PHYS 684.1
September 10, 2003

Homework #1

Due on September 17.

1. Read Edwards & Syphers chapter 3 to the end of 3.2.2.
2. The Lorentz transformations of the electromagnetic fields are given by

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}) \quad (1)$$
$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \quad (2)$$
$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \vec{v} \times \vec{E}_{\perp}) \quad (3)$$
$$\vec{B}'_{\parallel} = \vec{B}_{\parallel} \quad (4)$$

   where the $\parallel$ designates the component of the field parallel to the boost velocity $\vec{v}$ and $\perp$ is for the perpendicular. Using these expressions compute the electromagnetic fields of a singly charged particle traveling at the relativistic velocity v.

3. a) Show that the dipole field of a window-frame dipole enclosed by iron of infinite permeability $\mu_r = \infty$ is given by

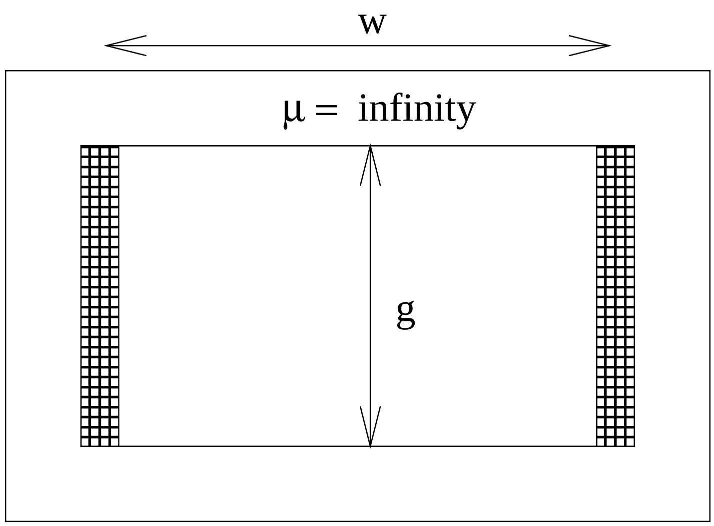
$$B = \mu_0 \frac{NI}{g} \quad (5)$$

   where N is the number of turns, I is the current in each turn, and g is the vertical gap size.

   b) If l and w are the dipole length and width, show that the inductance is

$$L = \mu_0 N^2 \frac{lw}{g} \quad (6)$$

   c) If the resistivity of the wire is ρ and the *TOTAL* coil cross sectional area on each side of the gap is is A, what is the total power dissipation in the magnet?



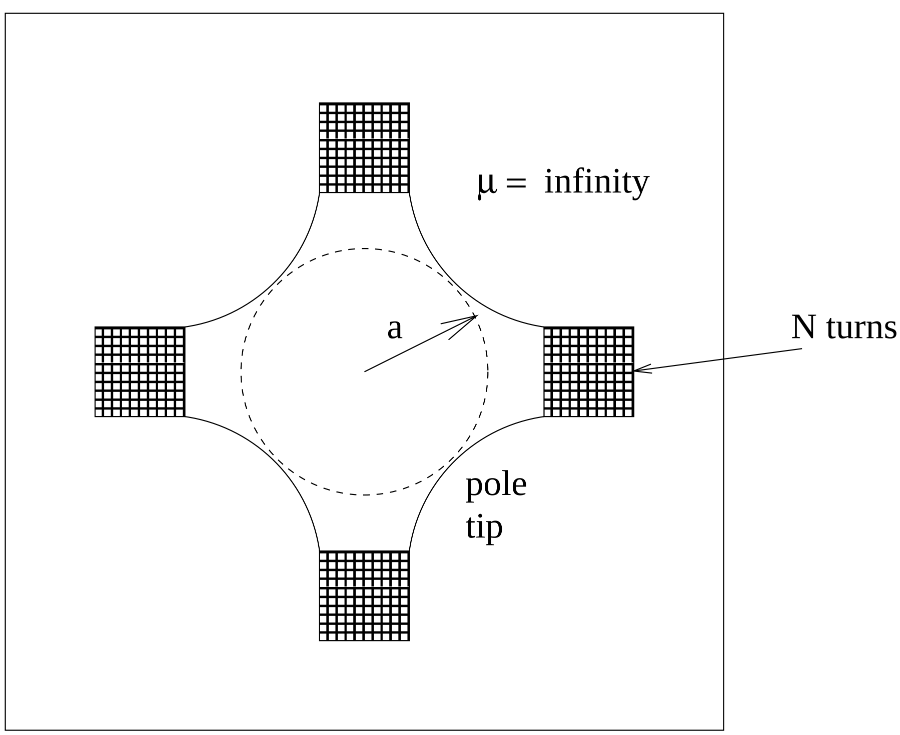
4. In a current free region one of Maxwell's laws states that $\nabla \times \vec{B} = 0$, so that the magnetic field can be derived from a magnetic scalar potential Φ through $\vec{B} = \nabla\Phi$.

   a) If the horizontal and vertical fields in a quadrupole are $B_x = B'\,y$ and $B_y = B'\,x$, where B' is the constant *field gradient*, what is the shape of an equipotential surface?

   b) Why is the pole tip of an ideal quadrupole ($\mu_r = \infty$) an equipotential surface?

   c) The radius of the inscribed circle touching the pole tips is a. Show that the field gradient is

$$B' = \mu_0 \frac{NI}{a^2} \tag{7}$$



5. a) Show that the general solution to the scalar magnetic potential

$$\Phi = C\, r^n \sin n\phi \tag{8}$$

satisfies Laplace's equation $\nabla^2\Phi = 0$.

b) Show that in cylindrical coordinates the magnetic field may then be written

$$B_r = Cnr^{n-1} \sin n\phi \tag{9}$$

$$B_\phi = Cnr^{n-1} \cos n\phi \tag{10}$$

c) If such a magnet is built from iron, how many poles does it have?