

## C-A D Engineering Design Support Documentation Cover Page

Title: <i>C3Inflector Thermal-Stress Analysis</i>	Author: V. Badea and S. Bellavia
Subject: Thermally induced stress state of the C3 inflector septum upon incident beam	
C-A Department Group: Mechanical	Approval: J. Tuozzolo
Documentation Type: Written Technical Paper <input checked="" type="checkbox"/> Design Calculations Design Review Presentation Design Certification Specification Procurement Documentation Safety/ALARA Review Presentation  Other: _____	Equipment Type: Magnets and Power Supplies Vacuum Systems RF Systems Cryogenic Systems Experimental Equipment Beam Instrumentation Water Systems Buildings, Structures, and Shielding  <input checked="" type="checkbox"/> Other: <u>Inflector</u> _____
Equipment Location: Tandem Van DeGraff Linac <input checked="" type="checkbox"/> Booster AGS RHIC 912 Experimental Area 919 Experimental Area RHIC Experimental Areas  _____ Buildings, Structures, and Shielding Other  _____	Associated information for cataloging (if available fill in the number): C-A D Design Room Job No.  _____ <input checked="" type="checkbox"/> C-A D Design Room Drawing No.  <u>D36-M-2256-5</u> C-A D Specification No.  _____ C-A D Experiment No.  _____ Other  _____

Discussion:

An analysis was performed to determine the effect of beam incident upon the Booster C3 Inflector Septum. This analysis included the thermal effect and the resulting stress state.

Parameters:

- Ti 6AL-4V alloy, .635mm thick x 146mm x 2.4m
  - EX = 113.8GPa
  - NUXY = .342
  - ALPX = 0.936 E-05
  - DENS = 4430.0 Kg/m<sup>3</sup>
  - KXX = 6.7 W/m-K
  - C = 544 J/Kg-K
  - Yield Stress = 126 ksi
  - Ultimate Stress = 134 ksi
- 4 x 25 Joule Pulses 3.0e-4sec @0.2sec intervals every 3.6 sec.  
(Condensed into 1 x 100 Joule Pulse for 1.0e-03 second every 3.6 second)

Assumptions:

- Energy absorbed/transferred by capacitance and conduction of inflector and attached structure only. No convection (vacuum) and radiation neglected (provides small safety factor).
- Conduction to support structure through edges of inflector must be somewhere between a perfect contact and no contact (insulated). Both extremes were analyzed. Fortunately, there was no need to determine the actual boundary condition as results at either extreme were close.

Hand Calculations (See appendices, attached):

- Calculations indicate a very slow thermal response, due to low conductance and high capacitance of the material.
- Buckling/ Flat Plate Instability seems likely in this thin section, which is undergoing thermal expansion with edges constrained. Hand calculations indicate this will begin when the compressive stress reaches approximately 2700 psi.

FEA Model:

- 7230 elements
- Transient Thermal: Shell57 – 3-D thermal elements with in-plane conduction 7,502 Degrees of Freedom (DOF)
- Steady State Linear Structural (at specified time intervals) SHELL63 structural elements with both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. 45,012 DOF's

Analysis method:

Due to the slow thermal response, it was expected that the 4 pulses of beam at 0.20 second intervals every 3.6 seconds could be modeled as a single pulse with equivalent energy at 3.6 sec intervals, as shown in Figure 1.

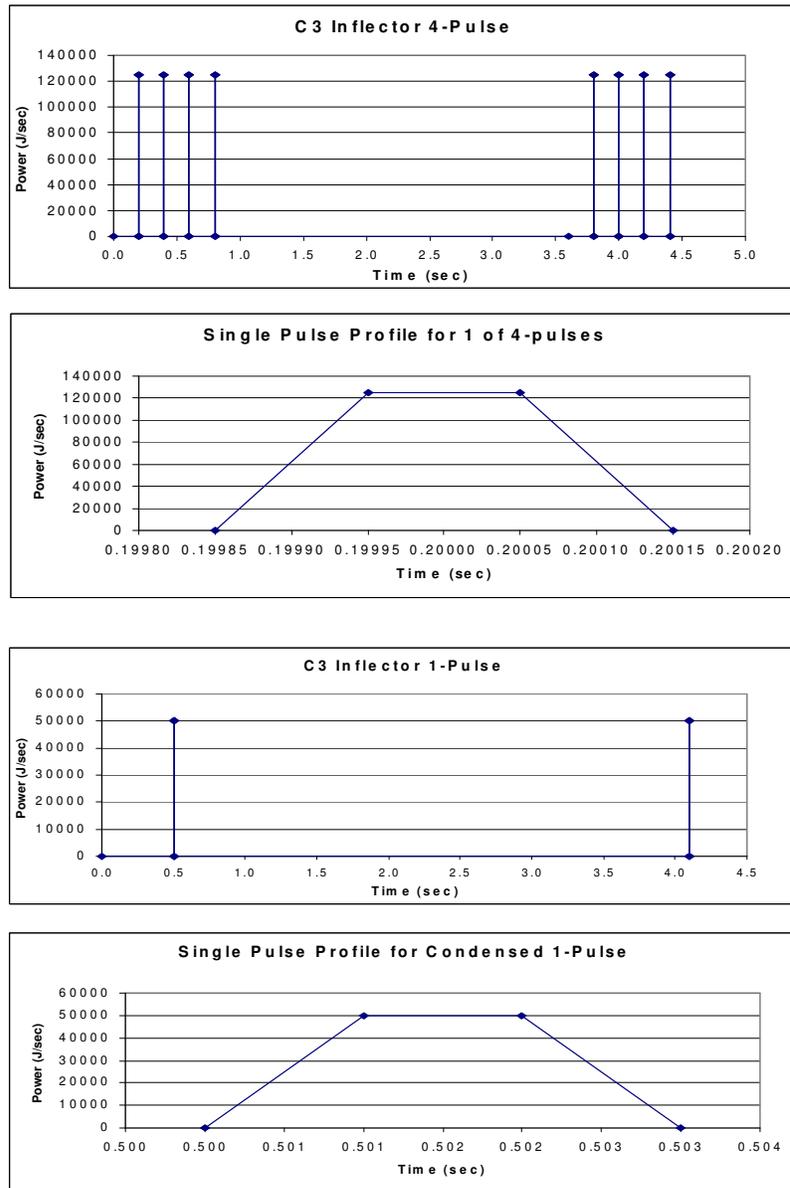


Figure 1. Beam Pulse Profiles

This would save significant computation time, and more importantly, disk space, which was reaching it's limit for this analysis. The analysis was performed for a 60 second period using the 4-pulse and again using the combined single pulse. As can be seen from figures 2 and 3, the single pulse is nearly identical to the 4-pulse scenario.

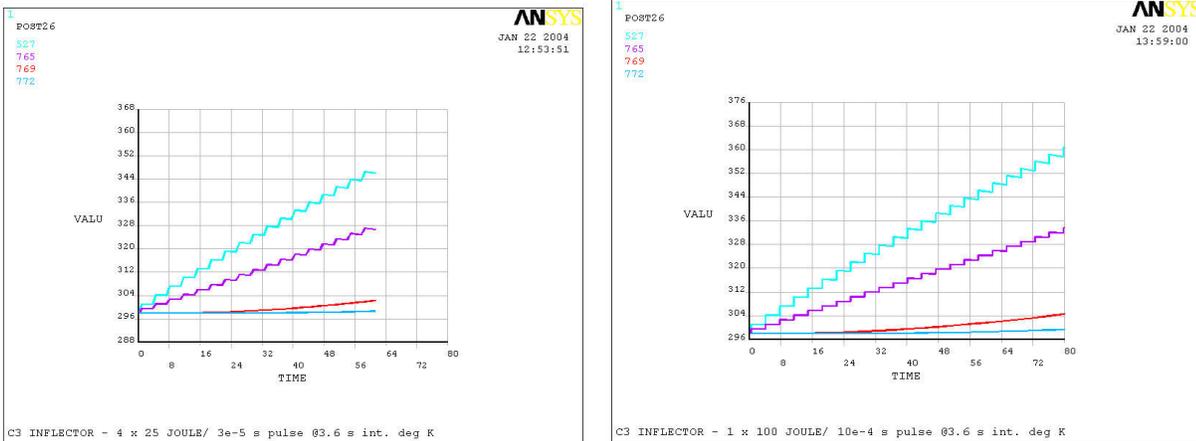


Figure 2. Transient Response of Selected Points on structure for 4x25 Joule pulse and 1 x 100 Joule pulse.

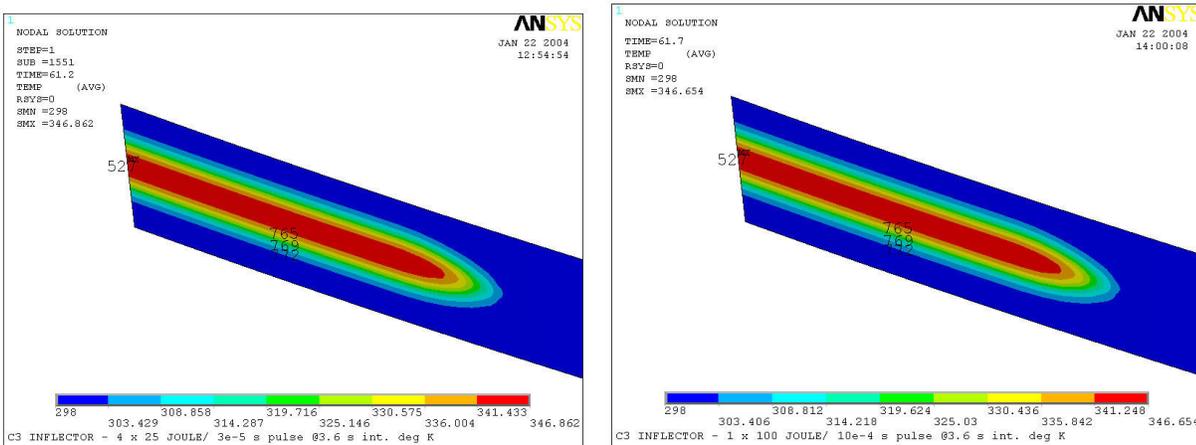


Figure 3. Temperature Contour at 60 seconds for 4-pulse and 1-pulse

The single pulse profile was then used for a 900 second thermal transient, recording all pertinent data to a file at each pulse so that the stress state during this transient could also be computed. This was done for two extreme cases: (1) insulated edges and (2) perfect thermal contact edges.

**Analysis Results:**

Both cases, i.e., no contact and perfect contact, were run simultaneously. Initially, the response is non-linear, as expected. However, after some time, the temperature, and associated stress states begin to linearize, allowing for an extrapolation to the yield stress of the material (126 ksi). This

occurs at 27 minutes for the no-contact (insulated) edges case at 650 deg C peak temperature, and 30 minutes for the perfect contact case at 560 deg C peak temperature.

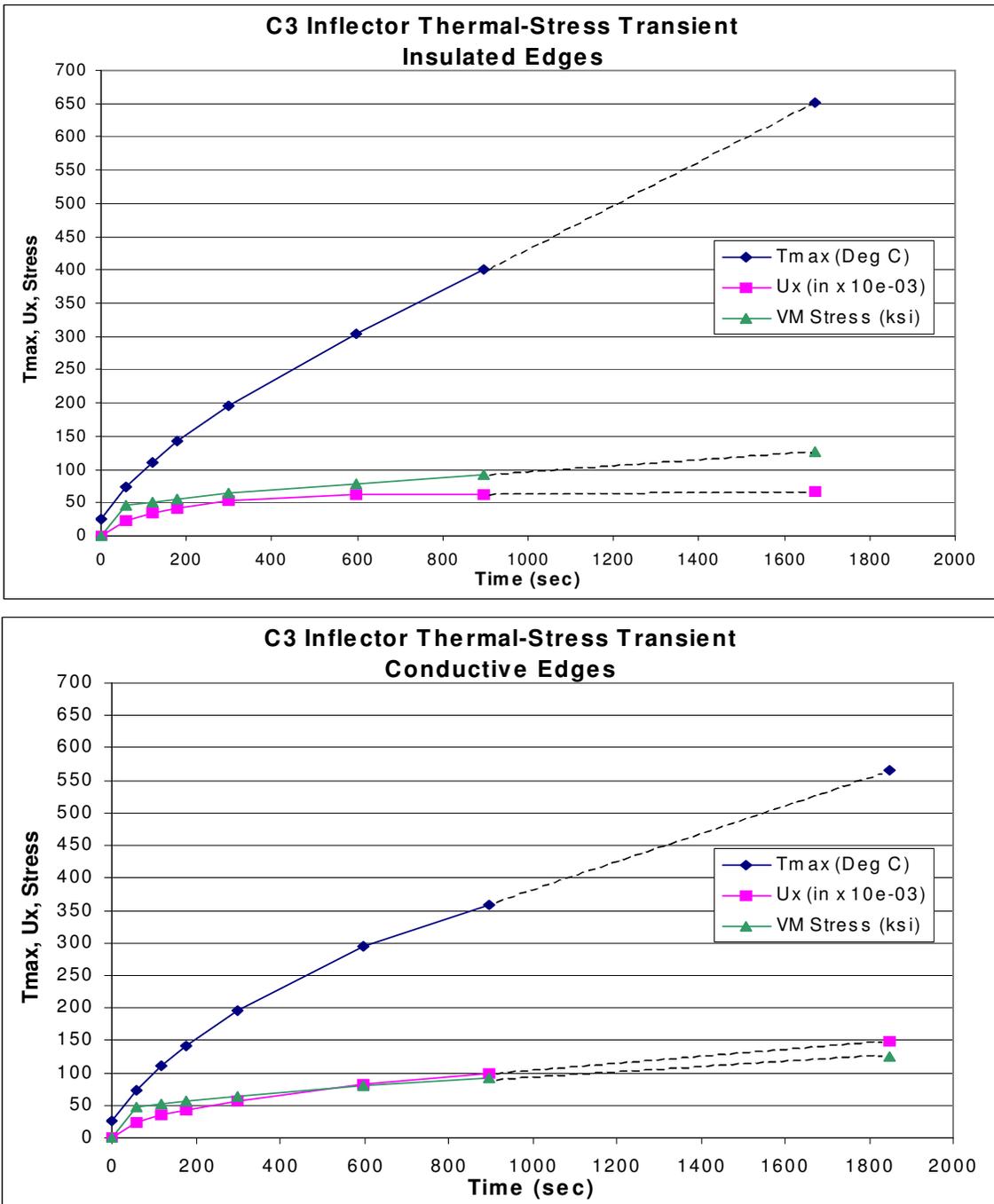


Figure 4. Thermal Transient Response from FEA model

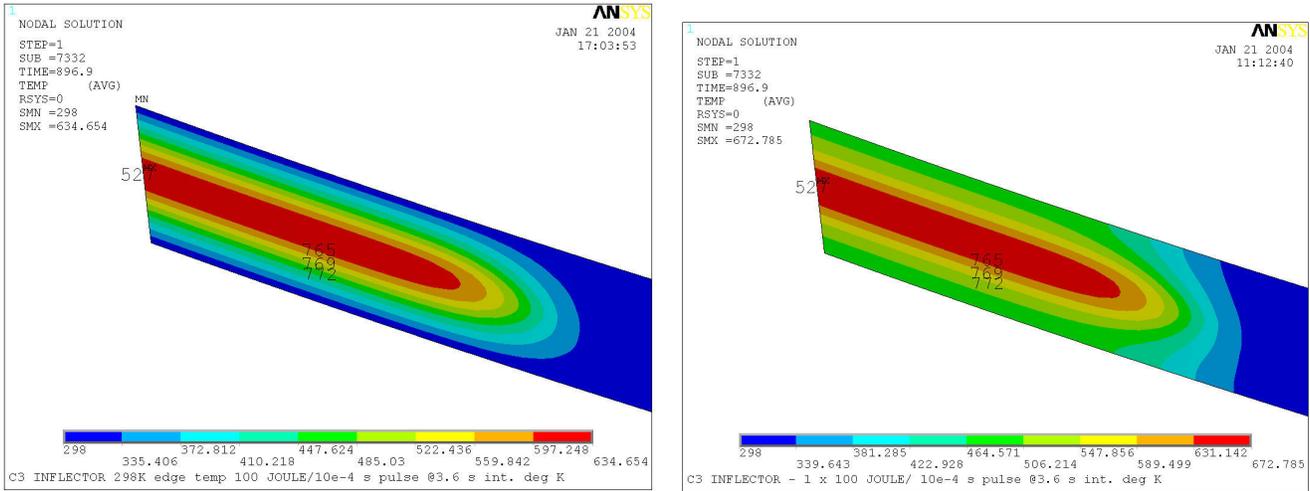


Figure 5. Temperature at 900 seconds for perfectly insulated and perfectly conductive edges

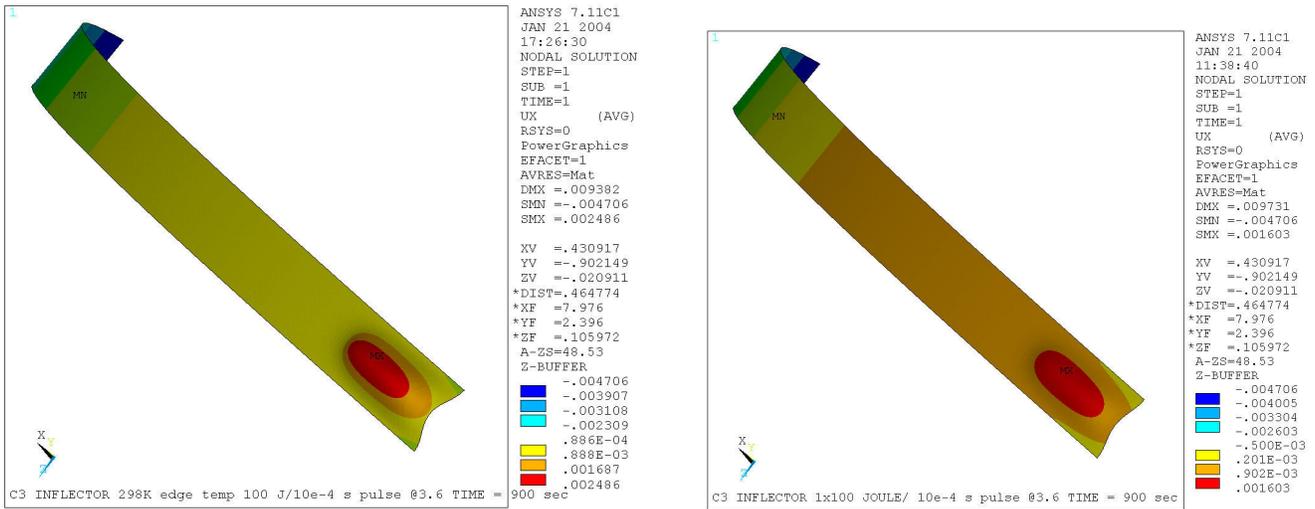


Figure 6. Linear Displ. at 900 seconds for perfectly insulated and perfectly conductive edges (See comment on instability, below).

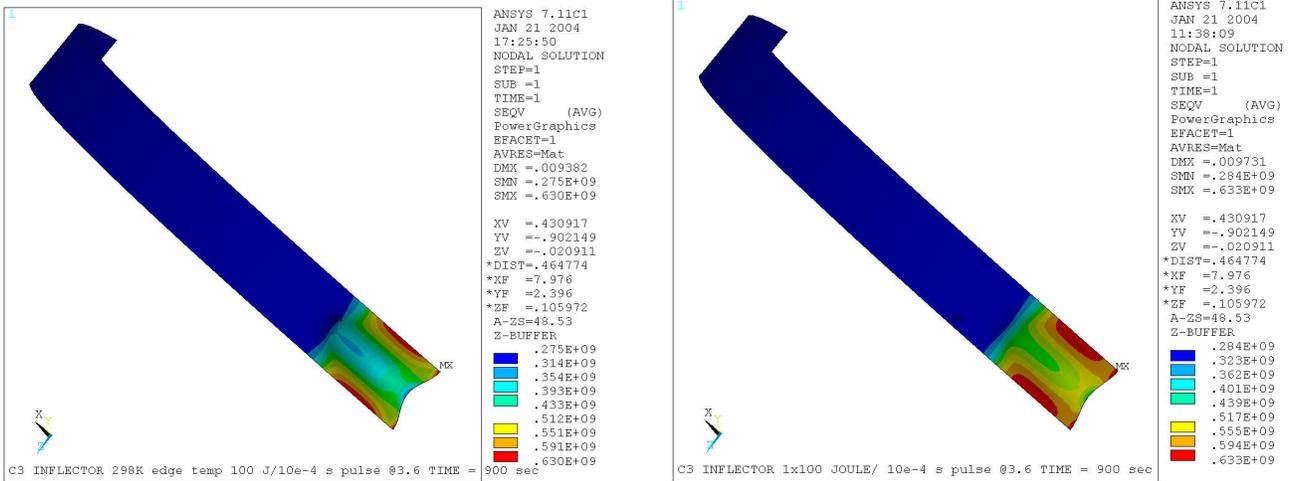


Figure 7. Von Mises Stress at 900 seconds for perfectly insulated and perfectly conductive edges

Comment on Buckling/Instability:

It should be noted that the compressive stress, ( $\sigma_z$ ) required for buckling/flat plate instability (similar to flexing a credit card) occurs at approximately 40 seconds. However, this buckling does not necessarily imply failure, and can completely recover once the temperature is removed. It will result in deflections greater than indicated by the linear stress FEA model, which currently does not include non-linear instability.

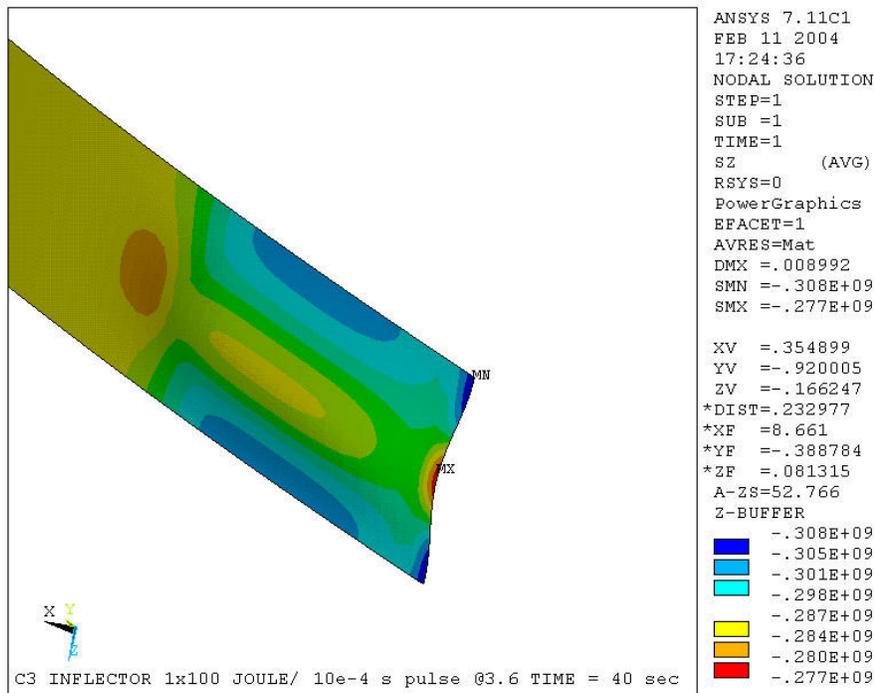


Figure 8. Stress gradient across the C3 inflector septum ( $\sigma_z$ ) at t= 40 seconds.

Initial Conclusion:

The approximate yield failure can be predicted using a linear stress analysis applied at the thermal state at selected solved thermal transient time points.

At 900 seconds (15 minutes) into the transient, the peak temperature is less than or equal to 673K (400 C). This results in a maximum Von Mises Stress of approximately 630 MPa (92 ksi). From the extrapolated curves, it appears that the yield stress of 126 ksi occurs at approximately 30 minutes of incident beam corresponding to a temperature of 600 C. As mentioned, the actual deflection will be greater than the linear model prediction due to localized and global instability of the section. This larger deflection may or may not result in higher total stresses. Due to the uncertainty of the peak stress which arises from operating in the buckled state, as well as fatigue and other temperature related material issues, a safety factor needs to be applied for any recommendation on operation in this mode.

Initial Recommendation:

Applying a safety factor of 2.0 on the yield stress (as a function of time), it is recommended to limit beam incident on the C3 Inflector Septum to less than 15 minutes, otherwise it may suffer permanent deformation, and possibly fail.

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Note: The following was performed after the above analyses to address the issue of extrapolation of the data as well as the effects of operation in the buckled state, which seems likely.

Eigen-Buckling Analysis using FEA:

A linear Eigen-Buckling analysis was performed using the same model as previous. This model takes the pre-stress condition of the static analysis and determines the buckling modes and associated deflections and stresses that occur while subject to the same loads in the buckled state.

Eigen-Buckling Analysis Results:

As shown in Figure 9, below, after 15 minutes of incident beam, the maximum deflection is 6.5 mm (.25 inches), about 2.5X the linear static result, and the stress is 812 Mpa (118ksi), or about 1.3X the linear static result.

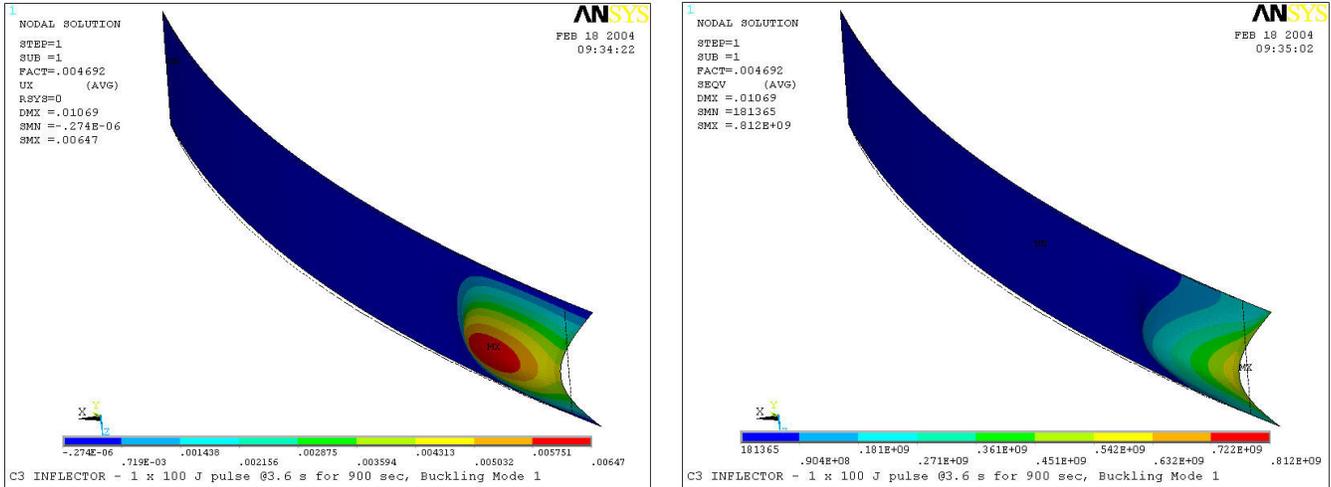


Figure 9. Eigen-Buckling Displacement and Von Mises Stress due to thermal loads

**Final Summary/Conclusion:**

As stated earlier, it is recommended to keep the Inflector Septum at or below the yield stress. This can be achieved by limiting the incident beam to 15 minutes or less.

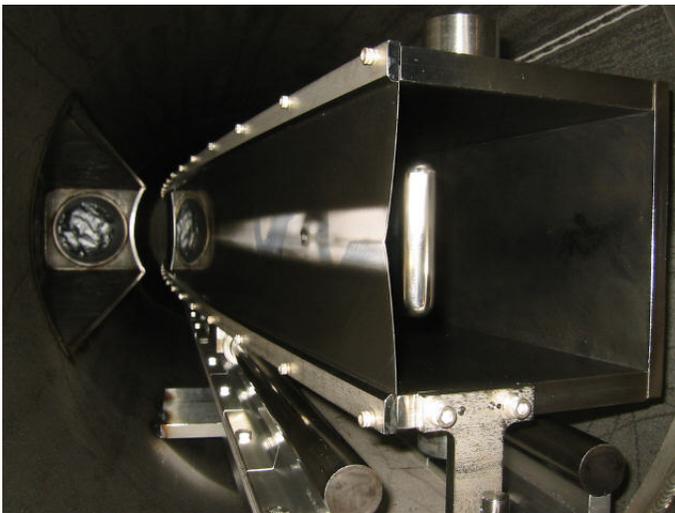
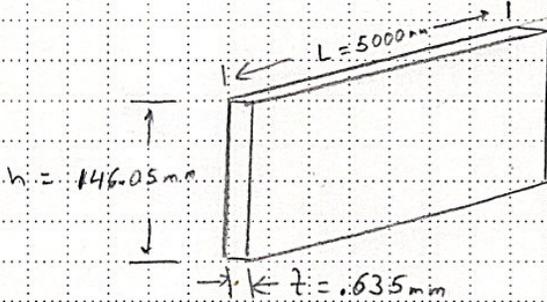


Photo by Leif Ahrens

ENGINEERING ANALYSIS

MODEL	SUBJECT	INDEX
ANALYST	CHECKER	DATE
S. BELLAVIA		1/21/04
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Approximate Thermal Time Constant:



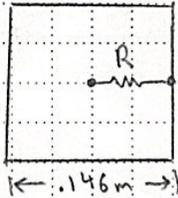
Titanium:

$k = 7.4 \text{ W/m-k}$   
 $c_p = .54 \text{ kJ/kg-k}$

$\rho = 4.85 \frac{\text{g}}{\text{cm}^3} = 4850 \frac{\text{kg}}{\text{m}^3}$

$\tau_{\min} = [RC]_{\min} = \frac{(m c_p)_{\min}}{(A k)_{\max}}$

If take a square section, and evaluate R from ctr. to any edge:



$R = \frac{x}{kA} = \frac{(.146\text{m}/2)}{(7.4\text{W/m-k})(.000635 \times .146\text{m})}$

$R = 106.4 \frac{\text{k}}{\text{watt}} \text{ or } \frac{\text{K-sec}}{\text{Joule}}$

Mass of Square,  $M = \rho V = 4850 \frac{\text{kg}}{\text{m}^3} \times .000635 \times (.146\text{m})^2$

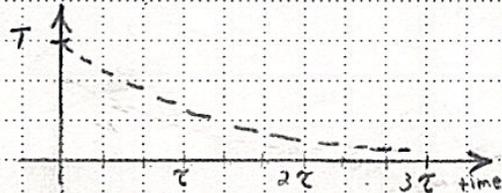
$m = .0656 \text{ kg}$

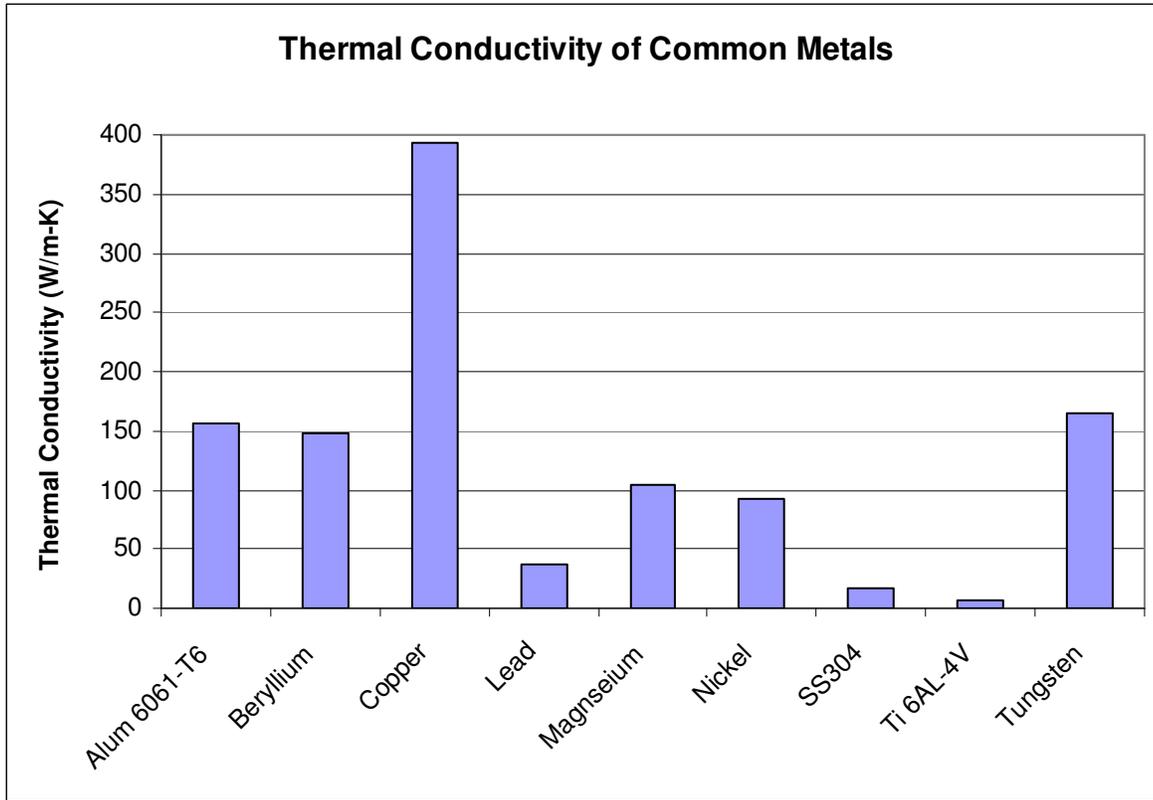
so that,

$\tau_{\min} = \left( \frac{106.4 \frac{\text{K-sec}}{\text{Joule}} \times 1000 \frac{\text{J}}{\text{kJ}}} \right) \times \left( .0656 \text{ kg} \times .54 \frac{\text{kJ}}{\text{kg-k}} \right)$

$\tau_{\min} = 376.9 \text{ sec}$

$\tau_{\min} \approx 1 \text{ hour}$

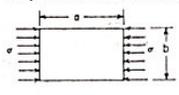




### ENGINEERING ANALYSIS

MODEL	SUBJECT <i>C3 Inflector Instability</i>	INDEX
ANALYST <i>S. BELLAVIA</i>	CHECKER	DATE <i>2/11/04</i>
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**TABLE 35 Formulas for elastic stability of plates and shells**  
 NOTATION:  $E$  = modulus of elasticity;  $\nu$  = Poisson's ratio; and  $t$  = thickness for all plates and shells. All angles are in radians. Compression is positive; tension is negative. For the plates, the smaller width should be greater than 10 times the thickness unless otherwise specified.

Form of plate or shell and manner of loading	Manner of support	Formulas for critical unit compressive stress $\sigma'$ , unit shear stress $\tau'$ , load $P'$ , bending moment $M'$ , or unit external pressure $q'$ at which elastic buckling occurs																																		
1. Rectangular plate under equal uniform compression on two opposite edges $b$ 	1a. All edges simply supported	$\sigma' = K \frac{E}{1 - \nu^2} \left(\frac{t}{b}\right)^2$ <p>Here <math>K</math> depends on ratio <math>\frac{a}{b}</math> and may be found from the following table:</p> <table border="1" style="font-size: small;"> <tr> <td><math>\frac{a}{b}</math></td> <td>0.2</td> <td>0.3</td> <td>0.4</td> <td>0.6</td> <td>0.8</td> <td>1.0</td> <td>1.2</td> <td>1.4</td> <td>1.6</td> <td>1.8</td> <td>2.0</td> <td>2.2</td> <td>2.4</td> <td>2.7</td> <td>3</td> <td><math>\infty</math></td> </tr> <tr> <td><math>K</math></td> <td>22.2</td> <td>10.9</td> <td>6.92</td> <td>4.23</td> <td>3.45</td> <td>3.29</td> <td>3.40</td> <td>3.68</td> <td>3.45</td> <td>3.32</td> <td>3.29</td> <td>3.32</td> <td>3.40</td> <td>3.32</td> <td>3.29</td> <td>3.29</td> </tr> </table> (For unequal end compressions, see Ref. 33)	$\frac{a}{b}$	0.2	0.3	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.7	3	$\infty$	$K$	22.2	10.9	6.92	4.23	3.45	3.29	3.40	3.68	3.45	3.32	3.29	3.32	3.40	3.32	3.29	3.29
	$\frac{a}{b}$	0.2	0.3	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.7	3	$\infty$																			
$K$	22.2	10.9	6.92	4.23	3.45	3.29	3.40	3.68	3.45	3.32	3.29	3.32	3.40	3.32	3.29	3.29																				
	1b. All edges clamped	$\sigma' = K \frac{E}{1 - \nu^2} \left(\frac{t}{b}\right)^2$ <table border="1" style="font-size: small;"> <tr> <td><math>\frac{a}{b}</math></td> <td>1</td> <td>2</td> <td>3</td> <td><math>\infty</math></td> </tr> <tr> <td><math>K</math></td> <td>7.7</td> <td>6.7</td> <td>6.4</td> <td>5.73</td> </tr> </table> (Refs. 1, 6, 7)	$\frac{a}{b}$	1	2	3	$\infty$	$K$	7.7	6.7	6.4	5.73																								
$\frac{a}{b}$	1	2	3	$\infty$																																
$K$	7.7	6.7	6.4	5.73																																

For Ti GAL-4V:

$$E = 113 \text{ GPa} = 16.4 \times 10^6 \text{ psi}$$

$$\nu = .342$$

Let  $a = b = 5.75$  inches (Assume a square section same as width,  $a$ )  
 $t = .025$  inches

i.e. For simply supported edges:

$$\sigma' = 1.0 \times \frac{16.4 \times 10^6 \text{ psi}}{1 - (.342)^2} \left(\frac{.025}{5.75}\right)^2$$

$$\sigma' = 351 \text{ psi}$$

For Edges Clamped: (Closer to actual configuration)

$$\sigma' = 7.7 \times \frac{16.4 \times 10^6 \text{ psi}}{1 - (.342)^2} \left(\frac{.025}{5.75}\right)^2$$

$$\sigma' = 2703 \text{ psi}$$