Horizontal intrinsic resonances

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Some basic spin formulas:

$$\sigma_1 := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 := \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_0 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M(nx, ns, ny, \phi) := \sigma_0 \cdot \cos\left(\frac{\phi}{2}\right) - i \cdot \frac{(nx \cdot \sigma_1 + ns \cdot \sigma_2 + ny \cdot \sigma_3)}{\sqrt{nx^2 + ns^2 + ny^2}} \cdot \sin\left(\frac{\phi}{2}\right) \qquad sptune(M) := \frac{1}{\pi} \operatorname{acos}\left(\frac{tr(M)}{2}\right)$$

$$preaxis(M) := \left[\frac{i \cdot tr(\sigma_1 \cdot M)}{2 \sin(\pi \cdot sptune(M))} \quad \frac{i \cdot tr(\sigma_2 \cdot M)}{2 \sin(\pi \cdot sptune(M))} \quad \frac{i \cdot tr(\sigma_3 \cdot M)}{2 \sin(\pi \cdot sptune(M))} \right]$$

Formulas for depolarization from resonance crossing (Froissart-Stora, rms[ε is rms emittance], polarization at center):

$$pf(\varepsilon, \alpha) := \left[2 \exp\left[-\pi \frac{(|\varepsilon|)^2}{2\alpha} \right] - 1 \right] \qquad prms(\varepsilon, \alpha) := \frac{\left[1 - \pi \frac{(|\varepsilon|)^2}{\alpha} \right]}{\left[1 + \pi \frac{(|\varepsilon|)^2}{\alpha} \right]} \qquad phc(\varepsilon, \alpha) := \left[2 \cdot \sqrt{\frac{1}{\left[\frac{\pi (|\varepsilon|)^2}{\alpha} \right] + 1}} - 1 \right]$$

Parameter definition: horizontal alpha and beta function at first partial snake, partial snake strength as fraction of full snake, G, average gamma, horizontal rms emittance, horizontal tune, :

$$\alpha1 := -1.03 \quad \beta1 := 11.8 \quad ss1 := 0.14 \qquad G := 1.7928 \qquad \gamma := \frac{22.5}{G} \qquad emx := 0.0000025 \qquad d\upsilon h := 8.6$$

Second snake: horizontal alpha and beta function, partial snake strength as fraction of full snake, distance from first snake as fraction of circumference:

$$\alpha2 := -1.2 \quad \beta2 := 17 \quad ss2 := 0.059 \quad dc := \frac{1}{3} \qquad f(x) := 1$$

One-turn spin matrix, starting in front of first partial snake, and spin tune as function of Gγ:

$$f1(x) := \frac{1 + x}{\sqrt{x^2 - G^2}} \qquad f1(50) = 1.021 \qquad f(x) := 1 + \frac{93.5 \cdot G^2}{25.2 \cdot x^2} \qquad f(50) = 1.005$$

$$Mot(x) := ((M(0,0,1,2\pi \cdot (1 - dc) \cdot x) \cdot M(0,1,0,ss2 \cdot f(x) \cdot \pi) \cdot M(0,0,1,2\pi \cdot dc \cdot x) \cdot M(0,1,0,ss1 \cdot f(x) \cdot \pi)))$$

$$\upsilon s(x) := sptune(Mot(x)) \qquad Ph(x) := \sqrt{\left[\left(preaxis(Mot(x))^T \right)_0 \right]^2 + \left[\left(preaxis(Mot(x))^T \right)_1 \right]^2}$$

$$Pv(x) := \left(preaxis(Mot(x))^T \right)_2 \qquad Ppol(x) := \left(preaxis(M(0,1,0,ss1 \cdot \pi) \cdot Mot(x) \cdot M(0,1,0,-ss1 \cdot \pi))^T \right)_2$$

Resonance crossing speed correction when spin tune is not equal to Gγ (dυs/dθ = dGγ/dθ * dυs/dGγ):

$$dvsdgg(x) := \frac{(\upsilon s(x + .001) - \upsilon s(x))}{.001} \qquad dvsdgg(d\upsilon h) = 0.948 \qquad \alpha := .00004168 \cdot dvsdgg(d\upsilon h) \qquad \alpha = 3.95 \cdot 10^{-5}$$

Horizontal component of stable spin direction in front of partial snake:

$$\left[1.207 \cdot 10^{-15} \right.$$

$P := \text{preaxis}(\text{Mot}(45))^T$ $\qquad$ $Ph(d\upsilon h) = 0.343$ $\qquad$ $Ph(d\upsilon h) = 0.343$ $\qquad$ $P = \begin{bmatrix} -1 \\ 6.732\cdot 10^{-15} \end{bmatrix}$

rms angular divergence at partial snake, rms value of deviation of total horizontal orbit angle from 2π per turn at the partial snake:

$$\theta := \sqrt{\frac{\left(1+\alpha 1^2\right)\cdot emx}{\beta 1\cdot\gamma}} \qquad \theta = 1.865\cdot 10^{-4} \qquad \theta := \sqrt{\frac{\left(1+\alpha 1^2\right)\cdot emx}{\beta 1\cdot\gamma}}\cdot 2\cdot\sin(\pi\cdot d\upsilon h) \qquad \theta = 3.548\cdot 10^{-4}$$

Strength of a resonance driven by spin rotation of $G\gamma\theta$ around the vertical direction scaled by the horizontal component of the stable spin direction Ph at the partial snake. The resonance condition for this resonance is $\upsilon s = n \pm \upsilon h$.

$$eps(Gg) := \sqrt{\frac{Gg\cdot emx}{G}}\cdot\frac{2\cdot G\cdot Ph(Gg)\cdot\sin(\pi\cdot d\upsilon h)}{4\,\pi}\cdot\sqrt{\frac{\left(1+\alpha 1^2\right)}{\beta 1}}$$

Some depolarization values for the average $G\gamma$:

$prms(eps(G\cdot\gamma),\alpha) = 0.998918$ $\qquad$ $phc(eps(G\cdot\gamma),\alpha)^{84} = 0.955559$ $\qquad$ $pf(eps(G\cdot\gamma),\alpha) = 0.999459$

$G\gamma$ values for the horizontal resonances $\upsilon s = n \pm \upsilon h$:

$$Ggh(z) := \begin{cases} \left(\frac{z}{2} + 13 - d\upsilon h\right) & \text{if } \frac{z}{2} = \text{trunc}\left(\frac{z}{2}\right) \\ \frac{z}{2} - 4.5 + d\upsilon h & \text{otherwise} \end{cases}$$

$Ggh(1) = 4.6$ $\qquad$ $Ggh(2) = 5.4$

$Ggh(3) = 5.6$ $\qquad$ $Ggh(4) = 6.4$

$Ggh(83) = 45.6$ $\qquad$ $Ggh(84) = 46.4$

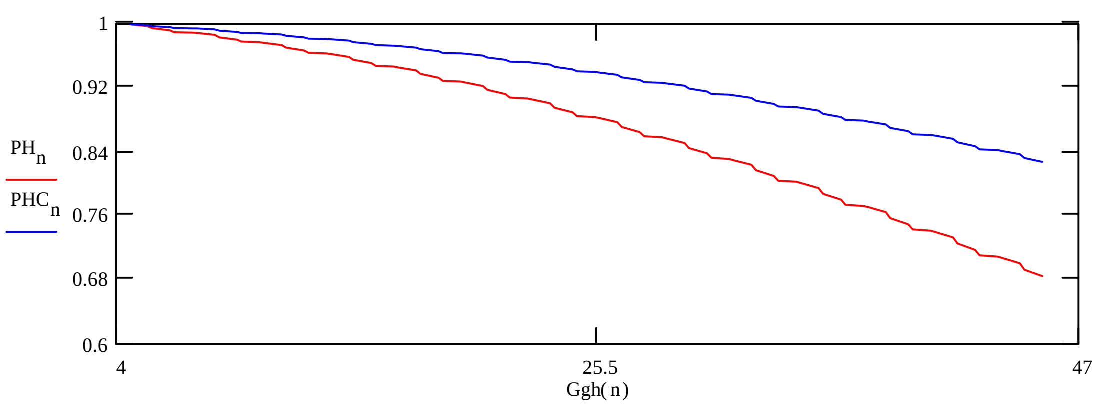
Calculation of integrated polarization loss over the whole AGS cycle for rms and center of beam:

$n := 1, 2 .. 82$

$PH_0 := 1 \quad PH_n := (prms(eps(Ggh(n)),\alpha))\cdot PH_{n-1}$ $\qquad$ $PHC_0 := 1 \quad PHC_n := (phc(eps(Ggh(n)),\alpha))\cdot PHC_{n-1}$

$PH_{82} = 0.685$ $\qquad$ $PHC_{82} = 0.828$

Polarization over the AGS cycle:



$Pinj := .80$

$Pfinal := PHC_{82}\cdot Pinj\cdot Pv(4.5)\cdot Ppol(45.5)$ $\qquad$ $Pfinal = -0.637$

$$Pfinal := PH_{82} \cdot Pinj \cdot Pv(4.5) \cdot Ppol(45.5) \qquad Pfinal = -0.527$$