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Tune Modulation Due to Gradient Ripple and the Dynamic Aperture

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RHIC PROJECT

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1. Introduction

A tracking study was done of the effects of a tune modulation, due to a gradient ripple in the quadrupoles, on the dynamic aperture in RHIC lattices. Using tracking runs of about 1×10^6 turns, the dynamic aperture was found to decrease roughly linearly with the amplitude of the tune modulation, and may be represented by

$$A = A_0 \left(1 - 42\ \Delta\nu\right) \tag{1}$$

where A_0 is the dynamic aperture for $\Delta\nu = 0$, and $\Delta\nu$ is the tune modulation amplitude.

Two RHIC lattices were studied. One with 6 insertions having $\beta^* = 6$m, and one with 6 insertions having $\beta^* = 2$m. Roughly the same result, Eq. (1), was found for both lattices.

The dependence of the dynamic aperture on the frequency of the gradient ripple was also studied. No appreciable dependence on the ripple frequency was found for the range covered in the ripple frequency and in the tune modulation amplitude.

2. Tracking Results

In this study the gradient in all the quadrupoles was varied according to

$$G = G_0 \left(1 + (\Delta G/G) \sin\left(2\pi t/T\right)\right) \tag{2}$$

$\Delta G/G$ is the amplitude of the gradient ripple and T is the period of the ripple. A 60 Hz gradient ripple was used. This gradient ripple produces a tune modulation of the form

$$\begin{aligned} \nu_x &= \nu_{x0} + \Delta\nu \sin\left(2\pi t/T\right) \\ \nu_y &= \nu_{y0} + \Delta\nu \sin\left(2\pi t/T\right) \end{aligned} \tag{3}$$

For the RHIC lattices used, the tune modulation amplitude $\Delta\nu$ and the gradient ripple amplitude $\Delta G/G$ are related by

$$\begin{aligned} \Delta\nu &= 80\ \Delta G/G \qquad \text{for } \beta^* = 2 \\ \Delta\nu &= 45\ \Delta G/G \qquad \text{for } \beta^* = 6 \end{aligned} \tag{4}$$

The dynamic aperture for 10^6 turns, A_{SL}, as a function of the tune modulation amplitude, $\Delta\nu$, was found by tracking particles for one particular distribution of field errors, for various values of a 60 cps field gradient ripple, $\Delta G/G$. The results are shown in Fig. 1 for the $\beta^* = 2$ and $\beta^* = 6$ lattices. No synchrotron oscillations are present, and the particle momentum is fixed at $\Delta p/p = 0$. The dynamic aperture appears to decrease roughly linearly with the size of the tune modulation amplitude, $\Delta\nu$.

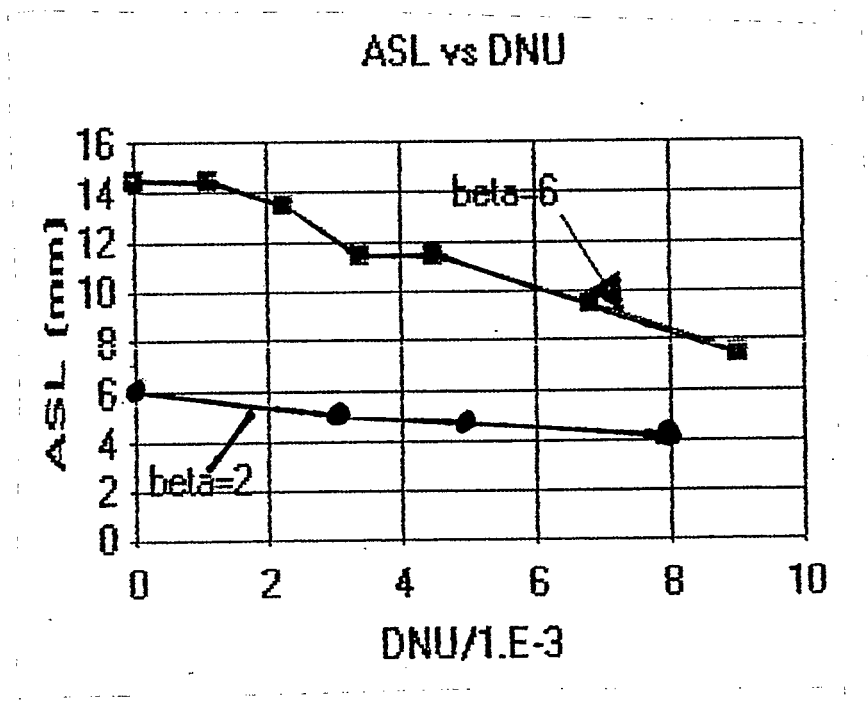

Figure 1: Dynamic aperture, A_{SL}, versus the tune modulation amplitude, DNU.

Figure 2 shows the survival time in turns as a function of the initial betatron amplitude x for several values of the tune modulation amplitude for the $\beta^* = 6$ lattice. One sees that for larger values of the tune modulation, $\Delta\nu$, it is necessary to track for a long time to find the dynamic aperture. Even at 10^6 turns, there is no certain indication that the dynamic aperture will not continue to decrease. For a 60 Hz gradient ripple, the period of the

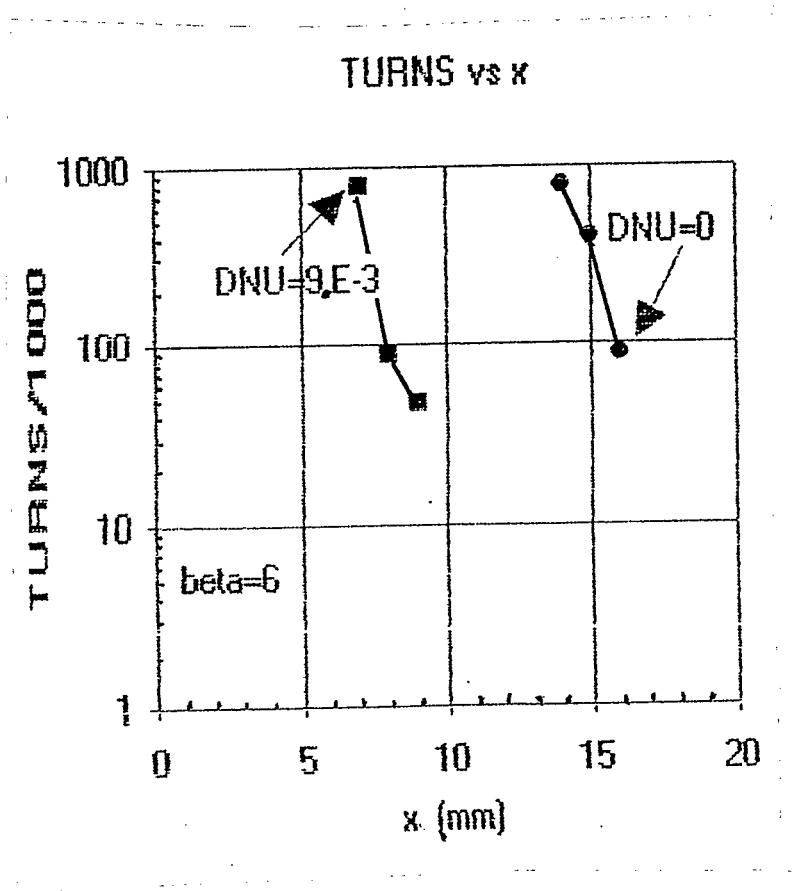

Figure 2: Survival time in turns, TURNS, versus the initial betatron amplitude x, for $\beta^* = 6$.

ripple is 1389 turns in RHIC. Thus a 10^6 turn run is about 720 ripple periods. Figure 3 shows the survival time in turns versus x for the $\beta^* = 2$ lattices.

In the range of $\Delta\nu$ covered in Figs. 1 through 3 there is only one low order resonance, which is below tenth order, that is reached by the tune modulation. This is the 5/6 resonance at $\nu = 28.83333$. The unperturbed tune is at $\nu_x = 28.826$ $\nu_y = 28.821$. The 5/6 resonance is reached at a tune modulation of $\Delta\nu \geq 7 \times 10^{-3}$. The results in Fig. 1 do not clearly show the presence of this resonance. The runs done with $\Delta\nu \geq 7 \times 10^{-3}$ sweep over the 5/6 resonance many times in 10^6 turns, yet the dynamic aperture found for $\Delta\nu \geq 7 \times 10^{-3}$ does not show much effect from the 5/6 resonance. Overall, Fig. 1 indicates

a roughly linear drop in the dynamic aperture, A_{SL} with $\Delta\nu$ which may be represented by

$$A_{SL} = A_0\,(1 - 42\ \Delta\nu) \tag{5}$$

where A_0 is the dynamic aperture for $\Delta\nu = 0$, and $\Delta\nu$ is the tune modulation amplitude. The same result Eq. (5) was found for both lattices, the $\beta^* = 6$ and the $\beta^* = 2$ lattices.

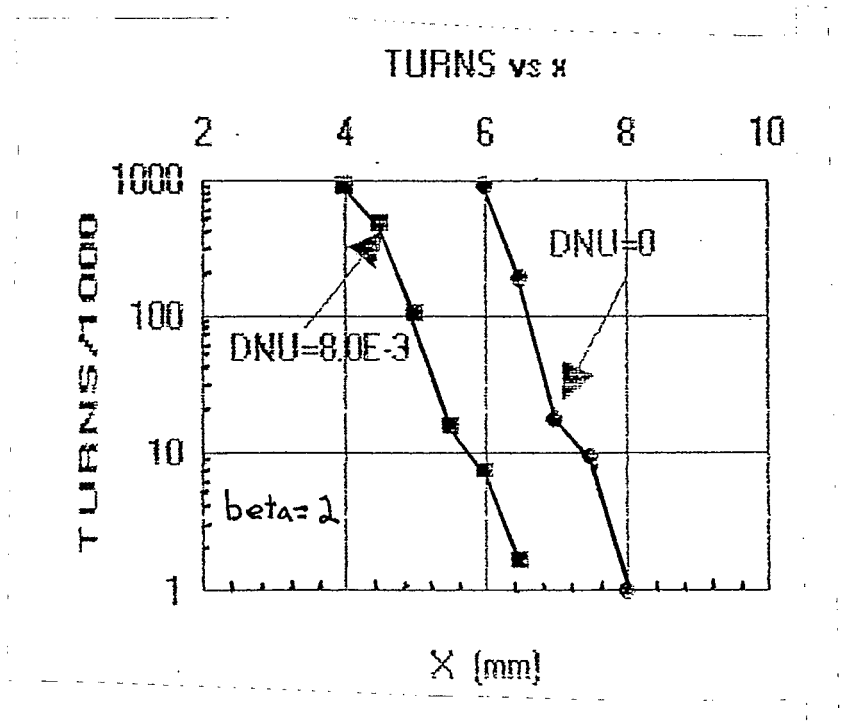

Figure 3: Survival time in turns, TURNS, versus the initial betatron amplitude, x, for $\beta^* = 2$.

One may speculate as to under what conditions the result, Eq. (5) may be roughly valid. One may conjecture that Eq. (5) for the dependence of the dynamic aperture on the amplitude of the tune modulation may be roughly valid under the following conditions:

1. The tune modulation does not sweep over the lower order resonances like the 1/3 or 1/4 resonances.
2. The field error multipoles, b_n or a_n, are roughly given by b_0/R^n, where $b_0 \simeq 2 \times 10^{-4}$ and, usually, R is roughly the magnet coil radius.
3. The tune modulation is generated by a gradient ripple and the ripple frequency is small compared to the particle revolution frequency in the accelerator.

It is difficult to compare the tracking results found in this study with those found in other tracking studies[2–4] because of the difference in procedures and in underlying assumptions. The general shape of the survival curves, Fig. 2 and Fig. 3 seems similar to those found in Ref. 2. Also the magnitude of the loss in dynamic aperture for a similar choice of ripple frequency and tune is in rough agreement with that found in Ref. 2.

3. Dependence of the Dynamic Aperture on the Gradient Ripple Frequency

The frequency of the gradient ripple was varied from $f_r = 15$ Hz to $f_r = 480$ Hz. The amplitude of the tune ripple was held constant at $\Delta\nu = 9 \times 10^{-3}$ for the $\beta^* = 6$ lattice. The results for the dynamic aperture are shown in Fig. 4 for the same distribution of field errors that was used in the previous studies. Within the uncertainties present in the concept and measurement of the dynamic aperture, no appreciable dependence on the

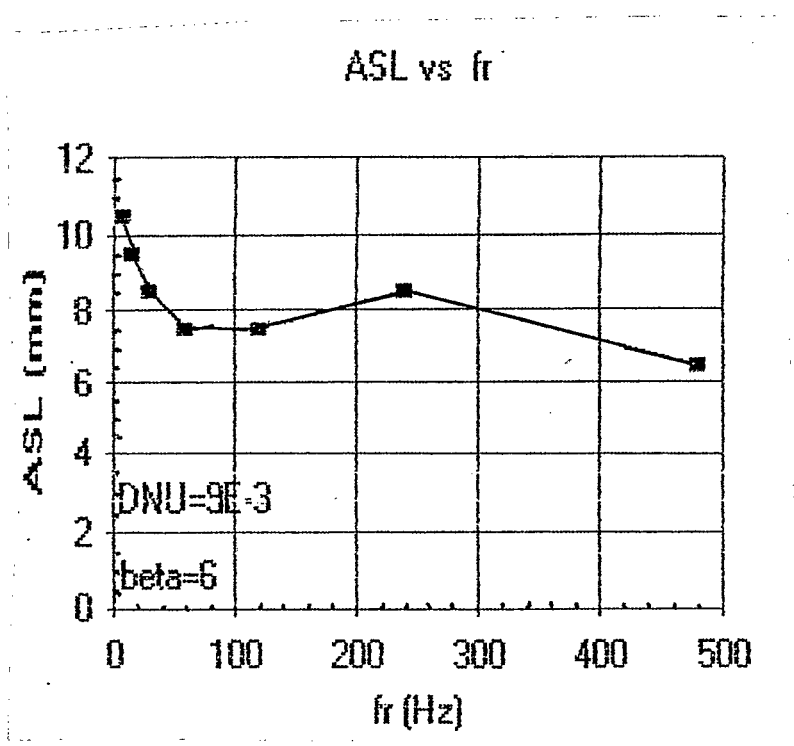

Figure 4: Dynamic aperture, A_{SL}, versus the gradient ripple frequency, f_r for a $\beta^* = 6$ lattice and a tune modulation amplitude $DNU = 9 \times 10^{-3}$.

ripple frequency, f_r is seen. There is a suggestion of a rise in the dynamic aperture at lower f_r. However this simulation study is starting to break down at the low f_r. In 10^6 turns, one has time for 720 ripple cycles for $f_r = 60$ Hz; at $f_r = 15$ Hz, one has time for only 180 cycles, and 10^6 turns may not be long enough to see the effect of a 15 Hz ripple. This breakdown would tend to overestimate the dynamic aperture at low f_r.

The procedure used in this simulation study also starts to breakdown when f_r gets large, of the order of the revolution frequency. In this study, the gradient in the quadrupoles is changed at the beginning of each turn (see Section 4). This may not be valid when the ripple period gets to be of the order of a few turns. At $f_r = 480$, the ripple period is 174 turns.

4. Comments on the Tracking

This tracking study was done without the presence of synchrotron oscillations and with a fixed $\Delta p/p$ of $\Delta p/p = 0$. Runs of about 10^6 turns were done to determine the dynamic aperture for 10^6 turns. The chromaticity was set at $C_x = C_y = 0$. Two RHIC lattices were used. One lattice has 6 insertion regions with $\beta^* = 2$m and one has $\beta^* = 6$m. The unperturbed tune was $\nu_x = 28.826$, $\nu_y = 28.821$.

Transfer matrices for each element were found using the large accelerator approximation that linearizes the equations of motion. The tracking is symplectic when synchrotron oscillations are absent. In introducing a ripple in the gradient of all the quadrupoles, the gradient in the quadrupoles was changed according to Eq. (2) at the beginning of each turn. A 60 cycle ripple has a period of 1389 turns for RHIC. At the beginning of each turn, after the gradient was changed, the transfer matrices of the magnets were recomputed using the new gradients in the quadrupoles.

Random and systematic field multipoles, up to order 10, were included in the study. The magnitude of these multipoles is given in Ref. 1. This study was done with the ORBIT program.[5] To find the dynamic aperture, tracking runs were done for various initial x_0, y_0 such that $\epsilon_x = \epsilon_y$ and $x_0' = y_0' = 0$.

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