MEASUREMENTS OF COHERENT TUNE SHIFT AND HEAD-TAIL GROWTH RATES AT THE SPS

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Abstract

A series of measurements of the coherent tune shifts with intensity and of head-tail growth rates have been performed with single proton bunches in the SPS, at 26 GeV. From these, the real and imaginary part of the transverse impedance can be estimated. This study, together with earlier and future measurements, will be used to experimentally document and follow up the effect of the impedance improvements on the SPS as injector to the LHC. A reproducibility at the 20% level was achieved for the value of the effective vertical impedance inferred from the coherent tune shift measurements.

1 INTRODUCTION

Several measurements, similar to those described in this article have been performed in the past, in the SPS. The main results are summarized in table 2. Most of these measurements, however, are quite old. Furthermore, they present a significant spread in the obtained vertical and horizontal broadband impedance parameters, covering about a factor of 3 from 12 to 48 MΩ/m in Z_v/Q).

In the present measurements, we aimed for an uncertainty below 20% in the impedence estimation. This would allow us to follow up and document experimentally the various steps of improvements planned to reduce the impedance of the SPS as injector into the LHC. As much as possible, we try to perform the measurements with the same bunch dimensions. This minimizes the model dependence and uncertainties due to variation in bunch parameters.

2 BEAM CONDITIONS

The measurements were all performed using single and relatively short bunches ($\sigma_z \approx 16$ cm or 5.5 ns) injected at 26 GeV in the SPS machine development (MD) cycle. Single short bunches were chosen for simplicity and in order to have a significant effect. The fixed beam energy of 26 GeV was rather imposed by beam availability. It would be useful in the future to confirm these measurements at a higher energy, to exclude any bias from space-charge effects [1].

The measurements were performed close to “standard tunes” ($Q_x = 26.62$, $Q_y = 26.58$). Chromaticity was carefully measured and corrected in order to be slightly positive (this was achieved with settings of typically $\xi_x = -0.16$, $\xi_y = +0.26$). The octupole components in the machine were compensated using octupole settings of typically $-.70$ for the radial and $-.75$ for the horizontal component. The damper was switched off and the tune measurements were done using 1 mm (nominal) kicks. With these settings and for small intensities ($\sim 10^{10}$ protons), one obtains rather clean sinusoidal oscillations with little damping, observable online over $2^{12} = 4096$ turns using the SPS tune application.

The variation of proton intensity in the range of 1 to $10 \cdot 10^{10}$ protons was performed in the PS. Ideally, the bunch dimensions and in particular the bunch length should not vary. The best compromise was achieved by adjusting the beam in the PS for the highest intensity first (close to $10 \cdot 10^{10}$), and then reducing it by vertical scraping. In this way, the bunch length and horizontal beam size remained nearly constant.

Longitudinal bunch parameters were also recorded on the PS side for every step in intensity. Typical numbers were: longitudinal emittance $\epsilon_z = 0.2\,\mathrm{eV\,s}$ $(2\,\sigma)$, total ($\sim \pm 2\sigma$) bunch length $l = 4$ ns and $\Delta p/p = 1.9 \cdot 10^{-3}$ ($\pm 2\sigma$).

On the SPS side, the 200 MHz rf was adjusted to obtain good capture and matching. Depending on intensity, this was achieved with voltages in the range of 0.5 - 0.8 MV.

In order to be independent of injection optimization and to have shorter bunches with a larger effect on the coherent tune shift, the rf was ramped adiabatically to 3 MV nominal (corresponding to about 2.5 MV measured) just before the time of the measurements. Details are given in Table 1.

Table 1: MD-cycle timing and RF-voltage

event	Δt start / Δt inj. (ms)	turns	$V_{\mathrm{rf}}(MV)$
cycle start	0		0.5
injection	972 / 0	0	0.5
rf-ramp start	1000 / 28	1214	0.5
rf-ramp end	1080 / 108	4685	3
Q_y meas. start	1080 / 108	4685	3
Q_x meas. start	1110 / 138	5986	3
cycle end	1700 / 728	31578	3

3 BUNCH DIMENSION

The vertical and horizontal bunch dimensions were recorded as a function of the proton intensity using wire-scanners. The results are shown in Figures 1, 2 and 3. Note that typical horizontal emittances from PS on the experiment of the 17/9/1999 were $0.36, 0.46, 0.4, 0.38$ [μm] at $2\,\sigma$. The horizontal measurements are scattered with max-

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Table 2: Broadband resonance parameters found in earlier transverse impedance measurements and calculations.

Z/Q in MΩ/m		year	studies performed
vertical	horizontal		
18		1980	head-tail growth rates (protons @ 270 GeV) [2]
47.7		1984	tune difference of high/low intensity bunches [3]
13 / 12.5	-8 / -5.2	1986	coherent tune shift (corrected for space-charge) / computed [4]
26.8	-16.88	1988	coherent tune shift @ 31.5 GeV [5]
(23 ± 2)		1993	leptons, TMCI threshold [6]

imal variations of about 40%. As we mentioned previousely, the PS beam was used at its maximum intensity and then it was scraped vertically to obtain the desired number of protons. This had an effect on the vertical dimension of the beam which was bigger at higher currents.

An approximately constant voltage of $V_{\rm rf} = 0.8$ MV was used on the first MD, on the 23/08/1999. A shorter and better controlled bunch length was obtained in the subsequent MD's using the voltage ramp described above. The bunch length was systematically recorded. The results at the time relevant for the tune measurements [1] are shown in Figure 4.

A good knowledge of the bunch length σ is needed to extract the parameters of the broad band impedance model. Since the bunch length is not constant we will use the average $\langle\sigma\rangle$ of all individual length measurements in our calculations. The r.m.s. spread in the measured bunch length is used as the error in the determination of σ and will lead to an error in the impedance estimate. These values are summarized in Table 3.

4 METHOD OF ANALYSIS OF TUNE AND GROWTH RATE MEASUREMENTS

The frequency analysis method is a refined Fourier analysis which can be applied on experimental or tracking data. More details about the mathematical details of the method can be found in papers of Laskar who introduced it in celestial mechanics [7] and accelerator dynamics [8].

The basic feature of the method is to produce a quasi-periodic approximation, truncated to order N,

$$f'(t) = \sum_{k=1}^{N} a_k e^{i\omega_k t} \ , \qquad (1)$$

with $f'(t), a_k \in \mathbb{C}$, of a numerical function $f(t) = q(t) + ip(t)$, usually representing in complex form the position and conjugate momenta associated with one of the degrees of freedom of a Hamiltonian dynamical system. This function can be either obtained by usual numerical integration or by real experimental data, recorded for a finite time span $t = T$. As we assume that the signal is quasi-periodic, the different frequencies of the series should be a linear combination of some base or fundamental frequencies $\omega_k = \mathbf{k}\cdot\boldsymbol{\omega}$.

[1] Closer to injection, for the capture voltage of $V \approx 0.62$ MV and $N_p = 2.5 \times 10^{10}$ we get $\sigma = 0.7$ ns, which is consistent with the value of 0.7 ns for the longitudinal σ given by the PS at these intensities.

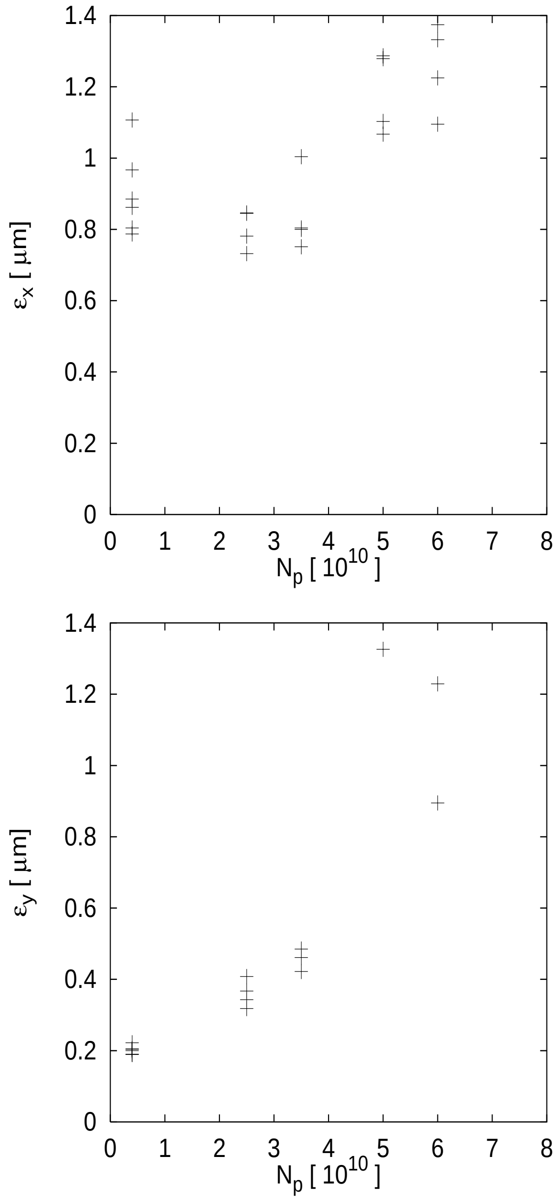

Figure 1: Proton horizontal emittance (top) and vertical emittance (bottom) as a function of bunch population, for an effective voltage $V_{\rm rf} \approx 2.5$ MV (measured on 23/08/1999).

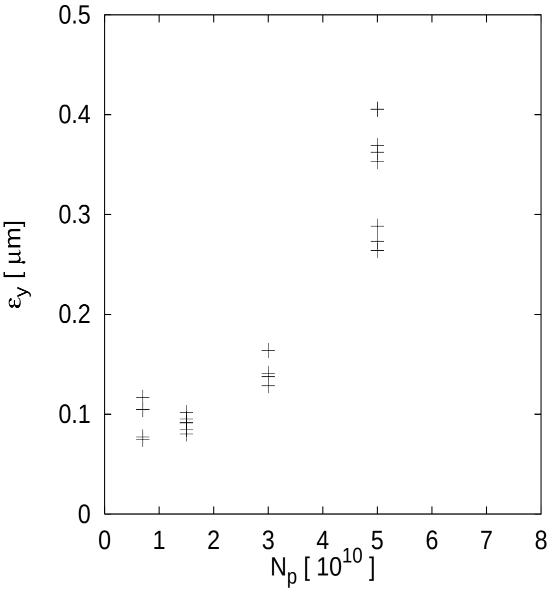

Figure 2: Vertical proton emittance ($4\sigma_y^2/\beta_y$) as a function of bunch population. Measured with a wire-scanner at a location with $\beta_y = 22$ m (measured on 17/9/1999).

Through an advanced filtering algorithm using the Hanning window, the method guaranties the asymptotic accuracy of the determination of the base tunes to be of the order of $1/T^4$ [9] for quasi-periodic signals, compared to $1/T$ of an FFT. Actually for the noisy signals associated with experimental data, we can expect an accuracy of the order of $1/T^2$ [10]. In that way, the horizontal and vertical coherent tune shifts can be efficiently estimated by applying the method to the raw data representing the coherent bunch oscillations.

Another interesting application of the method is the determination of the damping or growth rates, associated to some kind of collective instability, in a real accelerator. In fact, we may consider that the amplitudes of the series, instead of being constant, depend exponentially on time $a_k(t) = A_k e^{t/\tau}$, with $1/\tau$ denoting the growth or damping rate. A straightforward calculation of this rate can be achieved by estimating one of the amplitudes of the series (e.g. the one corresponding to the base frequency $a_1(t)$), for successive time spans (e.g. every 100 turns) and then fit an exponential to represent the function $a_1(t)$.

As example, we present in Fig. 5 one of the measurements effectuated in the SPS while the vertical chromaticity was slightly negative, producing a growth from the head-tail instability in the vertical plane. In Fig. 5, the actual measurement from the SPS acquisition system and the exponential fit with the calculated growth rate are plotted. We may note the good accuracy with which the growth rate is obtained (the R^2 of the fit is very close to 1).

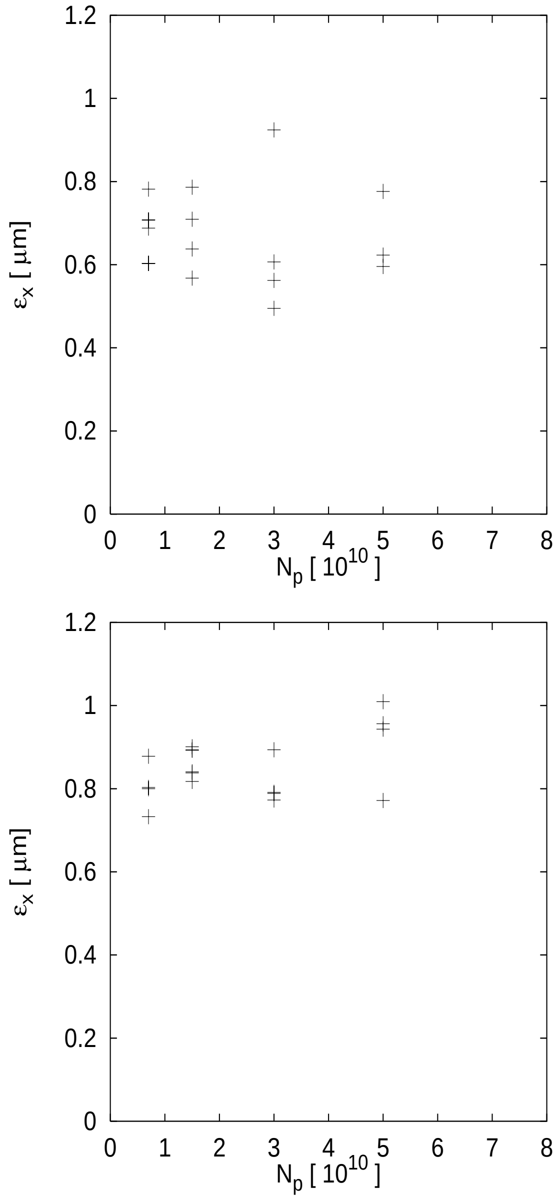

Figure 3: Horizontal proton emittance ($4\sigma_x^2/\beta_x$) as a function of bunch population, for the capture voltage (top) and after the ramp at $V_{\rm rf} \approx 2.5$ MV (bottom). Measured with the wire-scanner on 17/9/1999 at a location with $\beta_x = 97$ m and dispersion $D = 2.9$ m. Typical $\Delta p/p$ reported from PS: 1.6×10^{-3}, 2×10^{-3}, 1.6×10^{-3} and therefore $(D\Delta p/p)^2/\beta_x \approx 0.35$ at injection.

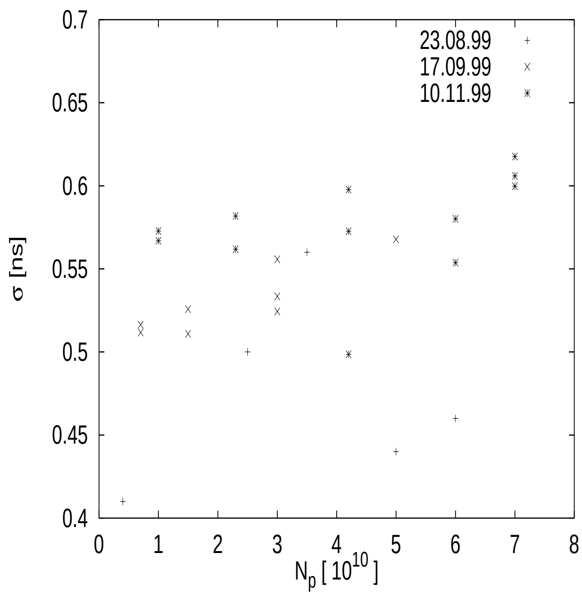

Figure 4: Longitudinal σ as a function of bunch population, as found by fitting the longitudinal profile with a Gaussian distribution. Measured when the actual voltage is $V \approx 2.5$ MV (or 3 MV nominal voltage).

5 HORIZONTAL AND VERTICAL TUNE SHIFT AS A FUNCTION OF BUNCH POPULATION

The vertical and horizontal tunes were obtained by kicking the beam and post-processing the time sequence (1024 to 4096 turns) of the beam position. Using the frequency analysis technique, the precision of the measurement was increased. We have measured the tunes after the adiabatic ramp for bunch population between 10^{10} and 5×10^{10}.

In Figures 6, 7, 8 and 9 we show the measured tune as a function of bunch population, for the horizontal and vertical plane. As expected from measurements performed in the past, with increasing current the vertical tune decreases and the horizontal tune increases. The slope of these plots is related to the imaginary part of the impedance. The difference in sign and magnitude between the two planes is due to the flat dimensions of the chamber: the horizontal mean radius is about 7 cm and the vertical mean radius of the SPS chamber is about 2.4 cm.

The data was fit to a straight line $f(x) = a \cdot x + b$. To obtain realistic errors for the slope, the uncertainties in each tune point were scaled to obtain $\chi^2 = 1$ for the fit.

5.1 *Summary Of Tune-shift Measurement*

In Table 3 we summarize the slopes found and the errors, as well as the σ of the longitudinal distribution. Note that the measurement on the 13/08/1999 was done without ramp of the rf-voltage, *i.e.* with longer bunches.

The bunch mode spectrum for these σ extends up to $f =$

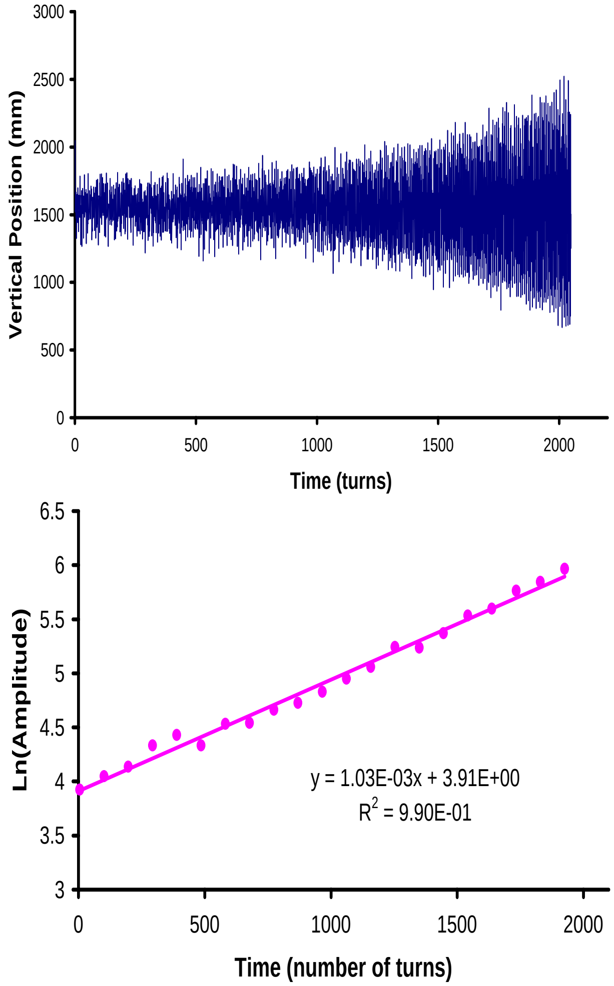

Figure 5: Vertical position of a bunch with slightly negative chromaticity, as measured by the SPS acquisition system (top) and growth rate obtained by fitting an exponential to the leading oscillating amplitude of the series issued by the frequency analysis method (bottom).

$1/(2\pi \cdot \sigma) \approx 300$ MHz.

6 GROWTH RATE AS A FUNCTION OF CHROMATICITY

For negative chromaticity, and operating above transition, the head-tail mode ($l = 0$) becomes unstable and drives the motion of the centroid of the beam. The amplitude of these oscillations increases exponentially in time. Analyzing this exponential growth, we get the growth rate $1/\tau$ which increases with $|\xi|$. The slope of this plot is related to the real part of the impedance.

During our most recent experimental attempt on the 10/11/1999, we studied the head-tail mode for low currents ($N_p = 1.6 \times 10^{10}$ protons per bunch). The chromaticity was reduced with respect to the previous settings by changing the strength of the sextupoles.

Table 3: Coherent tune shift measurements

date	$\Delta Q_x/\Delta N_p[10^{10}]$	$\Delta Q_y/\Delta N_p[10^{10}]$	σ [ns]
13/08/1999	$+0.00024 \pm 2 \times 10^{-5}$	$-0.0018 \pm 2 \times 10^{-4}$	0.77 ± 0.14
23/08/1999	$+0.00058 \pm 6 \times 10^{-5}$	$-0.0029 \pm 1 \times 10^{-4}$	0.47 ± 0.05
17/09/1999	$+0.00021 \pm 4 \times 10^{-5}$	$-0.0036 \pm 2 \times 10^{-4}$	0.53 ± 0.02
10/11/1999	$+0.00023 \pm 2 \times 10^{-5}$	$-0.0029 \pm 1 \times 10^{-4}$	0.58 ± 0.03

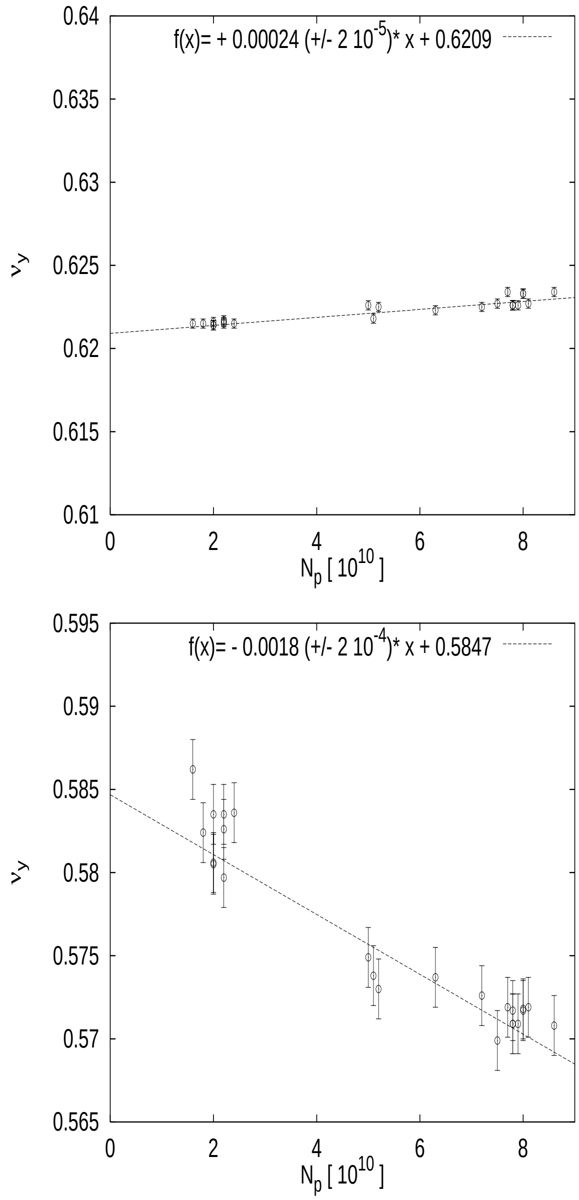

Figure 6: Horizontal (top) and vertical (bottom) tune as a function of the bunch population and fit with errors (measured on 13/08/1999 with RF voltage $V_{\rm rf}$=0.8 MV). Tune error bars are $e_y = 1.8 \times 10^{-3}$ and $e_x = 3 \times 10^{-4}$.

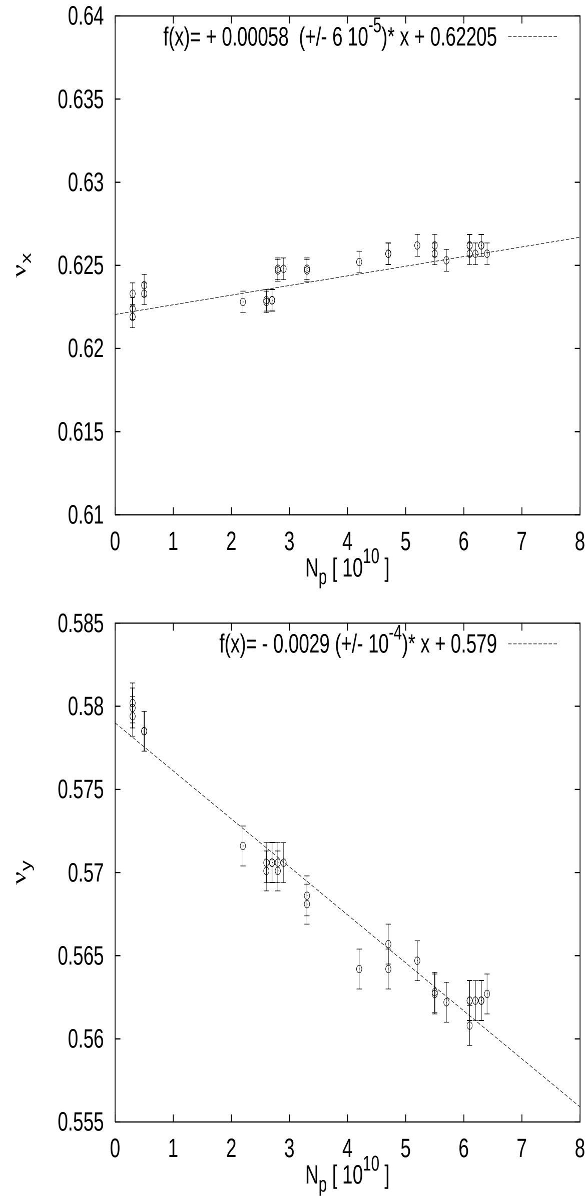

Figure 7: Horizontal (top) and vertical (bottom) tune as a function of the bunch population and fit with errors (measured on 23/08/1999 with RF voltage $V_{\rm rf}$=2.5 MV). Tune error bars $e_y = 1.2 \times 10^{-3}$ and $e_x = 6.5 \times 10^{-4}$.

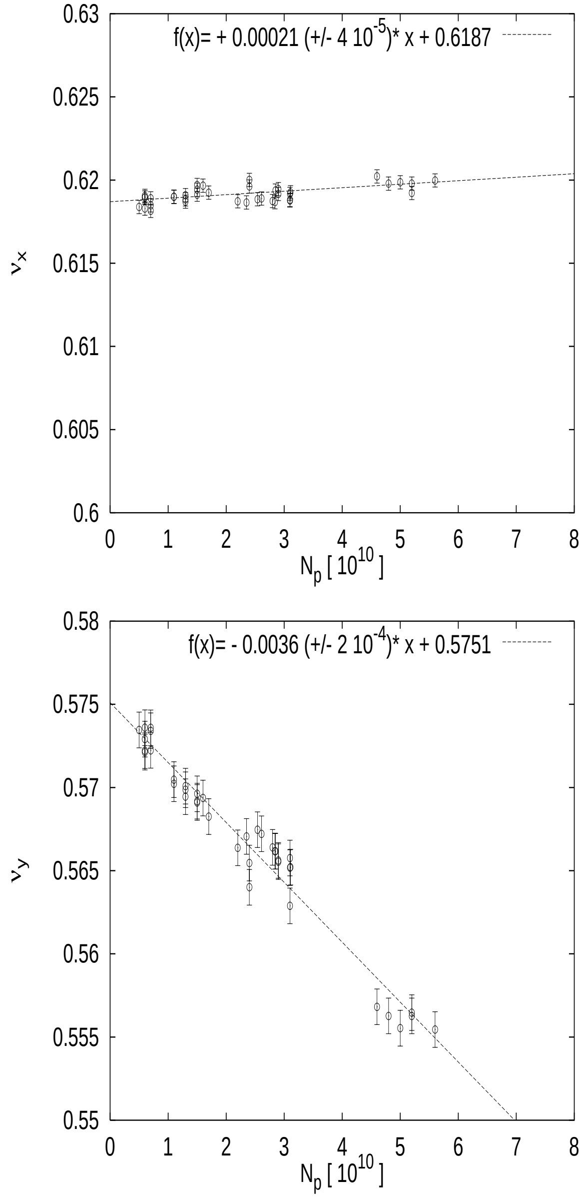

Figure 8: Horizontal (top) and vertical (bottom) tune as a function of the bunch population (measured on 17/09/1999 with $V_{\rm rf}$=2.5 MV). Tune error bars $e_y = 1 \times 10^{-3}$ and $e_x = 4 \times 10^{-4}$.

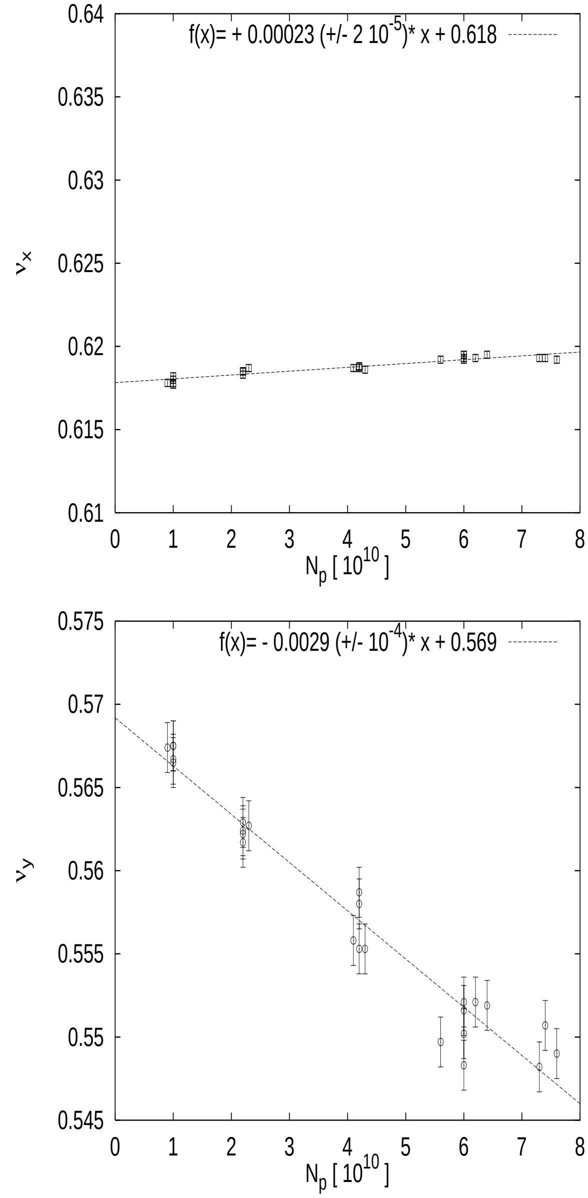

Figure 9: Horizontal (top) and vertical (bottom) tune as a function of the bunch population and fit with errors (measured on 10/11/1999 with $V_{\rm rf}$=2.5 MV). Tune error bars $e_y = 1.5 \times 10^{-3}$ and $e_x = 2 \times 10^{-4}$.

In Figure 10 we show the vertical growth rate as a function of the variation in the setting of the vertical chromaticity $\Delta\xi_v$, with respect to our setting used for the tune shift measurements. For negative chromaticity the bunch population was constant and equal to 1.6×10^{10}. The values at $\Delta\xi_v = 0$ were taken from the tune shift measurements which were performed with slightly positive chromaticity that lead to damping of the centroid motion. These points were measured with a bunch population of $N_p = 10^{10}$ and $N_p = 2.2 \times 10^{10}$ and their values were rescaled by the intensity ratio to compare with the measurements at $N_p = 1.6 \times 10^{10}$.

The zero crossing of the linear fit suggests that our standard setting $\Delta\xi_v = 0$ corresponds to a slightly positive chromaticity of $\xi = 0.011$.

A first attempt to measure growth rates was already undertaken earlier, on the 17/9/1999, with N_p between 3.5×10^{10} and 5×10^{10} and $\sigma = 0.53$ ns. The results are more scattered but are still shown for completeness in Figure 10 (bottom). The y-axis is scaled to the bunch population of 1.6×10^{10} and $\sigma = 0.58$ ns to be directly comparable to the linear fit of the measurements of the 10/11/1999.

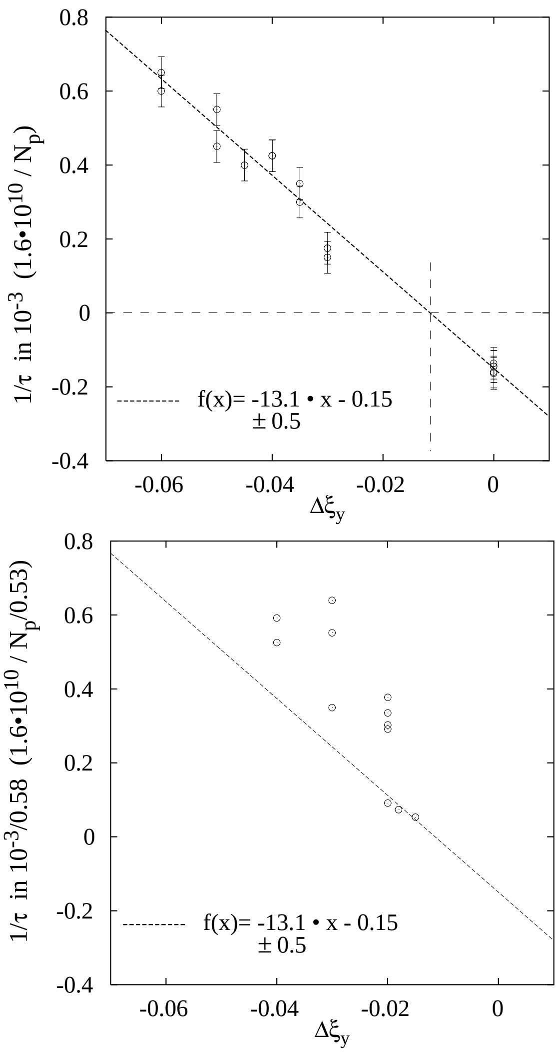

Figure 10: Growth rate of the head-tail mode instability (in units of 10^{-3} turns), as a function of the decrement of chromaticity. Top: measurement on the 10/11/1999. Error bars are $e = 0.043$ (in units of 10^{-3}). Bottom: Data of a first attempt on the 17/09/1999 and the straight line as obtained on the 10/11/1999.

7 FITTING THE RESULTS WITH A BROAD-BAND IMPEDENCE MODEL

For a single bunch, the longitudinal impedance has been modelled by an equivalent parallel LRC resonator circuit with resonance frequency $w_R = 1/\sqrt{CL}$, resistance R_s and quality factor $Q = R_s\sqrt{C/L}$. The Panofsky-Wenzel theorem requires that the same resonator gives a transverse impedance

$$Z_1^{\perp} = \frac{w_R}{w} \frac{Z_t}{1 + iQ\left(\frac{w_R}{w} - \frac{w}{w_R}\right)} \qquad (2)$$

where $Z_t = c/w_R R_s$. For $w \to w_R$ the impedance is purely resistive with $\Re(Z_1^{\perp}) = Z_t = c/w_R R_s$ and $\Im(Z_1^{\perp}) = 0$.

Let ξ be the chromaticity, η the slip factor, w_0 the revolution frequency, $w_\beta = Q_\beta w_0$ the betatron frequency and Q_β the betatron tune (including the integer part). Defining $w_\xi = \frac{\xi w_\beta}{\eta}$ and $w_p = pw_0 + w_\beta$ with p an integer number we can evaluate the effective transverse impedance [11]

$$(Z_1^{\perp})_{eff} = \frac{\sum_{p=-\infty}^{\infty} Z_1^{\perp}(w_p) h_l(w_p - w_\xi)}{\sum_{p=-\infty}^{\infty} h_l(w_p - w_\xi)}. \qquad (3)$$

where we take h_l as defined for a Gaussian beam model

$$h(w_p) = e^{-w_p^2 \sigma_z^2 / c^2} \qquad (4)$$

with $\sigma_z = c\sigma$ the bunch length, c the speed of light and σ the r.m.s of the Gaussian distribution in units of time.

Then the tune shift is given by

$$\Delta Q = \frac{\Omega - w_\beta}{w_0} \approx \frac{1}{w_0} \frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z} \Im(Z_1^{\perp})_{eff} \qquad (5)$$

with N_p the number of particles per bunch, e charge of the particle, E the particle energy, and $T_0 = 2\pi/w_0$ the revolution period. Similarly the growth rate (in turns^{-1}) is given by

$$\frac{1}{\tau} \approx -T_0 \frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z} \Re(Z_1^{\perp})_{eff} \qquad (6)$$

The real part of the effective impedance is different from zero if the chromaticity is not zero. Above transition, this leads to a negative growth rate (damping) for positive chromaticity, and to a positive growth rate otherwise.

In Table 4 we summarize parameters, relevant to our experiment.

Table 4: Parameters and their values

E	26.017 GeV	beam energy
T_0	23.05 μs	time for one revolution
Q_β	26.6	betatron tune
η	5.55×10^{-4}	phase slip factor
N_p	$1 - 8 \times 10^{10}$	number of protons in the bunch

7.1 Tune Shift

We fit the broad band resonator with a quality factor $Q = 1$ and a resonance frequency $w_R = 2\pi \times 1.3$ GHz.

The ratio $\Delta Q/\Delta N_p[10^{10}]$ is directly proportional to the impedance $Z_1^{\perp}$. For each plane we determine the impedance such that $\Delta Q/\Delta N_p[10^{10}]$ equals the slope found in our measurements. In Table 5 we summarize the impedances inferred from the tune shifts. The uncertainty reflects both the error of the fitted slope and the spread in the measured bunch length σ.

The averages and uncertainties from combining the four measurements are also given. The four numbers of Z_v are

Table 5: Impedance results obtained by fitting coherent tune shifts with a broad-band model.

date	Z_v in MΩ/m	Z_h in MΩ/m
13/08/1999	25 ± 6	-3.3 ± 0.7
23/08/1999	24 ± 2	-4.8 ± 0.7
17/09/1999	33 ± 3	-2.0 ± 0.4
10/11/1999	30 ± 2	-2.4 ± 0.3
average	28 ± 2	-2.6 ± 0.2

all compatible with the mean within 20%. This makes us confident that the measurements presented here are in fact relevant to document and follow the improvements of the SPS as LHC injector. The effect in the horizontal plane is much smaller, and has clearly the opposite sign.

The uncertainties given above are effectively only from the scatter in the data, as relevant for a comparison of data taken under similar conditions. The model dependence should be considered in addition when this is compared to results obtained with different methods or under different conditions.

7.2 Growth Rate

On the experiment of the 10/11/1999 (see Figure 10), we found that the growth rate increases linearly with the decrement of chromaticity. This can be understood as follows. If the bunch is longer than the range of the wake field ($\sigma c > c/w_R = 3.6$ cm for $w_R = 2\pi \times 1.3$ GHz) then $(Z_1^{\perp})_{eff} \approx Z_1^{\perp}(w_\xi)$. The growth rate $1/\tau$ which is proportional to $\Re(Z_1^{\perp})_{eff}$ is then

$$\begin{aligned} \frac{1}{\tau} \approx & - T_0 \frac{N_p e c^2}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z} \frac{Z_1 w_\xi}{w_R} \\ = & - T_0 \frac{N_p e c^2 Z_1}{2E/eT_0 w_\beta 2\sqrt{\pi}\sigma_z w_R} \frac{\xi w_\beta}{\eta} \end{aligned} \quad (7)$$

which increases linearly with $-\xi$.

Using the complete formula (Eq. 3) and assuming $Q = 1$, the impedance that fits the measured dependence on the chromaticity is $Z_t = 8.3 \pm 0.6$ MΩ/m. This impedance is 3.7 times smaller than the impedance found by fitting the coherent tune shift.

We can fit both measurements simultaneously with $Z_t = 108$ MΩ/m by changing the quality factor to $Q = 3.6$. In this broad band model $Z_t/Q = 30$ MΩ/m.

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