Talk dates back to 11/90

Local, Linear, Transverse Coupling in Storage Rings

Peter Bagley, Fermilab
Work was done at CESR, Cornell Univ.

Dave Rubin
Michael Billings
Raphael Littauer

My main references

Edwards and Teng

Parametrization of Linear Coupled Motion in Periodic Systems

IEEE Trans. on Nuc. Sci. Vol NS-20 No 3 June 1973

Peggs

Coupling and Decoupling in Storage Rings

IEEE Trans. on Nuc. Sci. Vol NS-30, No. 4, Aug. 1983

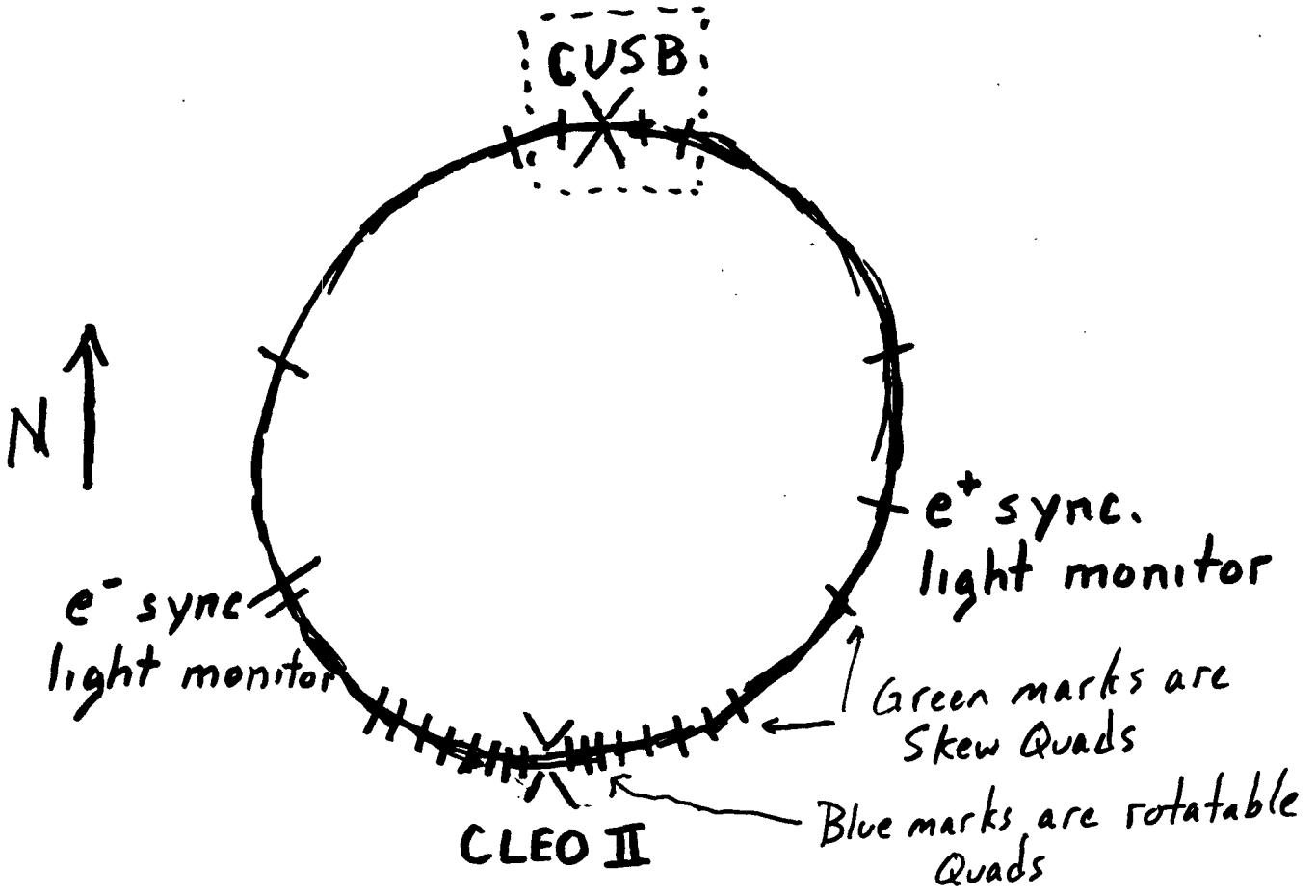
The Projection Approach to Solenoid Compensation

Particle Accelerators 1982, Vol. 12 pp. 219-229

Billing

The Theory of Weakly Coupled Transverse Motion in Storage Rings

CESR Note CBN85-2

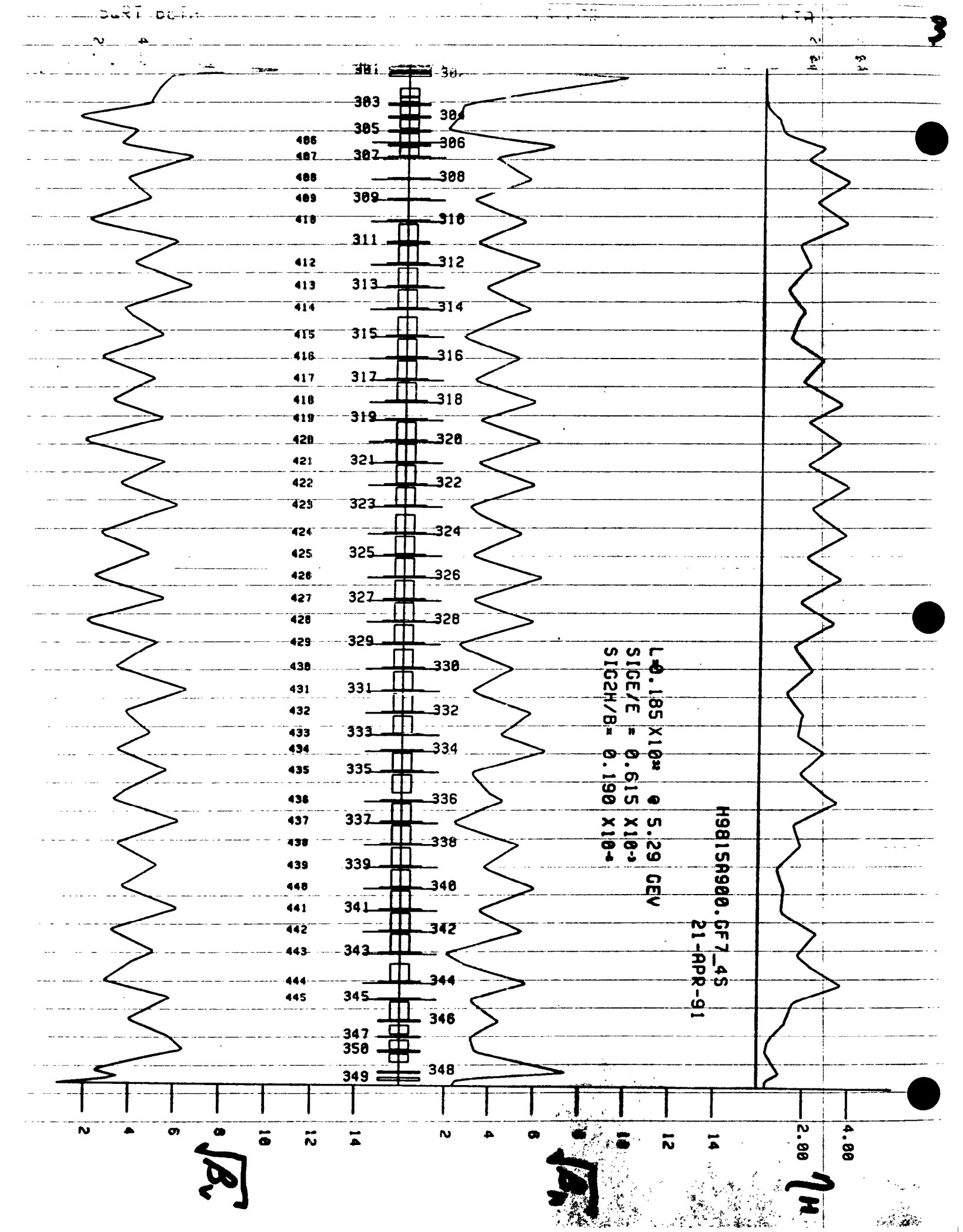


Circumference = 768. m

e^+e^- storage ring

Υ energy region

(~ 5 GeV/beam)


H9815A900.CF7_4S
21-APR-91
L=0.185 X10**
SIGE/E = 0.615 X10-3
SIG2H/B= 0.190 X10-4
ηH

```
ETA= VERT   HORZ    ETA    LUMINOSITY            NU    CHROMATICITY                PAGE       0            09:19    21-APR-91
  0.0155   1.0902  0.004   0.1852E+32   VERT  9.6514  -24.74  SIG2H/B 0.18986E-06  ALP/P  0.015694  H9815A900.GF7_4S
  0.4746   5.5019  0.361   0.1454E+31   HORZ  8.5871  -16.66  SIGE/E  0.61494E-03  CIRC  768.42603M
                                                                                                                GEV=        5.2890
REA:  IR1 LARGE APERTURE   IR1 UPTO NORMAL      RF  STRAIGHT      NORMAL LATTICE  IR2 LARGE APERTURE          MEV/TURN    1.0290
PER V   72.542  1ST 351    49.693  1ST 307    38.134  1ST 309    49.545  1ST 350    31.474  1ST 349           OPT COUP    0.1192
ETA V  103.010  1ST 351    41.492  1ST 307    20.845  1ST 309    41.183  1ST 350    12.144  1ST 349   PARTIC /BUNCH 0.1315E+12
PER H   96.231  1ST 302    76.081  1ST 308    74.347  1ST 310    78.516  1ST 322    75.522  1ST 348           LIMIT       L0
ETA H   97.910  1ST 302    42.892  1ST 203    27.823  1ST 310    40.701  1ST 334    54.703  1ST 348           POLIZ    180.3 MIN
ETA      0.019  1ST 302     3.701  1ST 308     3.661  1ST 310     3.794  1ST 322     0.958  1ST 348           NBUNCHES= 1
OD,EVEN # SEXT STRENGTHS= -0.2193  0.1016

adiation integral values:[wigglers on]  I1=0.1235E+02  I2=0.1002E+00  I3=0.2087E-02  I4=0.3039E-02  I5=0.5965E-03  U0= 1.1047
H>[at wigglers]=  0.4466E+00  Emittance=0.2448E-06  [Averages at RF]  Bv=15.95045  Bh=17.89410  Eta= 3.03838

 104 201 103 0/  110 351 111 301 112 302 113 203 114 204 115 303 116
02 400 101 304 102 202 400 101 305 102 202 406 101 306 102 202 124  307 407 117 118 308 408 109 309 409 109 310 410 1 311
1 312 412 120 1 313 413 1 314 414 121 1 315 415 1  316 416 1 317 417 1 318 418 105 319 419 1 320 420 1 321 421 1
22 422 1 323 423 122 1 324 424 1 325 425 1  326 426 126 1 327 427 1 328 428 126 1 329 429 123 1 330 430 126 1
31 431 1 332 432 126 1 333 433 106 334 434 134 201 103  335 435 139 201 103 135 336 436 136 201 103 337 437 1 338 438 126 1
39 439 1 340 440 1 341 441 1 342 442 1 343 443  150 208 151 344 444 127 128 345 445 104 208 151
46 400 153 206 145 347 146 207 107 350  147 203 143 141 348 148 349 140/

301 0.6000 -0.428090   334 0.6000  0.211503                  201 6.57273  87.89150   101 0.71230          134 0.56830
302 0.9500  0.524788   335 0.6000 -0.182631                  202 3.23649  31.65434   102 0.49515          135 2.61376
303 0.6000 -0.270600   336 0.6000  0.220960                  203 2.94526 140.62640   103 0.15714          136 0.68354
304 0.6000  0.421296   337 0.6000 -0.213221                  204 1.64350  87.89150   104 0.87304
305 0.6000 -0.379460   338 0.6000  0.209680                                          105 6.06000
306 0.6000  0.388279   339 0.6000 -0.162880                  206 3.28698  87.89150   106 5.37000          139 1.02550
307 0.6000 -0.295172   340 0.6000  0.182159                  207 3.17612  34.81718   107 0.50175          140 2.03000
308 0.6000  0.183343   341 0.6000 -0.203877                  208 6.57184  58.60328                        141 1.71999
309 0.6000 -0.250077   342 0.6000  0.238529                                          109 7.38945          142 0.50000
310 0.6000  0.210809   343 0.6000 -0.189860                                          110 0.64000          143 1.65200
311 0.6000 -0.241999   344 0.6000  0.262620                                          111 0.38070
312 0.6000  0.196555   345 0.6000 -0.219200                                          112 0.55930          145 0.85380
313 0.6000 -0.190217   346 0.6000  0.186899                                          113 4.65370          146 1.10564
314 0.6000  0.199337   347 0.6000 -0.055174                                          114 0.37721          147 0.79013
315 0.6000 -0.206946   348 0.6000  0.538050                                          115 0.60947          148 2.02000
316 0.6000  0.227763   349 0.9500 -0.596467                                          116 0.23538          149 0.60000
317 0.6000 -0.232147   350 0.6000 -0.185151                                          117 5.70945          150 3.64560
318 0.6000  0.229979   351 1.2200 -0.837600                                          118 1.68000          151 0.18260
319 0.6000 -0.299469                                                                                       152 5.73981
320 0.6000  0.207700                                                                  120 0.46000          153 1.32600
321 0.6000 -0.264381                                                                  121 1.92000
322 0.6000  0.218378                                                                  122 2.00000
323 0.6000 -0.215507                                                                  123 1.08000
324 0.6000  0.200819                                                                  124 0.29801
325 0.6000 -0.270373
326 0.6000  0.225874                                                                  126 0.22500
327 0.6000 -0.301116                                                                  127 2.39981
328 0.6000  0.241073                                                                  128 3.34000
329 0.6000 -0.245740
330 0.6000  0.217103
331 0.6000 -0.201986
332 0.6000  0.162235
333 0.6000 -0.167200
```

Global Decoupling Recipe

1) Use normal (upright) quads to bring the tunes together.
   Typically there is a minimum tune split
2) Use skew quads to make the tunes equal
3) Use normal (upright) quads to return to the operating point.

Can also measure and correct η_v, but
η_v is also generated by vertical bends
No information about skew quads where
η_h is small.

Problems with Global Decoupling

It is not sensitive to $(\varphi_A + \varphi_B)$ coupling component.

Not unique. Can typically use any of several SQ's.

Measurement is not made at the operating point.

Minor Concerns:

- When return to the operating pt., change the $(\varphi_A - \varphi_B)$ component.
- Global Decoupling is not defined unless the machine is on the difference resonance
- Depends on how the machine is brought to the difference resonance.

Machine can be globally decoupled and still have a large σ_v. (local coupling)

Another analogy:

global decoupling $\Longleftrightarrow$ correcting the tune

local decoupling $\Longleftrightarrow$ correcting the β's

$\Longrightarrow$ Global Decoupling is still a very useful proceedure. If the operating point is near the difference resonance can quickly and easily greatly reduce the coupling.

Decomposition of Full Turn Transfer Matrix into Normal Modes

Edwards and Teng

$$T = \begin{pmatrix} M & n \\ m & N \end{pmatrix} = V U V^{-1}$$

$$= \begin{pmatrix} \gamma I & C \\ -C^+ & \gamma I \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \gamma I & -C \\ C^+ & \gamma I \end{pmatrix}$$

where $\gamma^2 + \det C \equiv 1$ $\quad \gamma \approx 1$ usually

$$C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \Rightarrow C^+ \equiv \begin{pmatrix} C_{22} & -C_{12} \\ -C_{21} & C_{11} \end{pmatrix}$$

$$A = \begin{pmatrix} \cos 2\pi\nu_A + \alpha_A \sin 2\pi\nu_A & \beta_A \sin 2\pi\nu_A \\ -\gamma_A \sin 2\pi\nu_A & \cos 2\pi\nu_A - \alpha_A \sin 2\pi\nu_A \end{pmatrix}$$

and similarly for B

$$W = \begin{pmatrix} a_A \sqrt{\beta_A} \cos\varphi_A \\ \frac{-a_A}{\sqrt{\beta_A}} (\sin\varphi_A + \alpha_A \cos\varphi_A) \\ a_B \sqrt{\beta_B} \cos\varphi_B \\ \frac{-a_B}{\sqrt{\beta_B}} (\sin\varphi_B + \alpha_B \cos\varphi_B) \end{pmatrix} = V^{-1} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = V^{-1} \chi$$

The matrix V translates from the normal mode coordinates to the horizontal and vertical coordinates.

This is true for energy displacements also

$$\begin{pmatrix} \eta_A \\ \eta_A' \\ \eta_B \\ \eta_B' \end{pmatrix} = V^{-1} \begin{pmatrix} \eta_x \\ \eta_x' \\ \eta_y \\ \eta_y' \end{pmatrix} = \begin{pmatrix} \gamma I & -C \\ C^+ & \gamma I \end{pmatrix} \begin{pmatrix} \eta_x \\ \eta_x' \\ \eta_y \\ \eta_y' \end{pmatrix}$$

Given the normal mode dispersions and twiss parameters, the normal mode emittances may be calculated from the usual recipe (as per Sands).

Actually must be careful with damping partition numbers. (Reference Raubenheimer)

PAC Chicago 1989

Separate the dependence on the twiss parameters.

Peggs $$\overline{W} = GW = \begin{pmatrix} G_A & 0 \\ 0 & G_B \end{pmatrix} W = \begin{pmatrix} a_A \cos\varphi_A \\ -a_A \sin\varphi_A \\ a_B \cos\varphi_B \\ -a_B \sin\varphi_B \end{pmatrix}$$

where $$G_A = \begin{pmatrix} \frac{1}{\sqrt{\beta_A}} & 0 \\ \frac{\alpha_A}{\sqrt{\beta_A}} & \sqrt{\beta_A} \end{pmatrix}$$

and similarly for G_B

$$\overline{V} = GVG^{-1} = \begin{pmatrix} \gamma I & G_A C G_B^{-1} \\ -G_B C^+ G_A^{-1} & \gamma I \end{pmatrix} = \begin{pmatrix} \gamma I & \bar{C} \\ -\bar{C}^+ & \gamma I \end{pmatrix}$$

X

Relative Phase and Amplitude

Calculate the horz. and vert. motion that results from a coherent excitation of the "A" mode.

Let the initial state be

$$\overline{W}_0 = (a_A, 0, 0, 0)$$

n turns later $\varphi_A = 2\pi n \nu_A$

$$X_n = V W_n = V U^n W_0 = G^{-1} \bar{V} \bar{U}^n \overline{W}_0$$

$$= \underbrace{\begin{pmatrix} \sqrt{\beta_A} & 0 & 0 & 0 \\ \frac{-\alpha_A}{\sqrt{\beta_A}} & \frac{1}{\sqrt{\beta_A}} & 0 & 0 \\ 0 & 0 & \sqrt{\beta_B} & 0 \\ 0 & 0 & \frac{-\alpha_B}{\sqrt{\beta_B}} & \frac{1}{\sqrt{\beta_B}} \end{pmatrix}}_{G^{-1}} \underbrace{\begin{pmatrix} \gamma & 0 & \bar{C}_{11} & \bar{C}_{12} \\ 0 & \gamma & \bar{C}_{21} & \bar{C}_{22} \\ -\bar{C}_{22} & \bar{C}_{12} & \gamma & 0 \\ \bar{C}_{21} & -\bar{C}_{11} & 0 & \gamma \end{pmatrix}}_{\bar{V}} \underbrace{\begin{pmatrix} a_A \cos\varphi_A \\ -a_A \sin\varphi_A \\ 0 \\ 0 \end{pmatrix}}_{\bar{U}^n \overline{W}_0}$$

$$= \underbrace{\begin{pmatrix} \sqrt{\beta_A} & 0 & 0 & 0 \\ \frac{-\alpha_A}{\sqrt{\beta_A}} & \frac{1}{\sqrt{\beta_A}} & 0 & 0 \\ 0 & 0 & \sqrt{\beta_B} & 0 \\ 0 & 0 & \frac{-\alpha_B}{\sqrt{\beta_B}} & \frac{1}{\sqrt{\beta_B}} \end{pmatrix}}_{G^{-1}} \begin{pmatrix} a_A \gamma \cos\varphi_A \\ -a_A \gamma \sin\varphi_A \\ -a_A (\bar{C}_{22}\cos\varphi_A + \bar{C}_{12}\sin\varphi_A) \\ a_A (\bar{C}_{21}\cos\varphi_A + \bar{C}_{11}\sin\varphi_A) \end{pmatrix} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$

$$= G^{-1} \begin{pmatrix} a_A \gamma \cos\varphi_A \\ -a_A \gamma \sin\varphi_A \\ a_A \sqrt{\bar{C}_{12}^2 + \bar{C}_{22}^2}\, \cos(\varphi_A + \Delta\varphi_A) \\ -a_A \sqrt{\bar{C}_{11}^2 + \bar{C}_{21}^2}\, \sin(\varphi_A + \Delta\varphi_A + \Delta\varphi_{AY}) \end{pmatrix}$$

Relative Phase and Amplitude

Calculate the horiz. and vert. motion that results from a coherent excitation of the "A" mode.

Let the initial state be

$$\overline{W}_0 = (a_A, 0, 0, 0)^T$$

n turns later $\varphi_A = 2\pi n \nu_A$

$$X_n = V W_n = V U^n W_0 = G^{-1} \overline{V} \overline{U}^n \overline{W}_0$$

Define $\Delta\varphi_A$ so that

$$\cos\Delta\varphi_A = \frac{-\bar{C}_{22}}{\sqrt{\bar{C}_{12}^2 + \bar{C}_{22}^2}} \qquad \sin\Delta\varphi_A = \frac{\bar{C}_{12}}{\sqrt{\bar{C}_{12}^2 + \bar{C}_{22}^2}}$$

Then

$$x = a_A \gamma \sqrt{\beta_A} \cos\varphi_A \equiv x_{amp} \cos\varphi_x$$

$$y = a_A \sqrt{\beta_B} \sqrt{\bar{C}_{12}^2 + \bar{C}_{22}^2} \cos(\varphi_A + \Delta\varphi_A) \equiv y_{amp} \cos\varphi$$

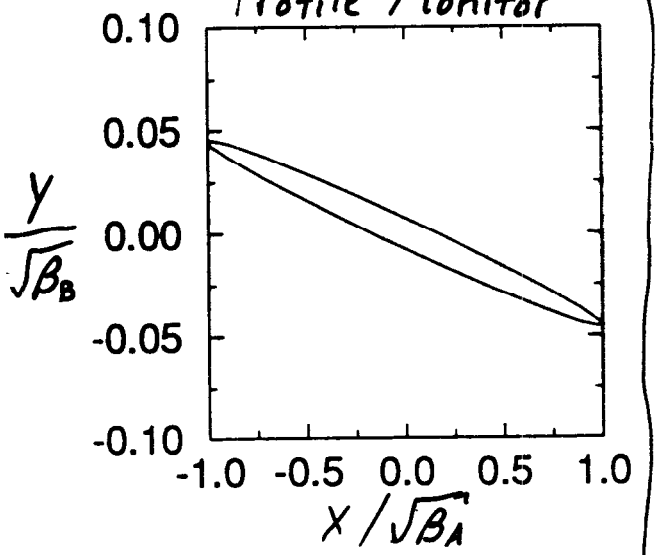
$$\left(\frac{y}{x}\right)_A \equiv \frac{y_{amp}}{x_{amp}} = \frac{1}{\gamma}\sqrt{\frac{\beta_B}{\beta_A}}\sqrt{\bar{C}_{12}^2 + \bar{C}_{22}^2}$$

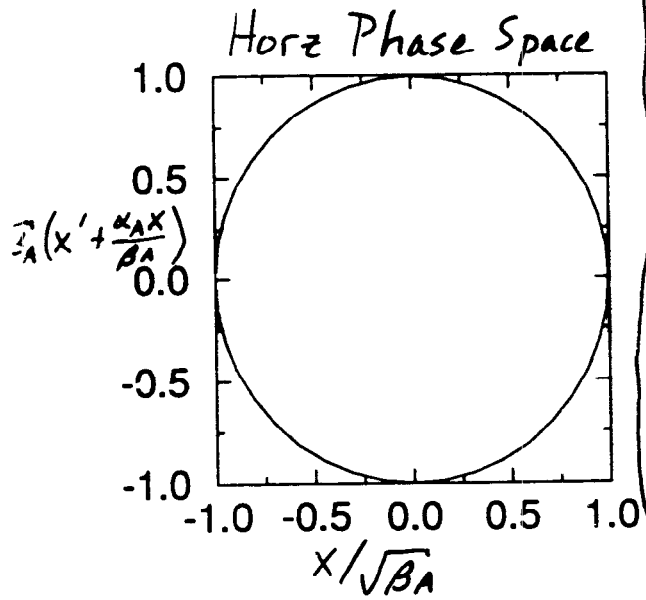
$$(\varphi_y - \varphi_x)_A = \Delta\varphi_A$$

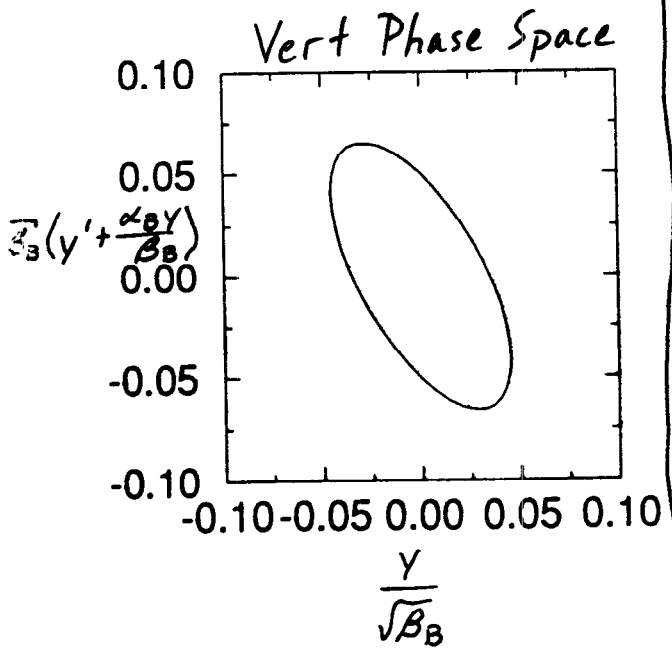
$$\bar{C}_{12} = \gamma\sqrt{\frac{\beta_A}{\beta_B}}\left(\frac{y}{x}\right)_A \sin\Delta\varphi_A \quad \rightarrow \text{out of phase} \rightarrow \text{blow up}$$

$$\bar{C}_{22} = -\gamma\sqrt{\frac{\beta_A}{\beta_B}}\left(\frac{y}{x}\right)_A \cos\Delta\varphi_A \quad \rightarrow \text{in phase} \rightarrow \text{tilt}$$

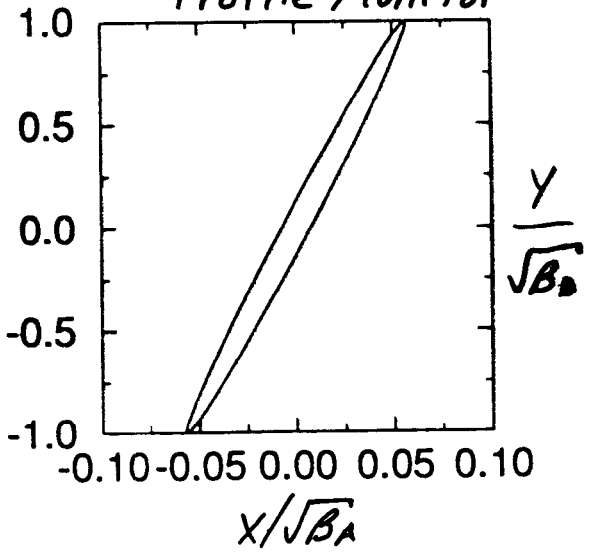
A mode only

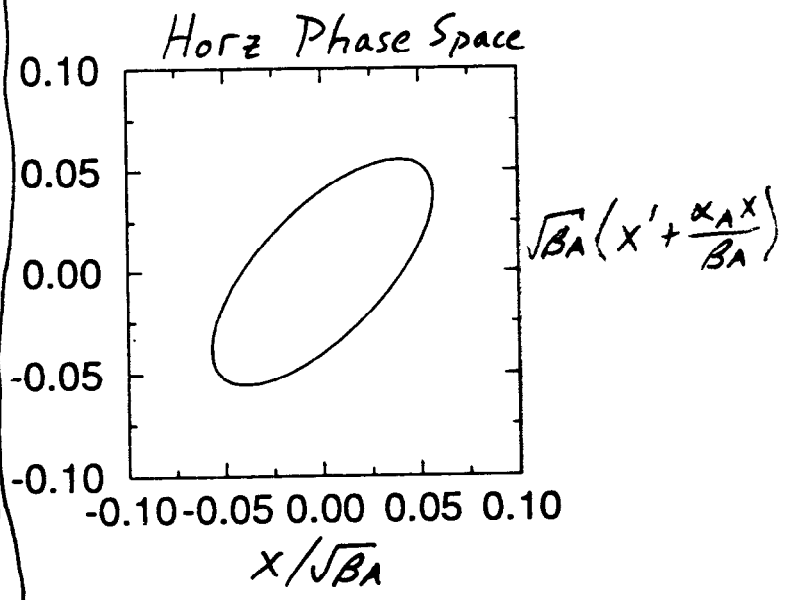


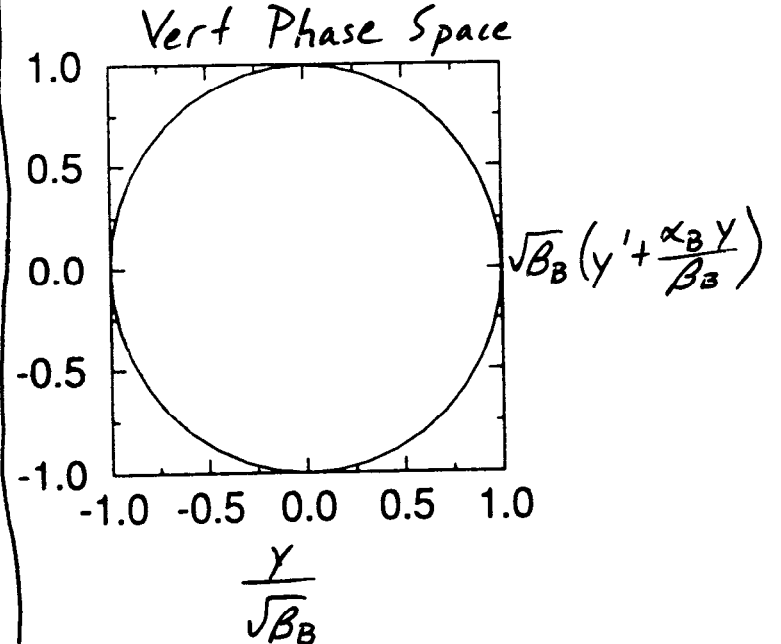




B mode only







$\bar{C}_{11} = .0565$

$\bar{C}_{12} = .0075$

$\bar{C}_{21} = .0328$

$\bar{C}_{22} = .0449$

$\gamma = .99885$

Generally must convolute the 2 ellipses.

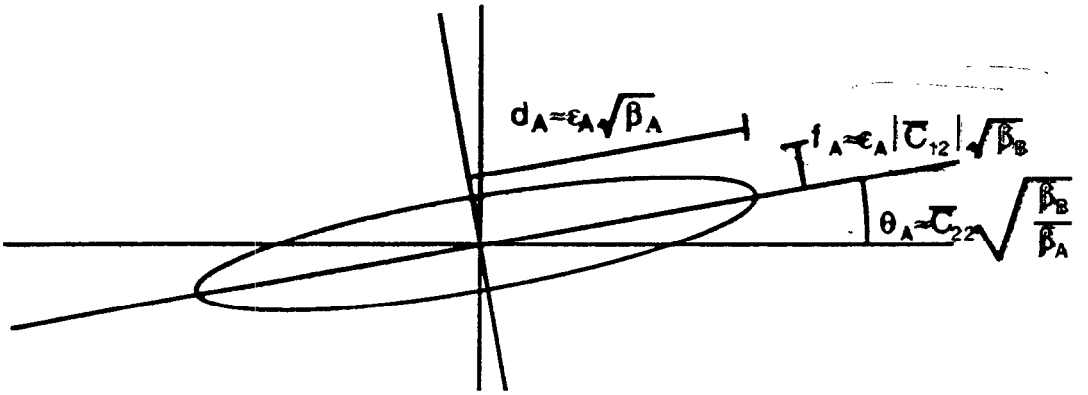


Fig.1A Beam ellipse for weak coupling and the "A" mode excited

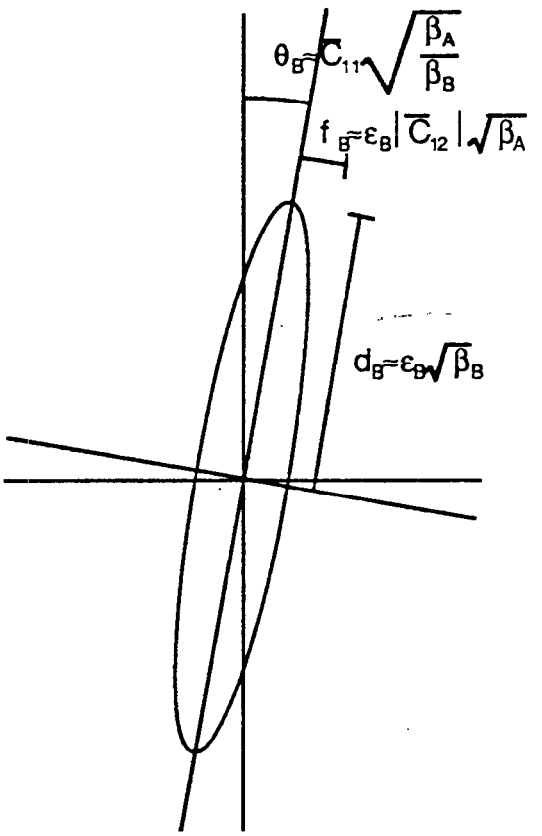


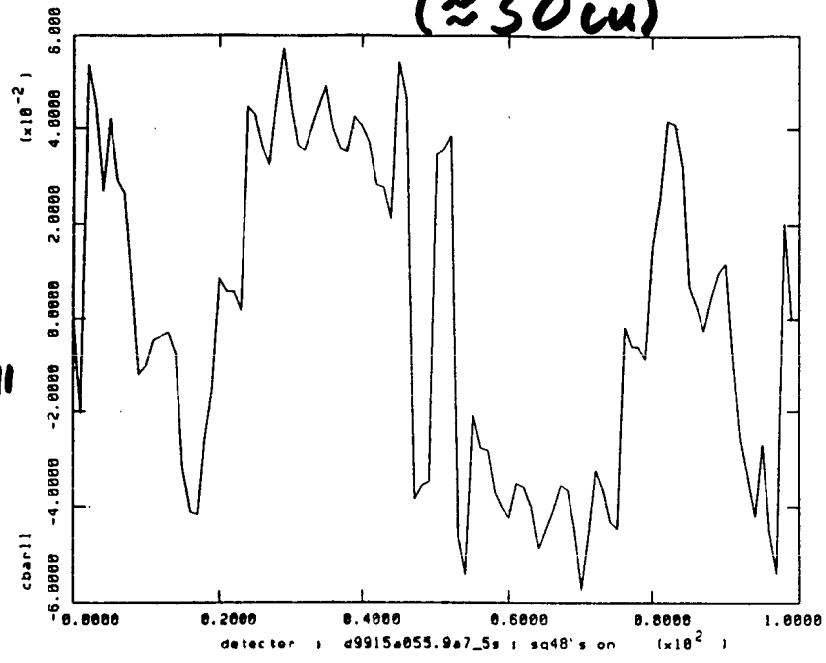
Fig.1B Beam ellipse for weak coupling and the "B" mode excited

For weak coupling

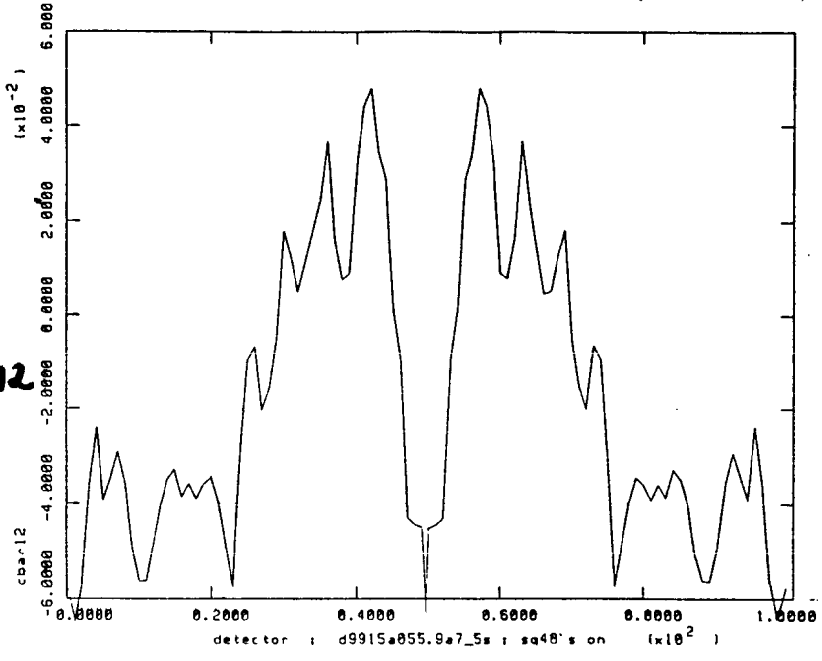
$$\sigma_v = \sqrt{\epsilon_B \beta_B + \epsilon_A \beta_B \bar{C}_{12}^2 + \left(\eta \frac{\sigma_\epsilon}{\epsilon}\right)^2}$$

$$\text{"}\epsilon_v\text{"} = \epsilon_B + \epsilon_A \bar{C}_{12}^2$$

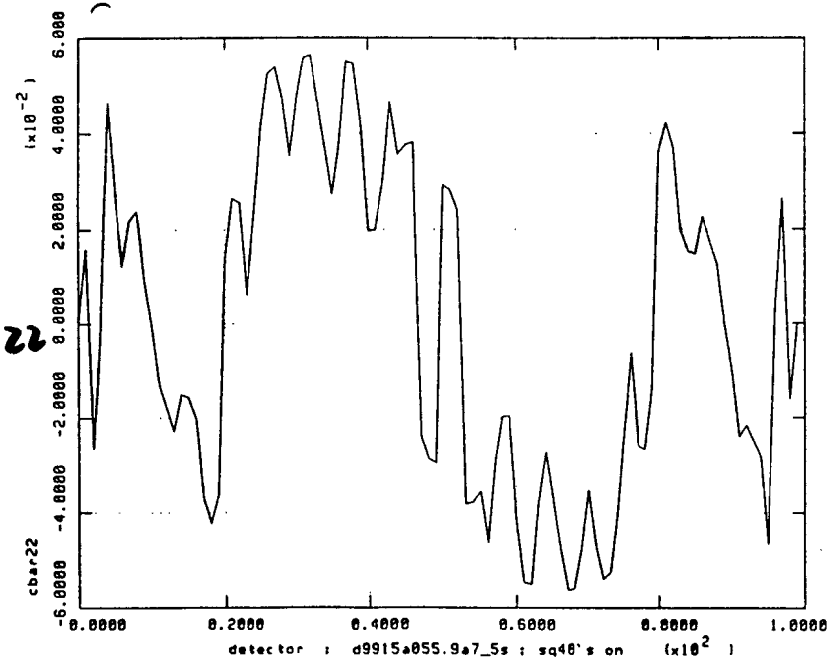
Skew Quads at 48 powered antisymmetrically (≈50 ea)



11



$\bar{C}_{12}$



-22

emittances

.1756e-6 m

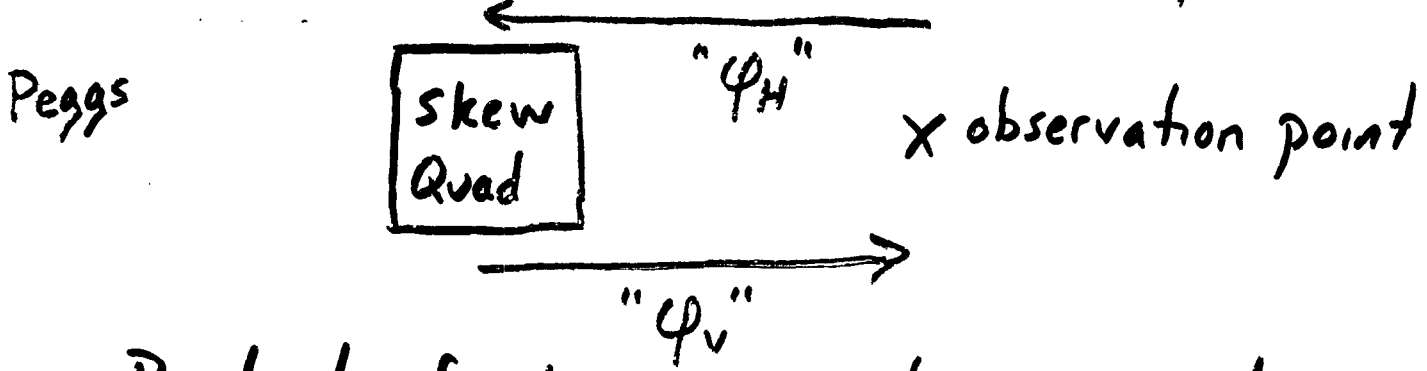
.4507e-9 m (.4260e-10m) for sol'n + comp. only

tunes 9.4058
9.3530

Propagation of $\bar{C}$

Want to look at the effect of an upstream skew quad.

Project the horz. motion backwards to the skew quad. The horz. displacement at the skew quad produces a vert. kick. Project the vert. kick forwards to the observation point.



Product of these projections give terms that propagate as $(\varphi_A - \varphi_B)$ and $(\varphi_A + \varphi_B)$

Since coupling propagates as $(\varphi_A + \varphi_B)$ and as $(\varphi_A - \varphi_B)$, get resonances at

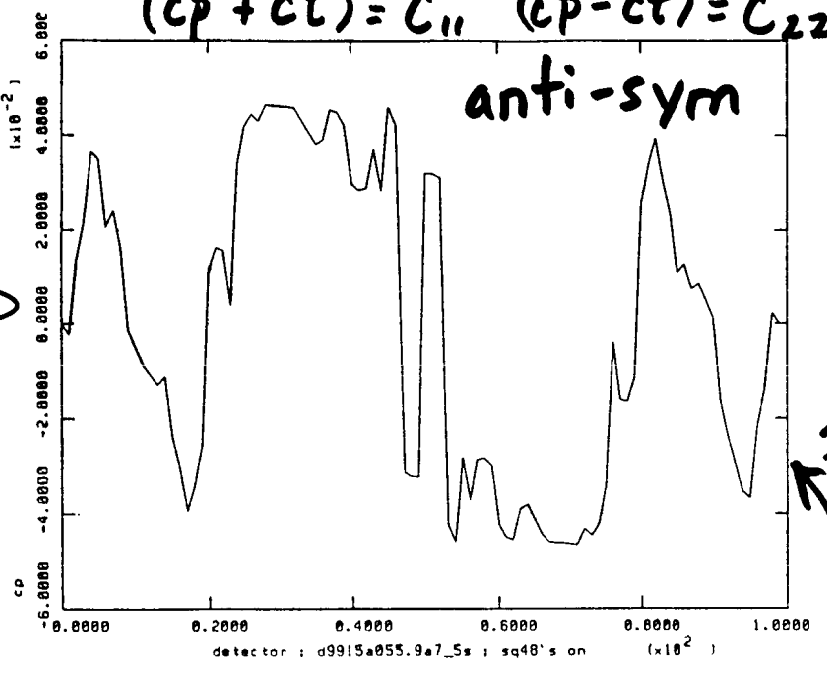
$$2\nu_A + 2\nu_B = n$$
$$2\nu_A - 2\nu_B = n \Rightarrow$$

Many machines run near here.

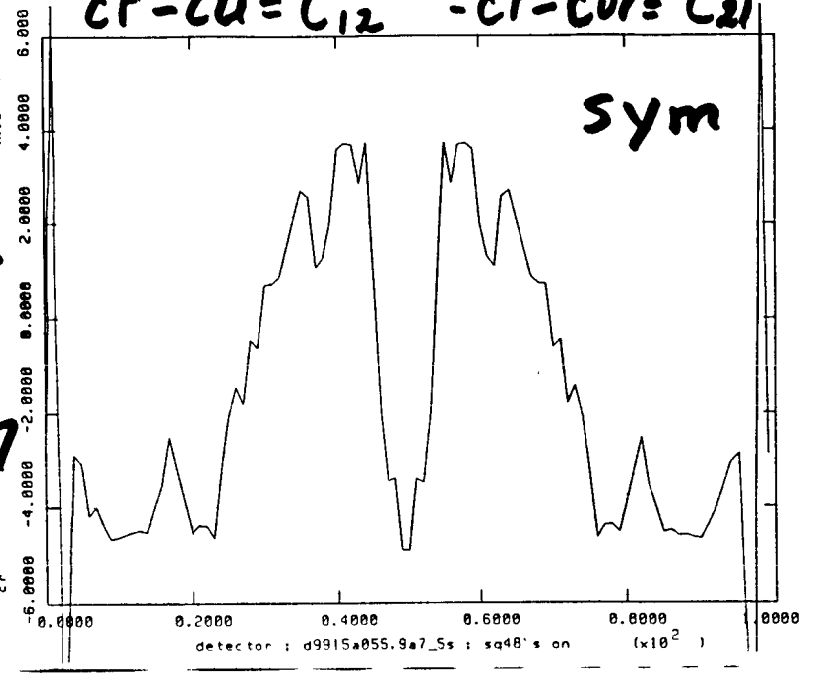
Analogous to how the β wave from a gradient error propagates.

Skew Quads at 48 powered antisymmetrically

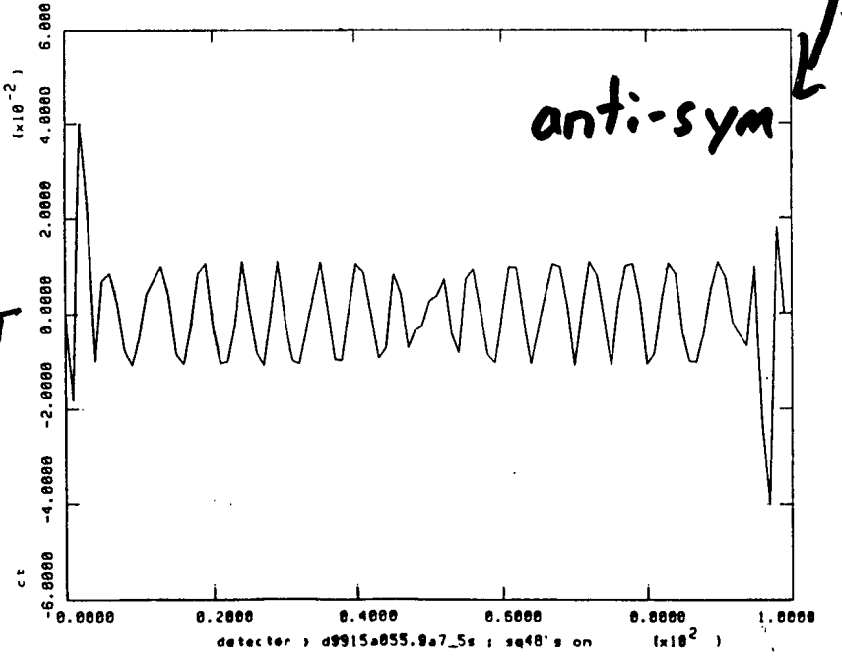
$(cp + ct) = \bar{C}_{11}$ $(cp - ct) = \bar{C}_{22}$

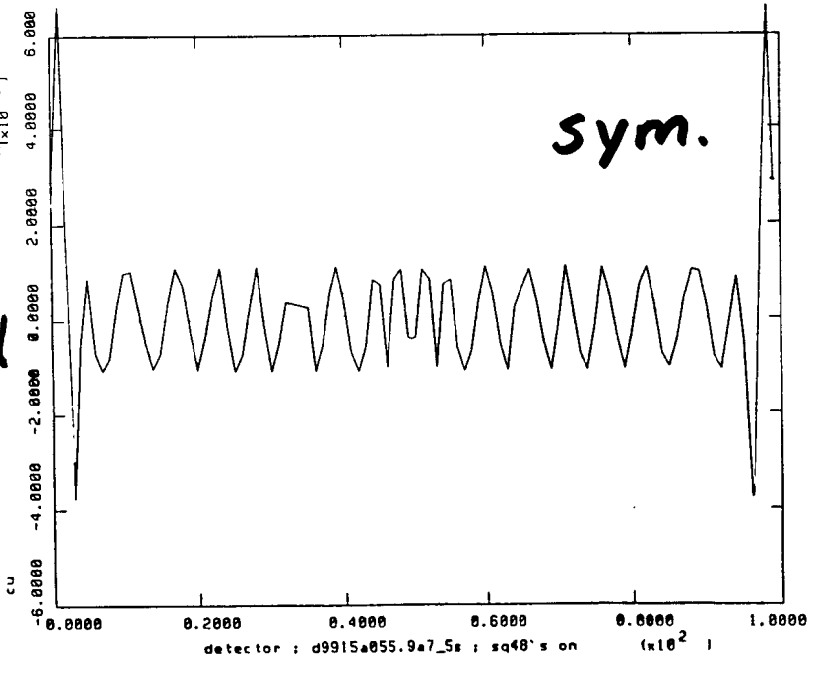


$cr - cu = \bar{C}_{12}$ $-cr - cu = \bar{C}_{21}$



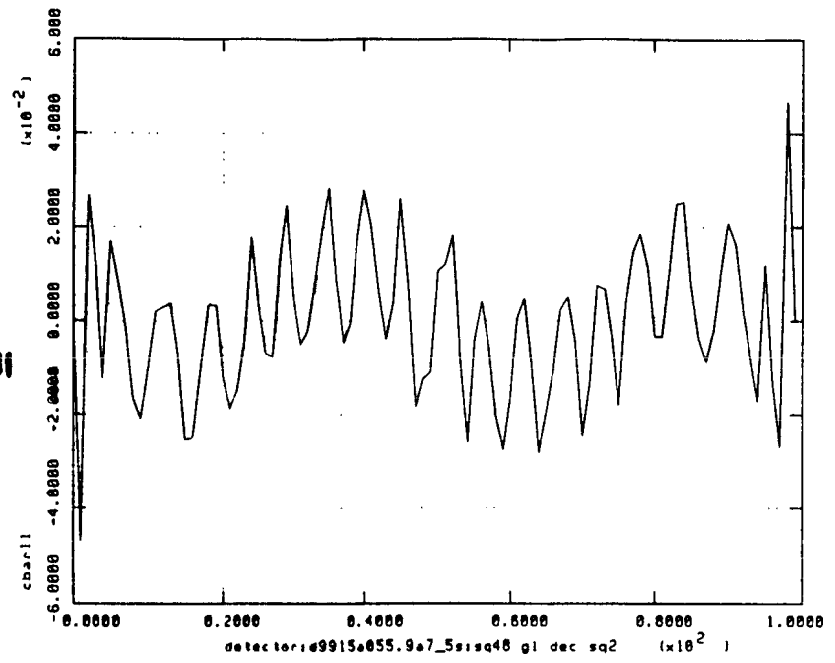
≈ factor of 4 due to prox. to coup. reson.



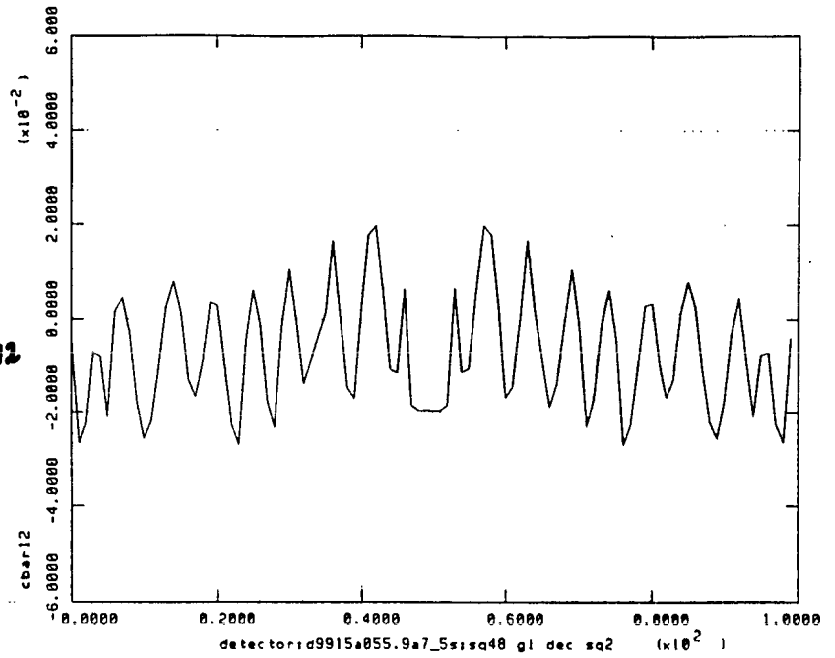


Skew quads at 2 and 10 powered anti-sym. Globally decoup

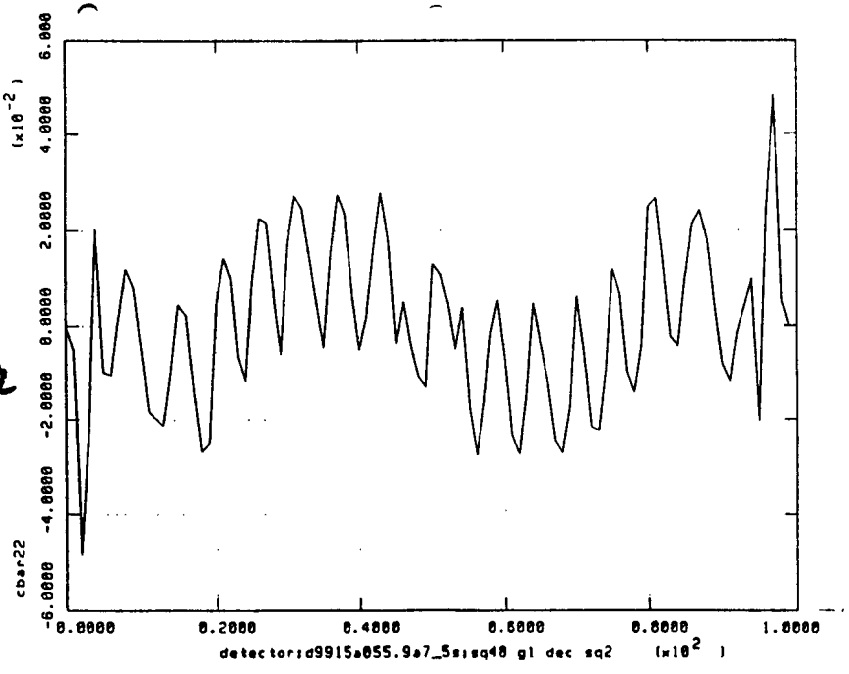
$\bar{C}_{11}$



$\bar{C}_{12}$



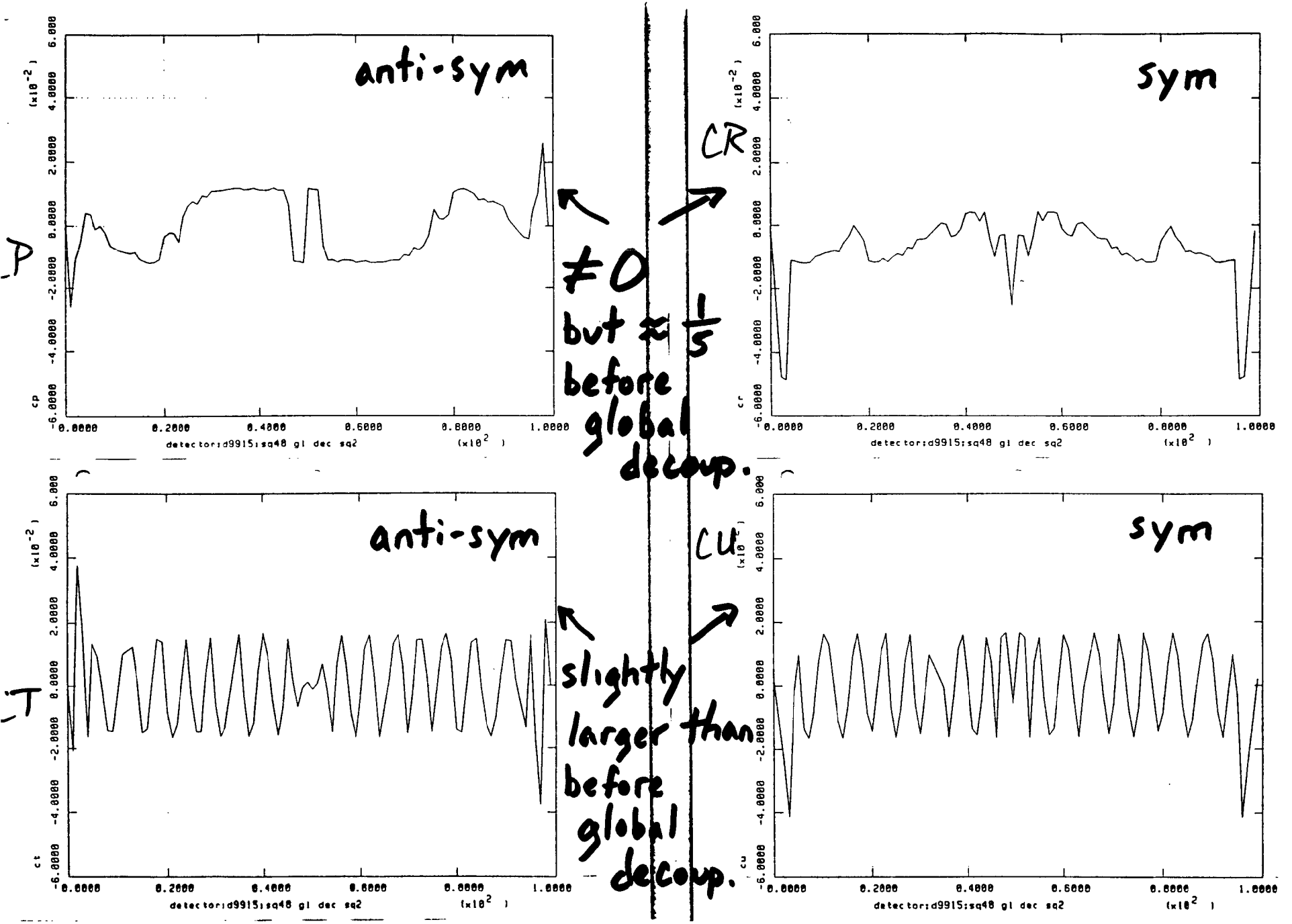
$\bar{C}_{22}$

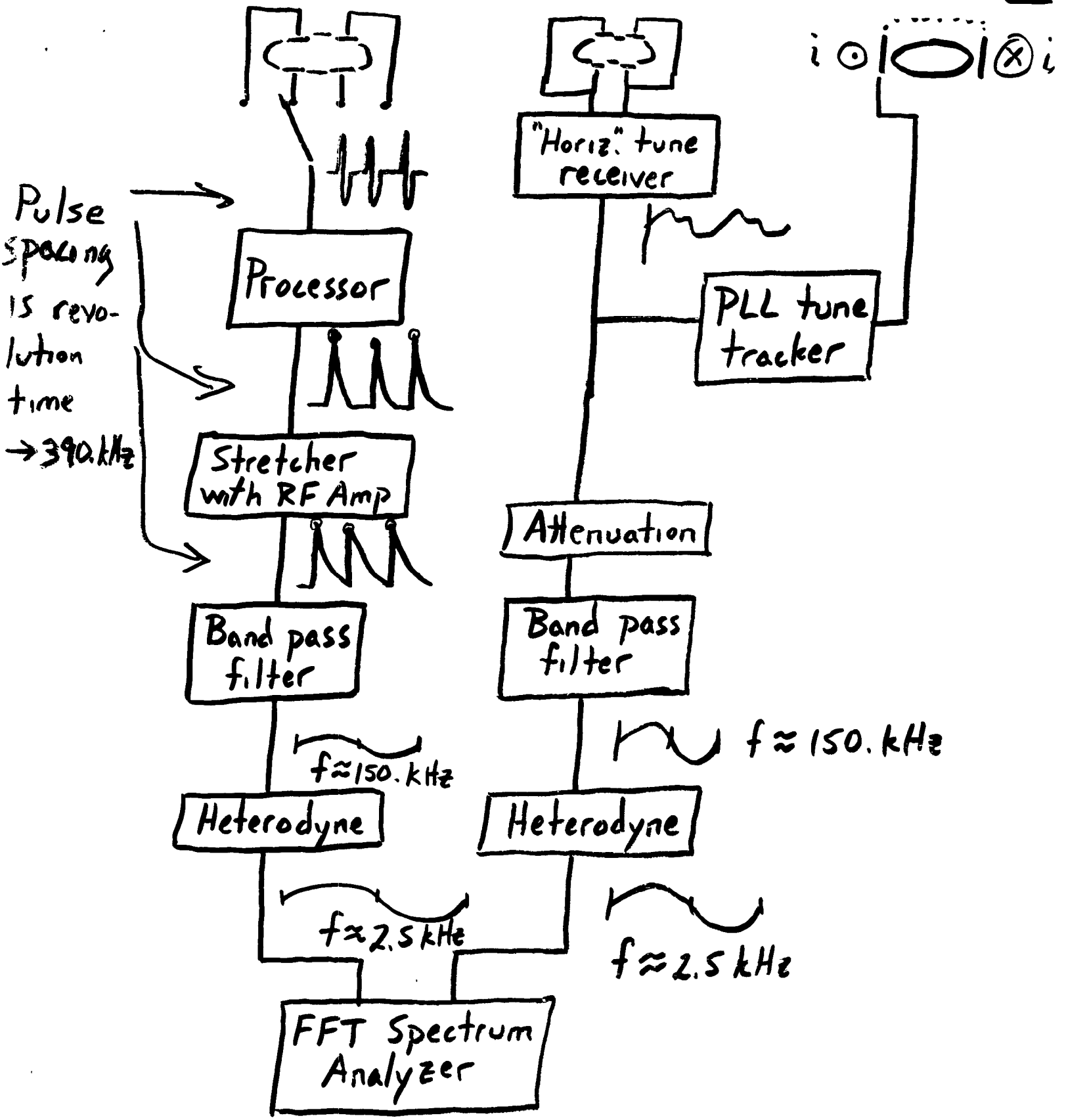


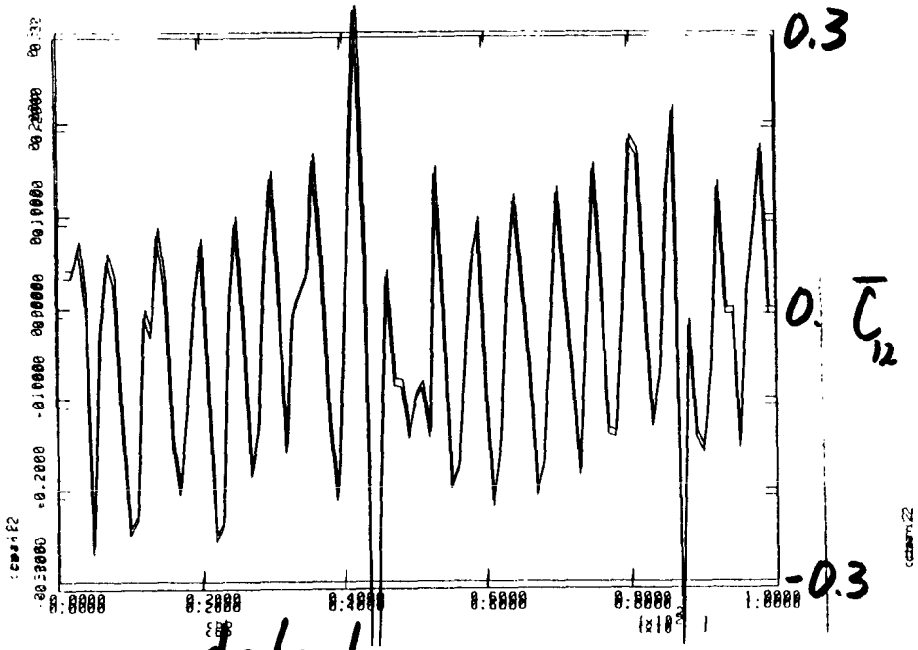
tunes .4058
.3530

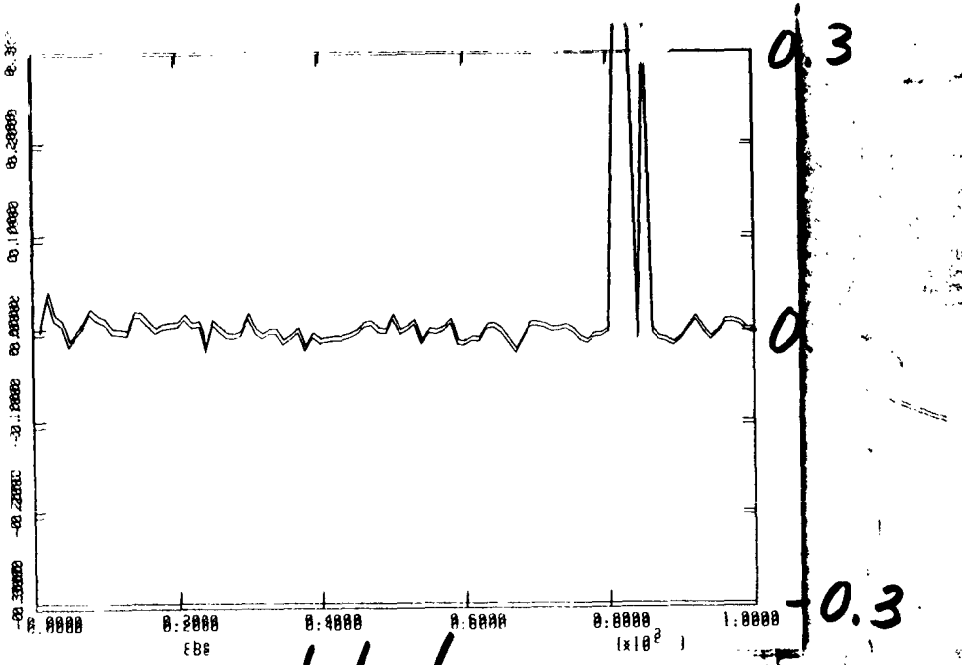
emittances .1761e-6 m
.1234e-9m (.4260e-10m)
for
sol'n + comp.
only

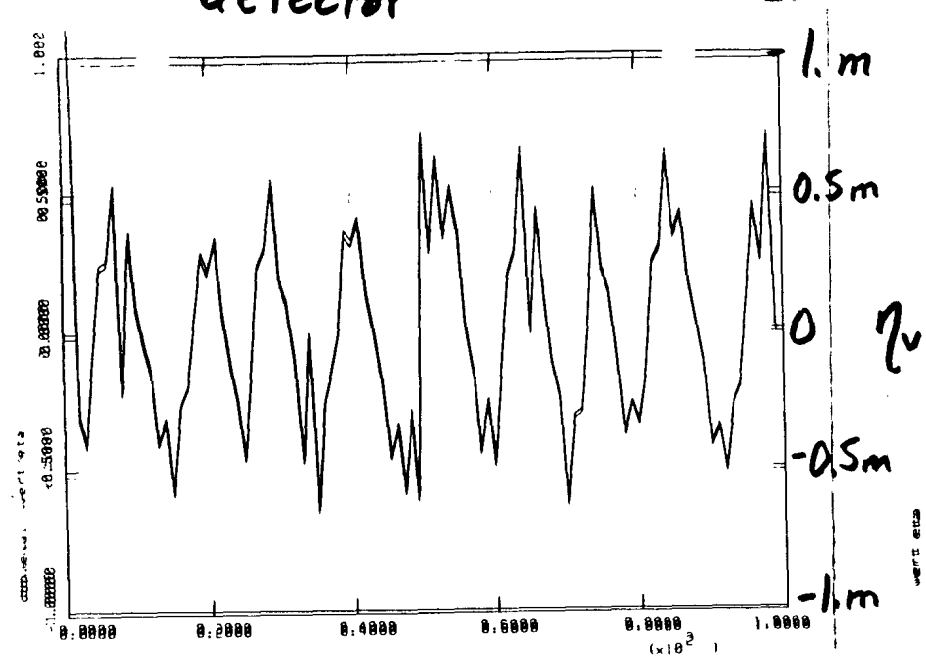
Show quads at [illegible] to perform anti-sym Globally decoup.

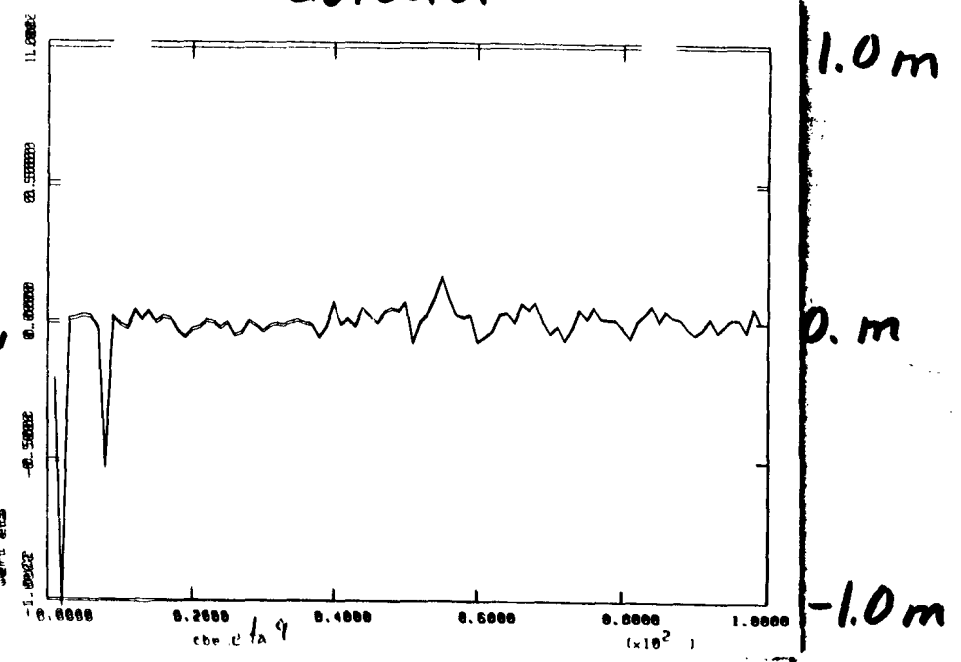



"Horiz." tune receiver
PLL tune tracker
Pulse spacing is revolution time → 390. kHz
Processor
Stretcher with RF Amp
Attenuation
Band pass filter
Band pass filter
f ≈ 150. kHz
f ≈ 150. kHz
Heterodyne
Heterodyne
f ≈ 2.5 kHz
f ≈ 2.5 kHz
FFT Spectrum Analyzer





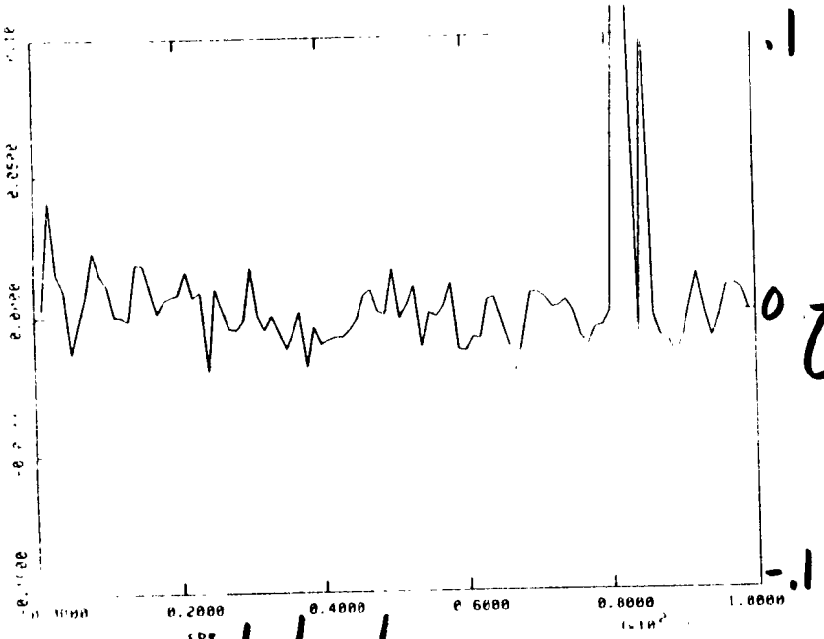




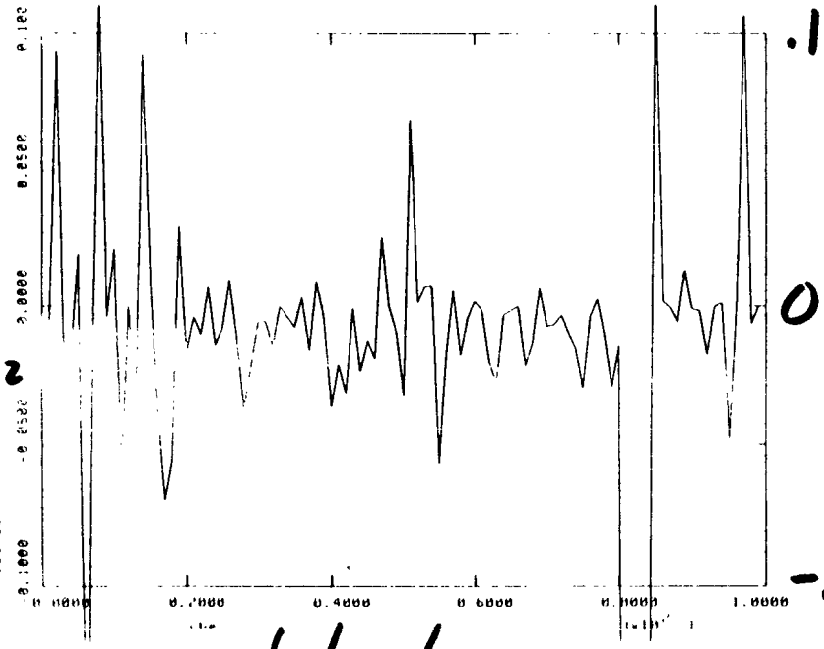
Before

10854 GLEO sol'n off

After

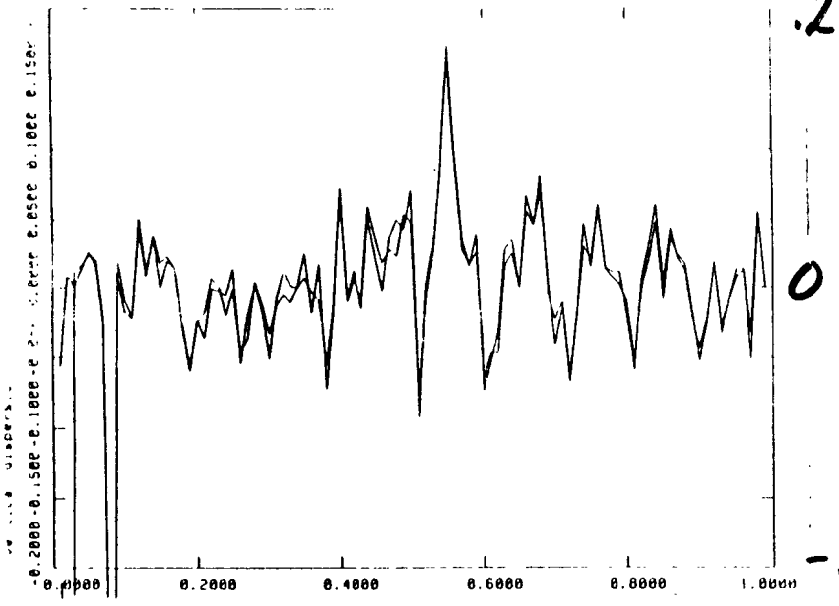


$\bar{C}_{22}$



detector

detector



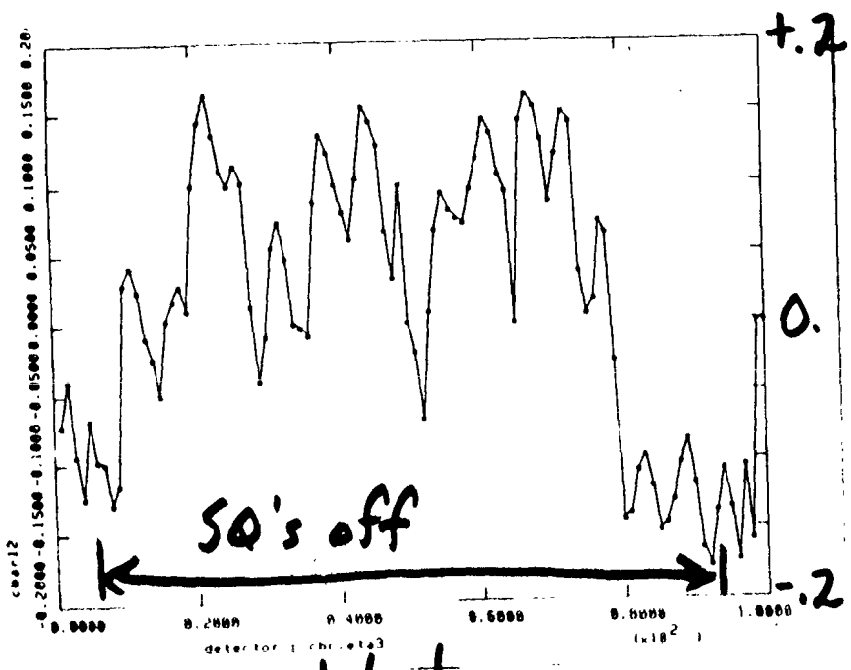
After

Note that $\bar{C}_{22}$ is much noiser than $\bar{C}_{12}$

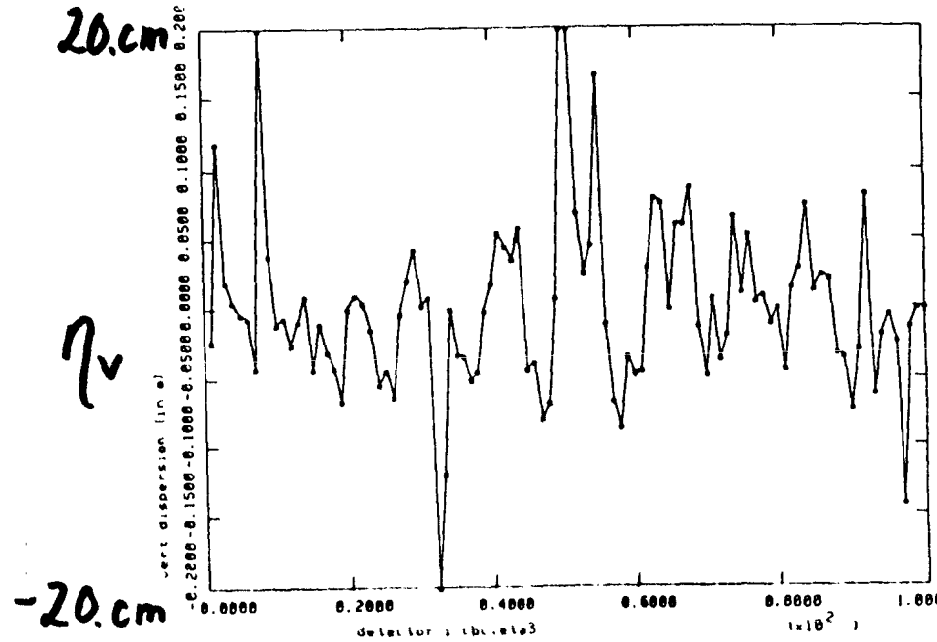
CLEO sol'n off

Before

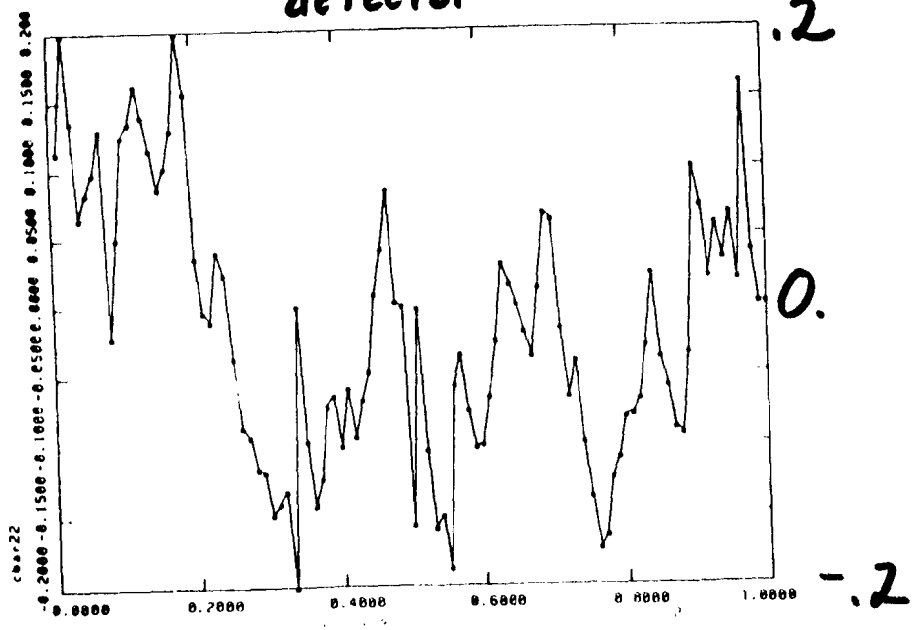
$\bar{C}_{12}$



η_v



$\bar{C}_{22}$

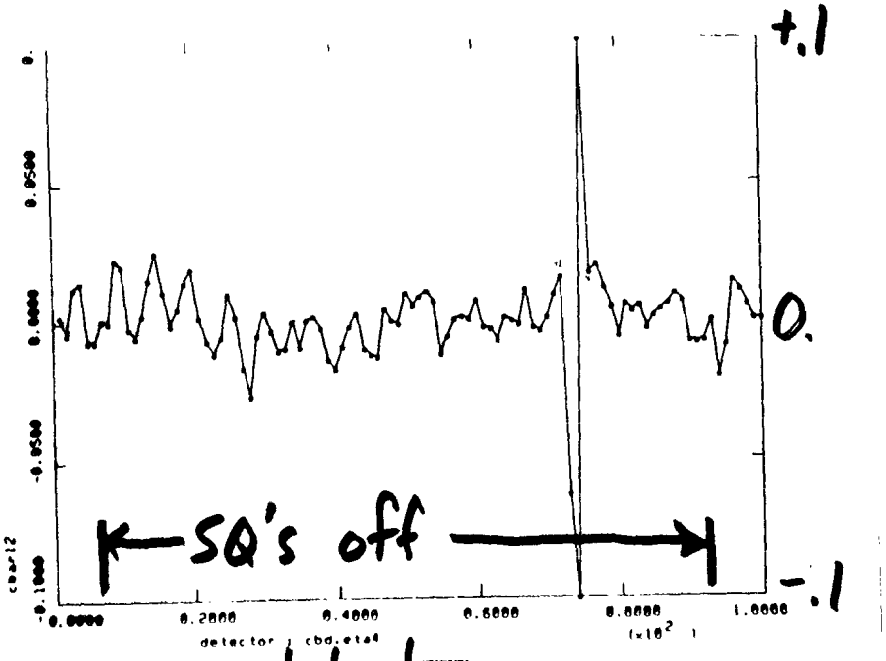


910325

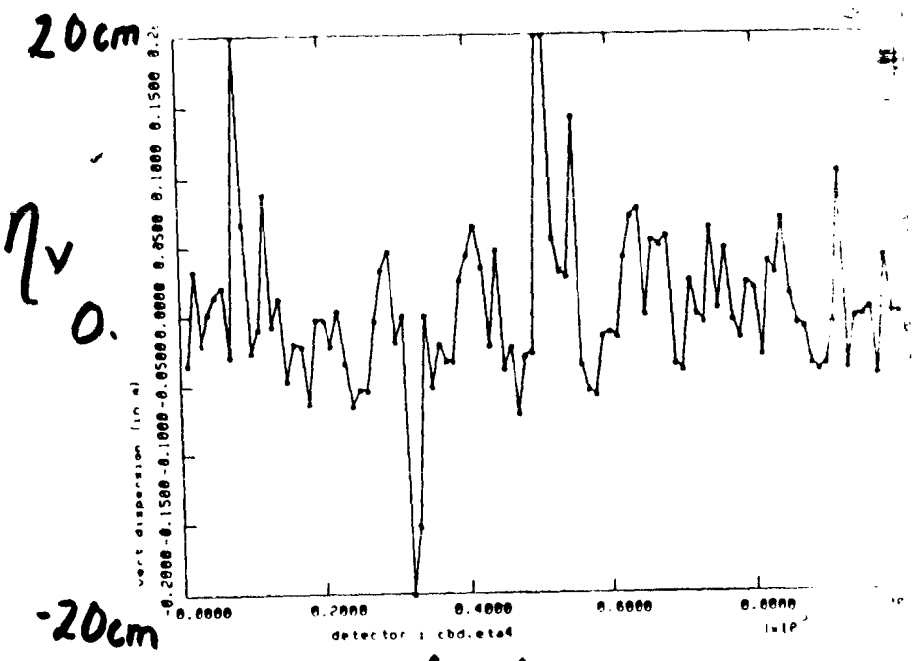
Cleo soln on

σ_v = 550 μm (sync light)

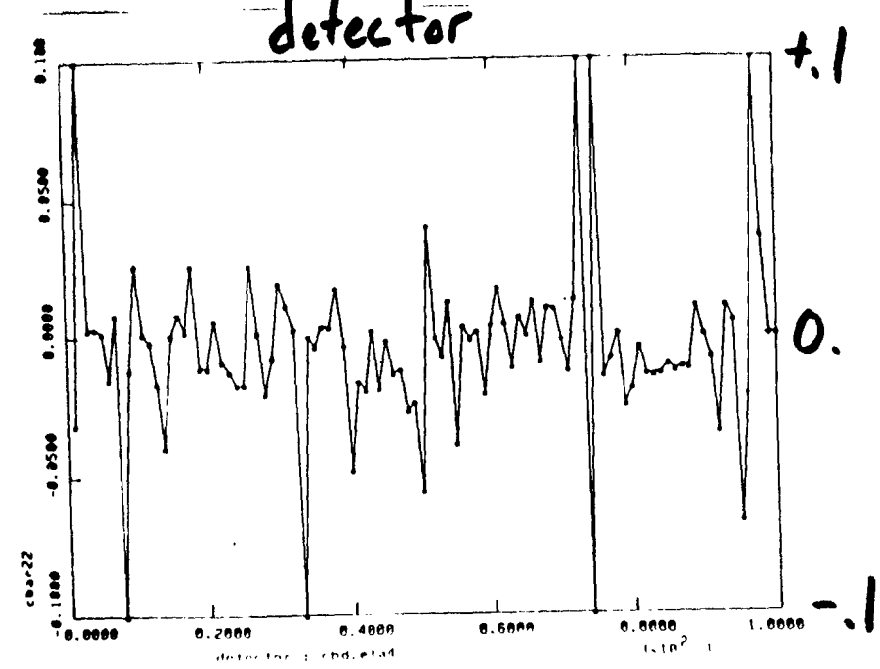
After



$\bar{C}_{12}$



η_V



$\bar{C}_{22}$

910325

Cleo soln on

$\sigma_V = 270 \cdot \mu m$ (sync/1+)

Conclusions

Less serious for proton machines than for electron machines.

We parametrize the coupling by 4 coupling parameters, the elements of the 2×2 matrix $\bar{C}$

These relate the normal modes to the horizontal and vertical motion.

The $\bar{C}_{ij}$ have components that propagate like

$$\varphi_A \pm \varphi_B$$

Global coupling is determined by the components of $\bar{C}$ that propagate as $\varphi_A - \varphi_B$.

It is not sensitive to the component that propagate as $\varphi_A + \varphi_B$

Only defined on the coupling resonance.

We have developed a method for measuring 3 of the 4 coupling matrix elements. We can measure one of these, the $\bar{C}_{12}$, very well.

With the measured $\bar{C}_{12}$ and η_v we can calculate changes in skew quad settings that will locally decouple CESR. This is now a standard procedure at CESR.

Betatron Phase Measurements in CESR*

D. Sagan, D. Rubin, and S. Greenwald
Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853

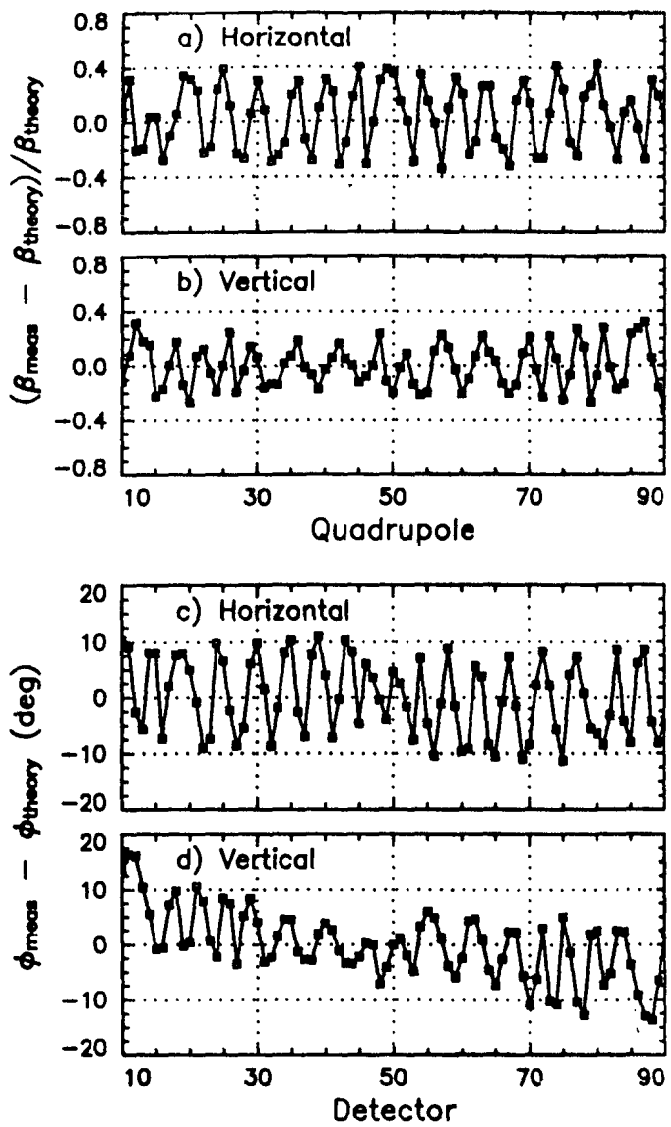


Figure 3: Initial beta and betatron phase relative to the theoretical values. The numbering system in CESR is such that the ith detector is close to the ith quadrupole.

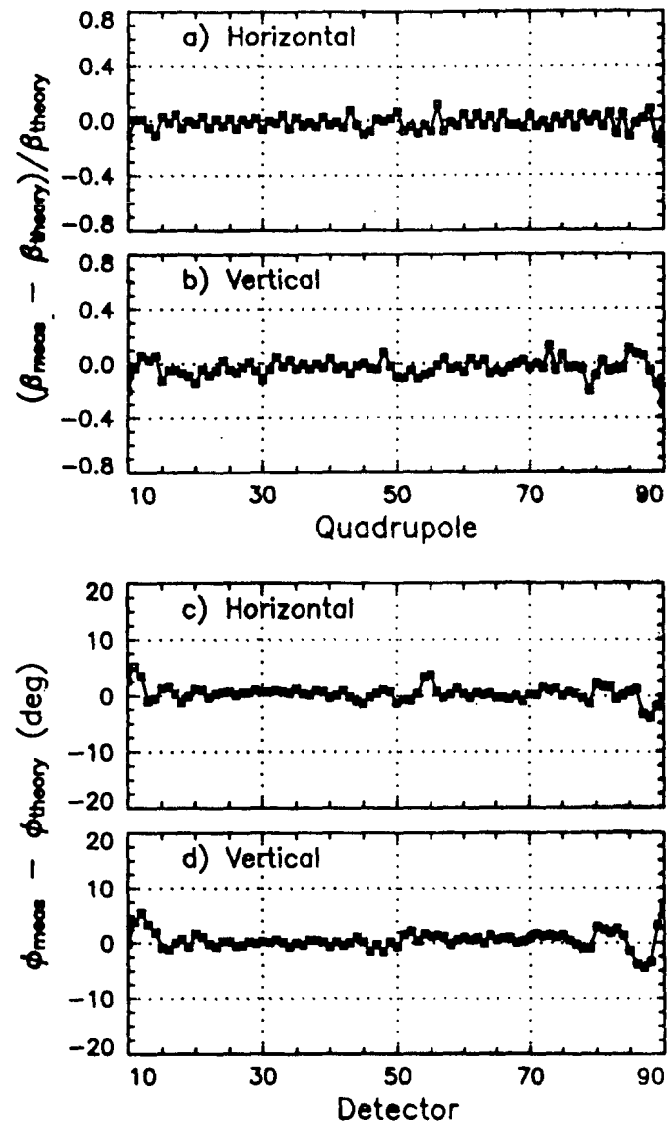


Figure 4: Beta and betatron phase relative to the theoretical values after correcting using only the phase data of the previous figure.

With electrons, we could coherently and continuously shake them really hard at the tune.

We can't do this with protons.

An obvious method is to kick the beam and look at the turn by turn oscillations.

We would certainly want chromaticities of ≈ 0 to get a nice long lasting signal.

Its not clear that this will be a clean enough signa

Horz and Vert BPM's are NOT at the same locations in the Tevatron.

If we assume no couplers between a detector and its neighbor, this should be easy to handle.

I think we've been in the middle of an upgrade for several years.

In principle, pieces are in place for turn by turn capability (1024 turns) at every BPM.

Still needs work

I believe that it will eventually be able to readout all the BPMs in a house on the same trigger. about 5 BPMs/plane/house

Least Significant Bit is 0.15 mm.

Ming-Jen Yang was doing some good work on this a few years ago, but has since not had time to work on it.

I think it will take a significant effort to get believable, consistent data.

F17 proton injection kicker Horz
.057 mrad at 1 TeV $\Rightarrow \simeq \pm 2.9$ mm peak displacements

Vertical pinger
at 1 TeV $\Rightarrow \simeq \pm 1.3$ mm peak displacements