ metres.

Measurements in CESR[2] with antisymmetric pretzels on and off show a perturbation of the dispersion with nodes at the collision points, as expected, with peak values which are in very good agreement with the predicted amplitudes, which are similar to those above. Here at last is a well founded measurement which implies that the pretzels in CESR should indeed be east-west antisymmetric.

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