STRING REFORMULATION OF SPACE CHARGE FORCE

R. Talman, BNL, 16 January, 2004

MOTIVATION AND REVIEW

Windows of opportunity for Cornell ERL

- ultra–small emittance
- ultra–short bunches

| either or both may be limited by space charge forces

Small emittance issues

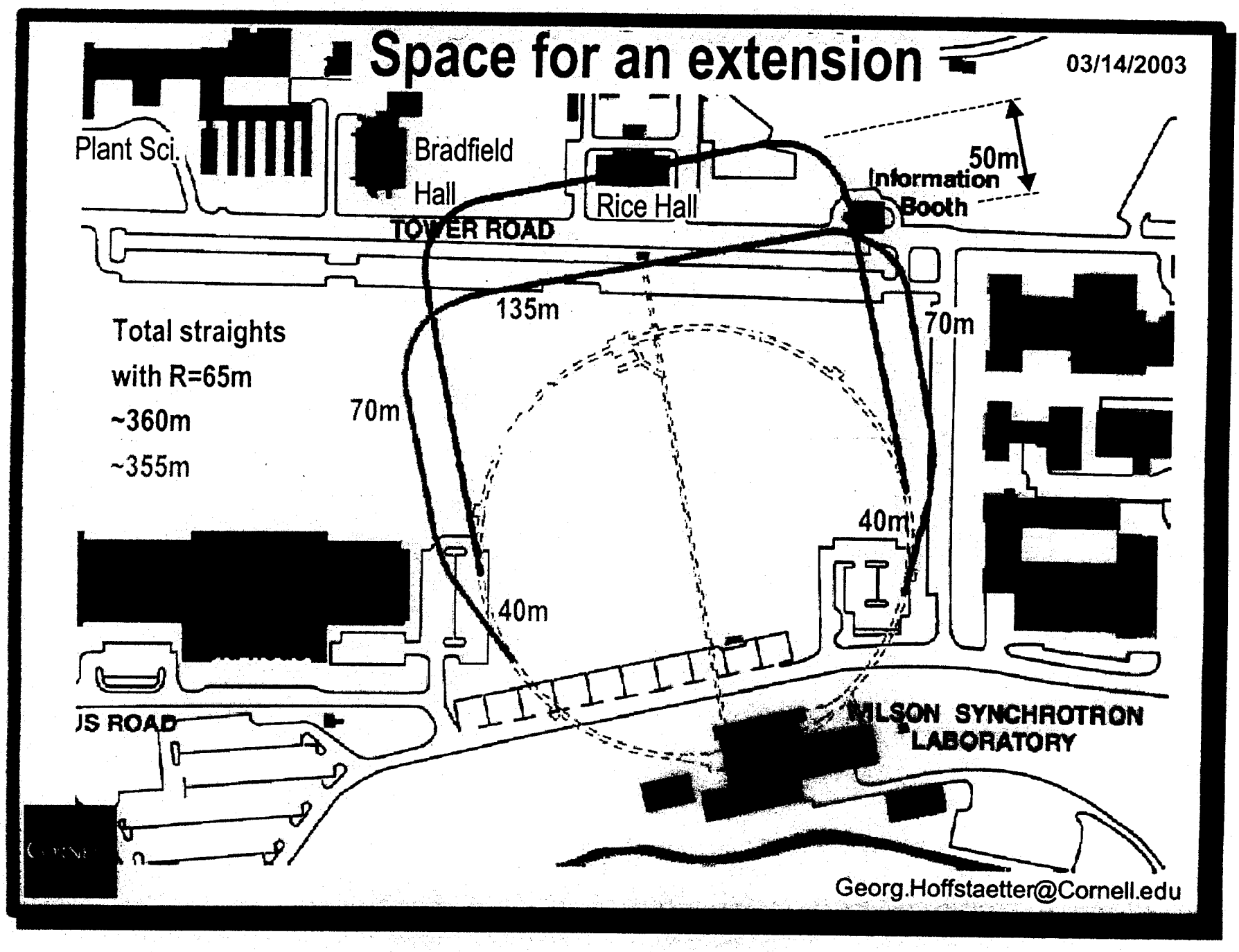
- minimum useful emittance, for x–ray source
- dependence of brilliance on beam current
- comparison of ERL with III'rd generation light source

Space charge effects in bending magnets

- coherent synchrotron radiation (CSR), longitudinal
- centrifugal space charge force (CSCF), transverse

STRING/POINT CHARGE REFORMULATION OF SPACE CHARGE FORCE

- represent space charge force as force between point and 'string'
- concentrate on (subtle) longitudinal force (in this talk)
  even though transverse is just as important
- self–force of curving string and extraction of CSR
- renormalization to remove (only logarithmic) divergence
- formulation of space charge problem as 'intrabeam scattering'


Space for an extension
03/14/2003
Plant Sci.
Bradfield Hall
Rice Hall
TOWER ROAD
50m
Information Booth
135m
70m
Total straights
with R=65m
~360m
~355m
70m
40m
40m
JS ROAD
WILSON SYNCHROTRON LABORATORY
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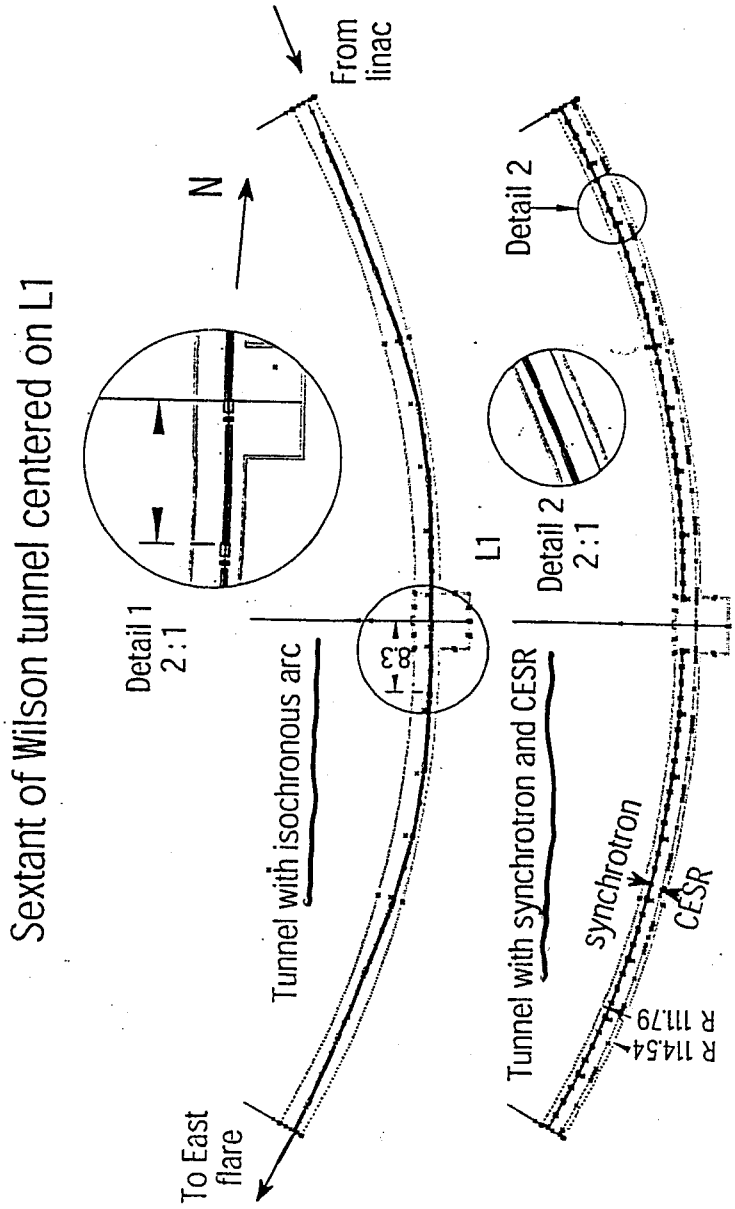
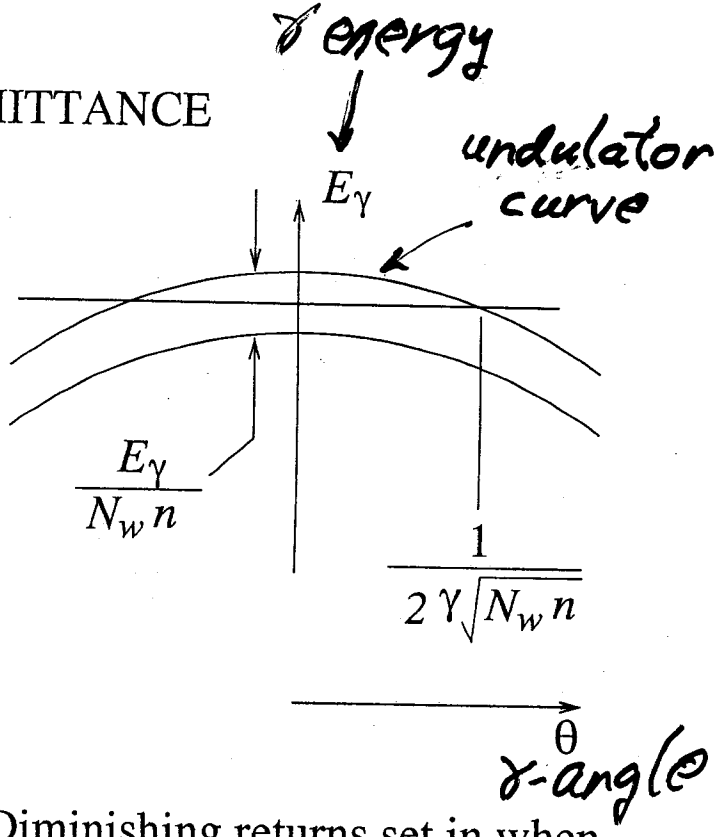


Figure 3.1: The left figure shows the (southeast) isochronous arc conveying beam from end of the final ERL linac section to the Wilson experimental hall. For comparison purposes the right figure shows the present tunnel layout. The southwest arc is similar.

MINIMUM USEFUL ELECTRON EMITTANCE



$$\text{useful electron fraction} = \frac{\text{thru–monochromator cone solid angle}}{\text{electron beam cone solid angle}}$$

(n: undulator harmonic (n=1 is best); N_w: number of wiggler periods)

$$= \frac{1/(4\, n\, N_w \gamma^2)}{(\sigma_x/\beta_x)(\sigma_y/\beta_y)} \qquad \sqrt{\varepsilon_x/\beta_x} \quad \sqrt{\varepsilon_y/\beta_y}$$

Reducing electron beam emittance increases brilliance. Diminishing returns set in when useful electron fraction is about 1.

$$\varepsilon_{\min,\gamma} = \frac{\beta}{4\, n\, N_w \gamma^2} \quad \text{e.g.} = \frac{2\ (\text{m})}{4 \times 200 \times 10^8} = 0.2 \times 10^{-10}\ \text{m}$$

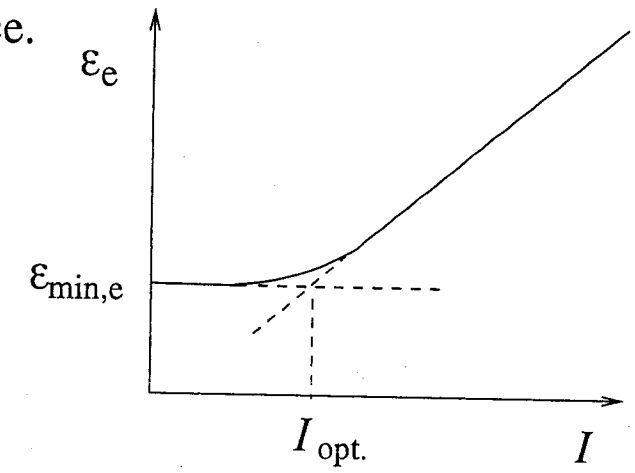
Good target to shoot for $\varepsilon_N \approx 0.2$ mm·mr

This is far smaller than can be obtained with a storage ring.

4

DEPENDENCE OF X–RAY BRILLIANCE ON LINAC BEAM CURRENT

Reducing electron beam current decreases emittance.



Diminishing returns set in when

$$\varepsilon_e \approx \varepsilon_{min,e}$$
$$I \approx I_{opt.}$$

target

For simplicity assume

$$\varepsilon_{min,e} \approx \varepsilon_{min,\gamma} = \frac{1}{4\,n\,N\,\gamma^2} \overset{\text{e.g.}}{=} \frac{1}{4 \times 100 \times 10^8} = 0.2 \times 10^{-10}\ \text{m}$$

For any experiment it is useful flux through sample that is the useful measure of beam quality. By convention 0.1 percent band width is "useful". The useful flux depends on detailed beam line design, but it correlates most closely with brilliance B . In region when diminishing returns has not set in, with n=1,

$$B \approx \frac{10^{18}}{\varepsilon_x\,(\text{nm})\ \varepsilon_y\,(\text{nm})} \frac{N_w\,K^2}{1 + K^2/2} \left[\frac{\text{photons}}{\text{s mm}^2\ \text{mr}^2\ 0.1\%\text{BW A}}\right] I\,(\text{A})$$

$\varepsilon_x \propto I$, $\varepsilon_y \propto I$, $I \propto I$, $\propto \frac{1}{I}$

For $I > I_{opt.}$ the brilliance (and hence useful flux) varies inversely with beam current

Increasing K much above 1 is not useful because of the denominator factor.

anti-intuitive

COMPARISON OF ERL LINAC AND GENERATION–III–RING AS SOURCE OF X–RAYS

	quality ratio	numerical	potential improvement	
1. Brilliance (flat x–ray beam) storage ring	$\frac{\varepsilon_x^{III}\varepsilon_y^{III}}{\varepsilon^2}$	$\frac{10^{-8}10^{-10}}{(10^{-10})^2}$	100	more brilliance
2. "Forward spike" (through monochromator) is ''matched'' to electron beam when • photon cone angle, $\frac{1}{\gamma}\frac{1}{\sqrt{nN_w}}$ is equal to • electron cone angle, $\sqrt{\frac{\varepsilon}{\beta}}$	$\rightarrow \frac{nN_w}{(nN_w)^{III}} = \frac{\varepsilon_x^{III}}{\varepsilon}$	$\frac{10^{-8}\text{m}}{10^{-10}\text{m}}$	100	bigger nN_w product
3. Transverse coherence: shortest wavelength with ''matching'' electron and photon waists • electron waist length β • photon mode length $\pi\frac{\beta\varepsilon}{\lambda}$	$\rightarrow \frac{\lambda_{\text{min.}}^{\text{coh.}}}{(\lambda_{\text{min.}}^{\text{coh.}})^{III}} = \frac{\varepsilon}{\varepsilon_x^{III}}$	$\frac{10^{-10}\text{m}}{10^{-8}\text{m}}$	0.01	shorter coherent wavelength
4. Longitudinal coherence length (including $\lambda/\Delta\lambda$ lengthening factor) • $l_{\text{coh.}} = nN_w\lambda(\Delta\lambda/\lambda)$; Also $N_w^{\text{max.}} \alpha \frac{E}{\Delta E}$ and other considerations are important				
5. Minimum pulse duration	$\frac{100\text{ fs}}{100\text{ ps}}$		0.001	shorter in time

COHERENT SYNCHROTRON RADIATION (CSR)

• harms emittance
• harms SC cavities

Far field method — Integrate Poynting vector

- straightforward (Schwinger etc)
- $P_{\text{incoh.}} = N \frac{2}{3} \frac{\omega e^2}{R} \gamma^4$ (rev. freq.) (Note: treats every particle equivalently, hence causes no current–dependent emittance growth)
- $P_{\text{coh.}} = N^2 \frac{\omega e^2}{R} \left(\frac{\sqrt{3}}{\alpha} \right)^{4/3}$ (angular bunch length) (Note: no γ^4) neglects "vacuum chamber screening"
- fractional effect $= \frac{1}{E} \frac{dE}{dt} \alpha \frac{N^2}{R^{2/3} l^{4/3}} \frac{1}{\gamma}$ (Favors bending at highest γ highest R)
- energy for $P_{\text{coh.}} = P_{\text{incoh.}}$: $\gamma = \left(\frac{3N}{2} \right)^{1/4} \left(\frac{\sqrt{3} R}{l} \right)^{1/3}$ (surprisingly large)

Near field method (plus conservation of energy) — self-field

- needed for studying internal–to–bunch particle dynamics
- longitudinal reactive force on accelerating point charge is infinite. (Point charges are incompatible with pure Maxwell theory.)
- to calculate excess longitudinal force due to curvature requires a renormalization procedure due (perhaps) to Saldin, Schneidmiller and Yurkov.
- with line charges the divergence is only logarithmic. Program TraFiC suppresses? the divergence by this approach, i.e. never calculating longitudinal force exactly on line charge.
- charge–dependent particle energy could, in principle, be compensated by beam line optics, but particle energy depending on position in bunch causes emittance growth (for non–vanishing dispersion). emittance growth ?

CENTRIFUGAL SPACE CHARGE FORCE (CSCF)

- In bending magnet a curvature effect tends to defeat the usual cancelation of electric and magnetic space charge forces. (RT, 1985.) The resulting force depends strongly on transverse position which can increase transverse emittance.

TABLE I. Numerical values.

Symbol	Meaning or equation	Value	Unit
γ		10^4	
R	Radius of orbit	80	m
ω_0	Revolution frequency	$2\pi \times 0.39 \times 10^6$	sec^{-1}
σ_L	Bunch length (s.d.)	2	cm
h	Bunch full height (uniform)	2.7	mm
q	Charge in bunch	$\frac{0.01\ A}{0.39 \times 10^6\ \text{Hz}}$	C
α	Damping (11)	40	sec^{-1}
R_0	(9)	455	V/m
R_1	(9)	−0.6	V/m·mm
R_2	(9)	−30	$\text{V/m} \cdot \text{mm}^2$
Z_2	(9)	−0.11	mm^{-2}
A_r	(10)	5	mm
A_z	(10)	1	mm
A_s	(10)	2	cm

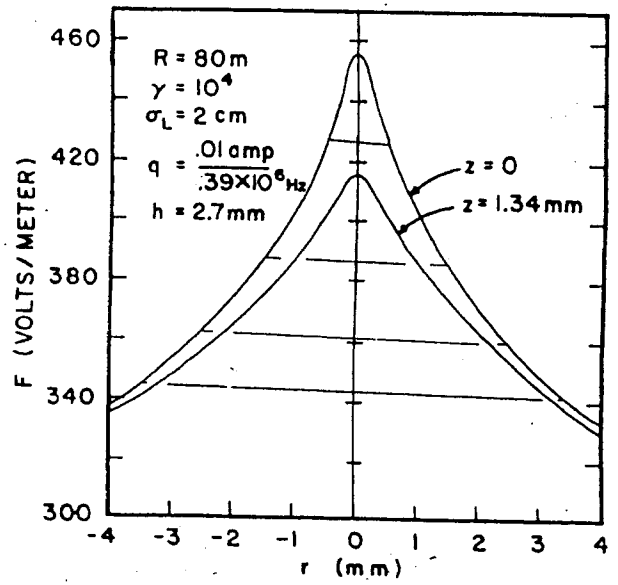


FIG. 1. Dependence of the transverse force F on radial position r and vertical position z.

"cancellation effect" controversial.

- (Lee, 1987) showed that (in azimuthally uniform, unbunched beam) the transverse force is accompanied by an energy–changing force. When the guide field force is augmented by the space charge force it perturbs both transverse force and dispersion. In their effect of tunes the transverse force and the dispersion effects tend to cancel. But emittance?
- (Derbenev and Shiltsev, 1996) extend Lee argument from tunes to emittance growth and also to separated function lattices. But is is meaningful to treat space charge as part of guide field for single passage (as contrasted to long term averaging)?

UNCORRELATED EMITTANCE GROWTH IN THE TTF-FEL BUNCH COMPRESSION SECTIONS DUE TO COHERENT SYNCHROTRON RADIATION AND SPACE CHARGE EFFECTS

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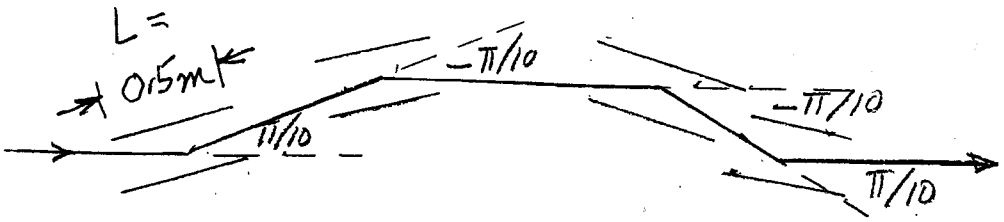

$E = 130\,MeV$

$\theta = \pi/10$

$L = 0.5\,m$

$R = \frac{L}{\theta} = 1.6\,m$

$\ell \overset{e.g.}{=} 0.25\,mm$

$Q \overset{e.g.}{=} 1nC \rightarrow N = 0.6\times10^{10}$

$$\gamma_{crit.} = \left(\frac{3\times0.6\times10^{10}}{2}\right)^{1/4}\left(\frac{\sqrt{3}\,1.6}{0.25\times10^{-3}}\right)^{\frac{1}{3}}$$

$= 6720$

$\therefore$ CSR is dominant.

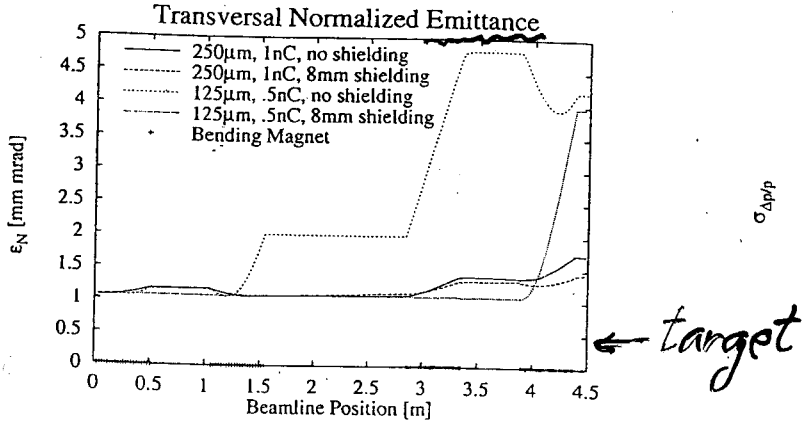

Figure 2: Normalized transversal statistical slice emittance for different parameter sets

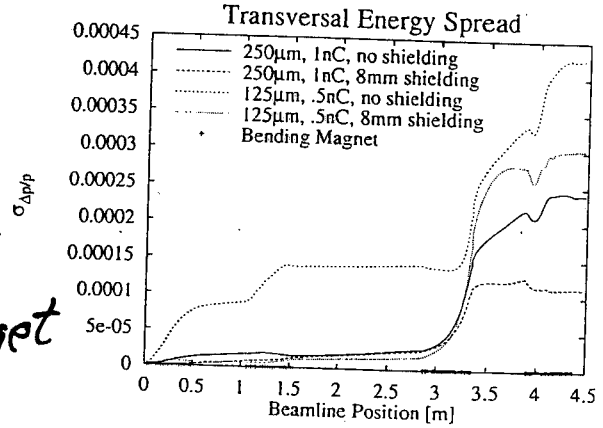

Figure 1: Induced transversal rms energy spreads for different parameter sets

$\Delta\varepsilon_N \approx 3\,mm\cdot mr$

$\Delta\delta \approx 4\times10^{-3}$

- also experimental investigations are underway

String Formulation of Space Charge Forces in a Deflecting Bunch

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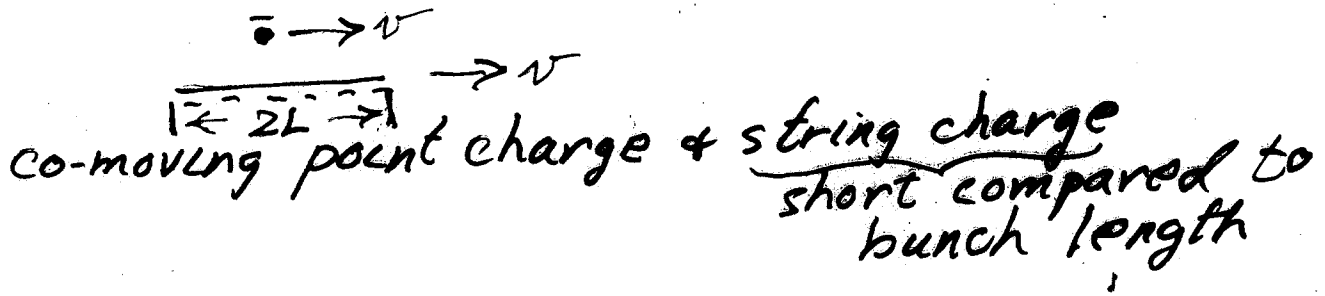
The force between two moving point charges, because of its inverse square law singularity, cannot be applied directly in the numerical simulation of bunch dynamics. Radiative effects make this especially true for short bunches being deflected by magnets. This paper describes a formalism in which the basic ingredient is the *total* force on a point charge co-moving with a longitudinally-aligned, uniformly-charged string and the calculation is treated as direct particle-to-particle intrabeam scattering, rather than as an effective field calculation. Electric and magnetic fields do not appear individually in the theory. Since the basic formulas are both exact and fully relativistic, they are applicable to beams of all particle types and all energies. But the theory is expected to be especially useful for calculating the emittance growth of the ultrashort electron bunches of current interest for ERL's (energy recovery linacs) and FEL's (free electron lasers). The theory subsumes CSR (coherent synchrotron radiation) and CSCF (centrifugal space charge force.)



In a numerical simulation of bunch dynamics, each of the N particles is to be treated as a point particle as far as its own dynamics is concerned, but as a line charge for the purpose of calculating the electromagnetic fields it generates. The forces caused by these fields can either be applied directly to each individual particle in the bunch or, for simulations in which N is very large, be recorded on a three dimensional grid, from which forces on individual particles are then calculated by interpolation. In either case the basic ingredient is the force on a point charge due to a charged string.

intrabeam scattering

effective field

no P.I.C.

Fundamental Formulas

For a longitudinally uniform charge distribution the charge and current densities can be expressed in terms of step function U;

$$\lambda(\alpha') = \lambda_0[U(\alpha' - \underline{\alpha}') - U(\alpha' - \overline{\alpha}')]$$
$$\mathbf{I}(\alpha') = \beta c \lambda_0 \hat{\mathbf{v}}'(\alpha')[U(\alpha' - \underline{\alpha}') - U(\alpha' - \overline{\alpha}')]. \quad (69)$$

The only dependence of λ on α' and t is through the string end locations—otherwise its value is either λ_0 or zero. The *direction* of $\mathbf{I}$ depends on α', but not on time.

It is well known that there is a substantial cancellation of magnetic and electric forces between parallel-traveling, relativistic particles. It is therefore appopriate to express explicitly the force $\mathbf{F}$ acting on the test charge as it passes point P;

Lorentz

$$\frac{\mathbf{F}(P,0)}{e} = \mathbf{E}(P,0) + \beta c \hat{\mathbf{s}} \times \mathbf{B}(P,0). \quad (70)$$

the complete Jefimenko equations are

retarded time

end effects

$$\mathbf{E}(P,t) = \frac{1}{4\pi\epsilon_0} \int \left[\frac{\lambda(z',t_r)}{r^2}\hat{\mathbf{r}} + \frac{\dot{\lambda}(z',t_r)}{cr}\hat{\mathbf{r}} - \frac{\dot{\mathbf{I}}(z',t_r)}{c^2 r} \right] dz',$$

Coulomb

Biot-Savart

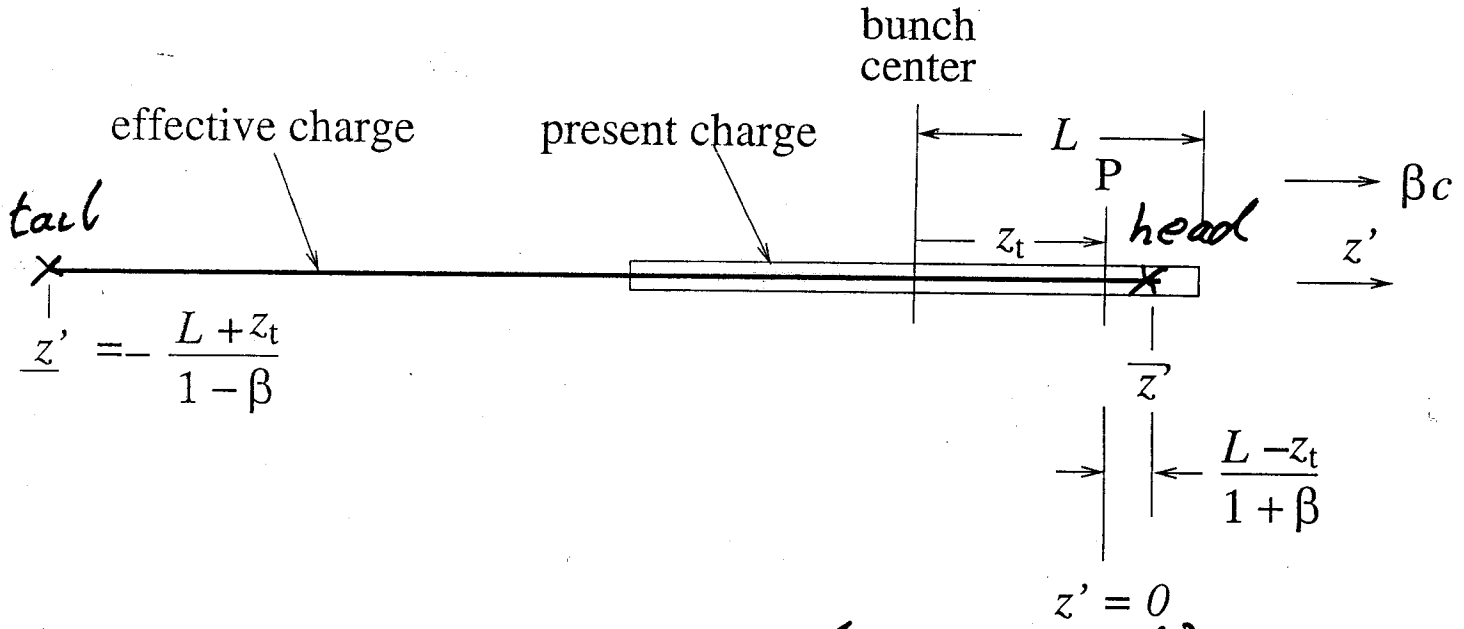
} static!

$$\mathbf{B}(P,t) = \frac{\mu_0}{4\pi} \int \left[\frac{\mathbf{I}(z',t_r)}{r^2} + \frac{\dot{\mathbf{I}}(z',t_r)}{cr} \right] \times \hat{\mathbf{r}}\, dz', \quad (31)$$

Self-Force of Moving Line Charge ("string")

- can use retarded ϕ, A, but getting $E + B$ is hard
- can use retarded E, B, but must include end terms

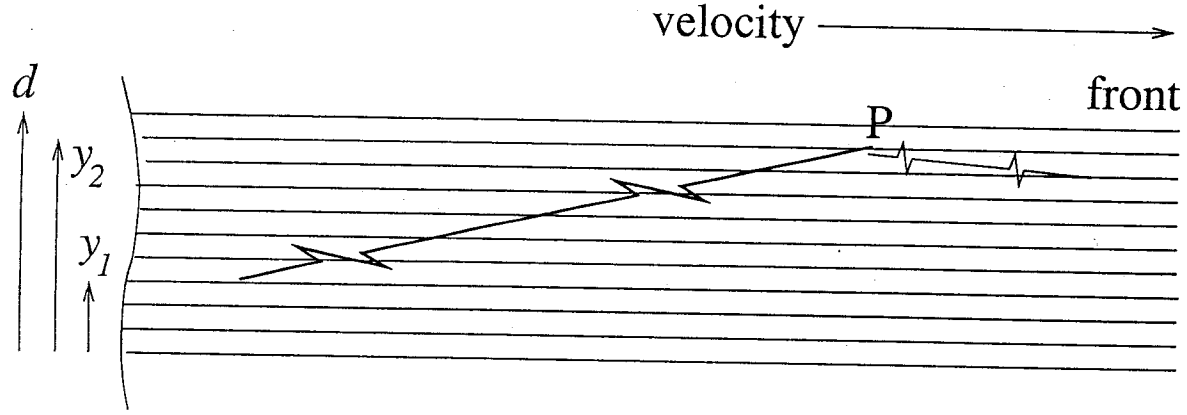
simple electro-magneto-statics



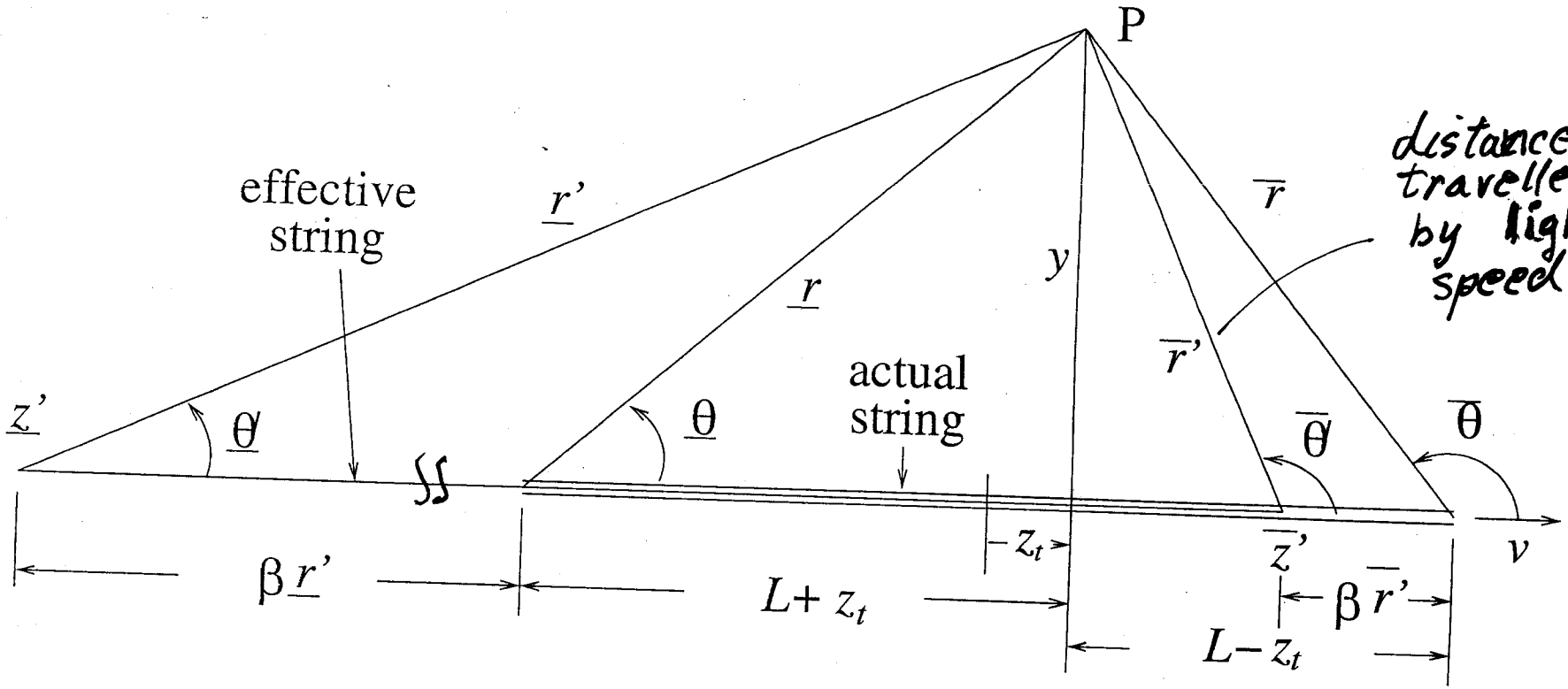
- string tension is infinite (unphysical)
- but only logarithmic
- total self-force vanishes

Self-Force of Moving Ribbon

- no divergence
- but d is artificial
- "regularization"

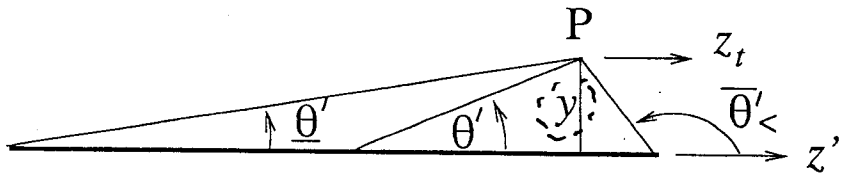


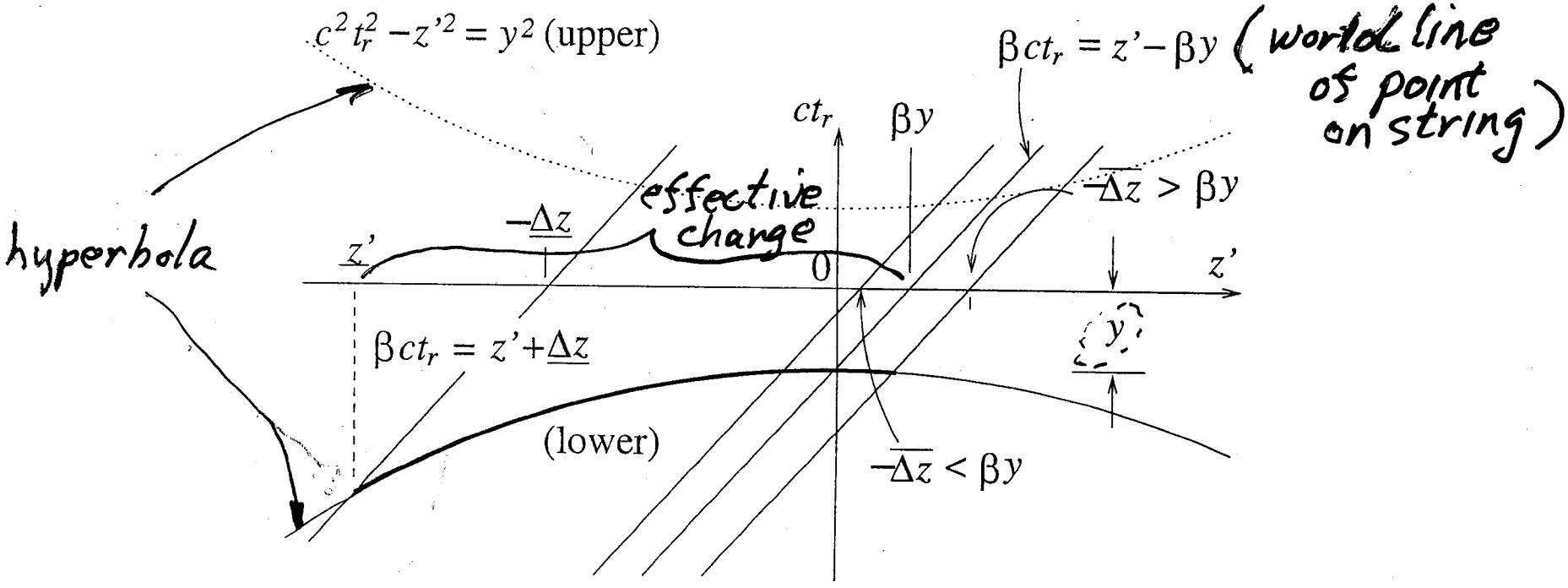
Off-axis fields



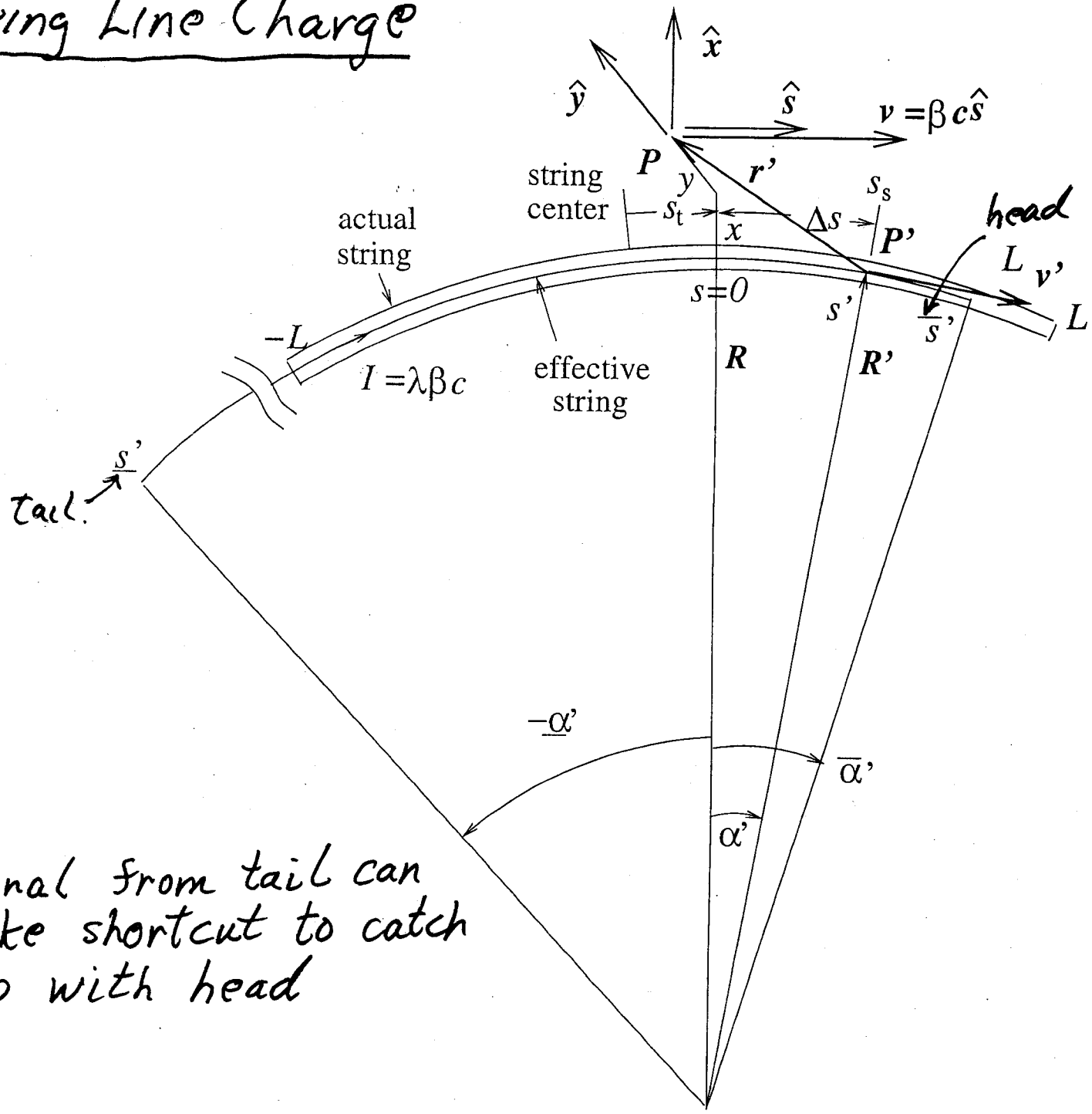
- figuring out retarded end points is half the battle

Graphical Representation of Retarded Time Calculation





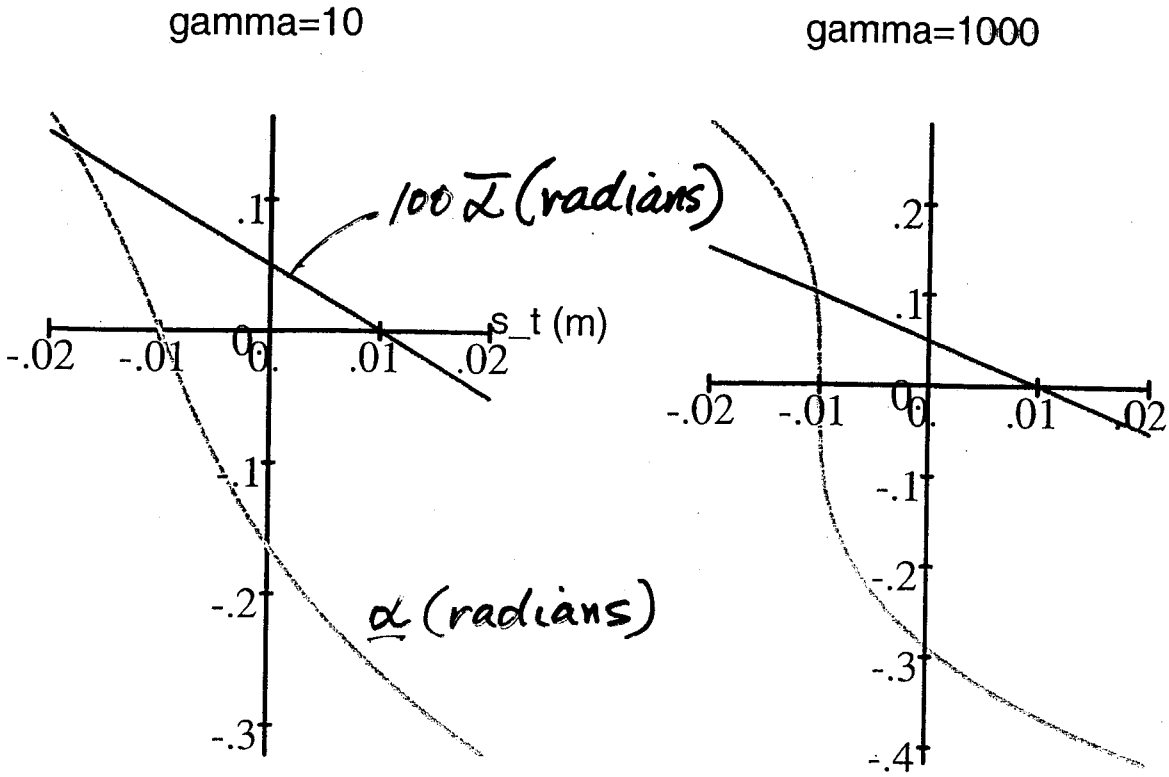
Curving" Line Charge



- signal from tail can take shortcut to catch up with head

Retarded Angles of End Points

• dependence on position s_t in bunch



Force Law (closed form).

$$\kappa = 1 - \beta\hat{\mathbf{v}}' \cdot \hat{\mathbf{r}}' = 1 + \beta \sin\alpha' \frac{R+x}{r'}.$$

Spelling out the components, the γ-independent part is

"ends"

$$\rightarrow \quad \frac{\mathbf{F}_0^{(\text{ends})}}{e} = \frac{\beta\lambda_0}{4\pi\epsilon_0}\left[\frac{1-\cos\alpha'}{\kappa r'^2}\begin{pmatrix} -R\sin\alpha' \\ R(1-\cos\alpha')+x \\ y \end{pmatrix} - \frac{R\sin\alpha'+\beta r'}{\kappa r'^2}\begin{pmatrix} \cos\alpha' \\ -\sin\alpha' \\ 0 \end{pmatrix}\right] \quad (\$\ x\ y)$$

$$= \frac{\beta\lambda_0}{4\pi\epsilon_0}\begin{bmatrix} -R\sin\alpha'/\kappa r'^2 - \beta\cos\alpha'/\kappa r' \\ (2R+x)(1-\cos\alpha')/\kappa r'^2 + \beta\sin\alpha'/\kappa r' \\ (1-\cos\alpha')\,y/\kappa r'^2 \end{bmatrix}_{\underline{\alpha}'}^{\overline{\alpha}'}.$$

The $1/\gamma^2$ term in Eq. (77) will typically be negligible but, if needed, it is

$$\rightarrow \quad \frac{\mathbf{F}_{1/\gamma^2}^{(\text{ends})}}{e} = \frac{\beta\lambda_0}{4\pi\epsilon_0\gamma^2}\begin{pmatrix} 0 \\ R(\cos\alpha'-1)+x\cos\alpha' \\ y\cos\alpha' \end{pmatrix}$$

"body"

$$\rightarrow \quad \frac{\mathbf{F}_0^{(\text{body})}}{e} = \frac{R\lambda_0}{4\pi\epsilon_0}\int_{\underline{\alpha}'}^{\overline{\alpha}'} \frac{d\alpha'}{r'^3}\left[(1-\cos\alpha')\begin{pmatrix} -R\sin\alpha' \\ R(1-\cos\alpha')+x \\ y \end{pmatrix} - R\sin\alpha'\begin{pmatrix} \cos\alpha' \\ -\sin\alpha' \\ 0 \end{pmatrix}\right]$$

$$= \frac{R\lambda_0}{4\pi\epsilon_0}\int_{\underline{\alpha}'}^{\overline{\alpha}'} d\alpha' \begin{pmatrix} -R\sin\alpha'/r'^3 \\ (2R+x)(1-\cos\alpha')/r'^3 \\ (1-\cos\alpha')\,y/r'^3 \end{pmatrix}$$

and the components of $\mathbf{F}_{1/\gamma^2}^{(\text{body})}$ are

$$\rightarrow \quad \frac{\mathbf{F}_{1/\gamma^2}^{(\text{body})}}{e} = \frac{R\lambda_0}{4\pi\epsilon_0\gamma^2}\int_{\underline{\alpha}'}^{\overline{\alpha}'} \frac{d\alpha'}{r'^3}\left[\cos\alpha'\begin{pmatrix} -R\sin\alpha' \\ R(1-\cos\alpha')+x \\ y \end{pmatrix} + R\sin\alpha'\begin{pmatrix} \cos\alpha' \\ -\sin\alpha' \\ 0 \end{pmatrix}\right]$$

$$= \frac{R\lambda_0}{4\pi\epsilon_0\gamma^2}\int_{\underline{\alpha}'}^{\overline{\alpha}'} d\alpha' \begin{pmatrix} 0 \\ (-R(1-\cos\alpha')+x\cos\alpha')/r'^3 \\ y\cos\alpha'/r'^3 \end{pmatrix}$$

X. EVALUATION OF INTEGRALS

The end effect forces have been given in closed form and the longitudinal body force integral evaluated for $x = 0$. For $x \neq 0$ the integals can also be evaluated in closed form, but they are more complicated. Depending, as they do, on r' as given by Eq. (53), the integrals appearing in Eqs. (72) and (73) depend on factors

$$\begin{aligned} \mathrm{E}(z,k) &= \int_0^z \sqrt{1-k^2t^2}/\sqrt{1-t^2}\,dt, \\ \mathrm{F}(z,k) &= \int_0^z 1/\sqrt{(1-k^2t^2)(1-t^2)}\,dt, \\ \mathrm{Pi}(z,\nu,k) &= \int_0^z 1/\sqrt{(1-\nu t^2)(1-t^2)(1-k^2t^2)}\,dt \end{aligned} \tag{99}$$

elliptic integrals. (quick algorithms are available)

$$I_s = \int \frac{\sin\alpha'}{(A+B\cos\alpha')^{3/2}}\,d\phi = \frac{2}{B\sqrt{A+B\cos\alpha'}}, \qquad I_0 = \int \frac{\cos\alpha'}{(A+B\cos\alpha')^{3/2}}\,d\phi,$$

$$\begin{aligned} I_{x,y} &= \int \frac{1-\cos\alpha'}{(A+B\cos\alpha')^{3/2}}\,d\phi \\ &= 2\frac{-B\sin(\alpha') + \sqrt{A-B}\,\sqrt{A-B+2B\cos^2(\alpha'/2)}\left(\mathrm{F}\big(\cos(\alpha'/2),\sqrt{\tfrac{-2B}{A-B}}\big) - \mathrm{E}\big(\cos(\alpha'/2),\sqrt{\tfrac{-2B}{A-B}}\big)\right)}{\sqrt{A-B+2B\cos^2(\alpha'/2)}\,B(A-B)} \end{aligned} \tag{100}$$

It is even possible to integrate first over y;

$$II_s = \int d\phi \int dy \frac{\sin\alpha'}{(A+B\cos\alpha')^{3/2}} = \frac{1}{2R(R+x)} \ln \frac{\sqrt{2R(R+x)+x^2+y^2-2R(R+x)\cos\alpha'} - y}{\sqrt{2R(R+x)+x^2+y^2-2R(R+x)\cos\alpha'} + y}$$

$$\begin{aligned} II_x &= \int d\phi \int dy \frac{1-\cos\alpha'}{(A+B\cos\alpha')^{3/2}} \\ &= -y\frac{x^2\Big(\mathrm{F}\big(\cos(\alpha'/2),D\big) - \mathrm{Pi}\big(\cos(\alpha'/2),C,D\big)\Big) + 4R(R+x)\mathrm{F}\big(\cos(\alpha'/2),D\big)}{R(R+x)(x+2R)^2\sqrt{4R(R+x)+x^2+y^2}} \end{aligned} \tag{101}$$

would be appropriate for "ribbons"

where

$$C = \frac{4R(R+x)}{(x+2R)^2}, \quad D = 2\sqrt{\frac{R(R+x)}{4R(R+x)+x^2+y^2}}. \tag{102}$$

VII. "RENORMALIZATION" OF THE LONGITUDINAL FORCE

It was established earlier that there is no net self-force for a straight string in field free region. This largely obviates the need for any renormalization. But for a curved string, even if it is subject to zero acceleration, since the ends are not quite parallel, a "small" logarithmically divergent self-force seems likely to survive. Furthermore, the present of divergent terms at string ends, even if they sum to zero in the total force, prevent the internal forces from being calculated reliably. For calculating effects due entirely to acceleration, forces present even with acceleration absent need to be subtracted or otherwise accounted for.

The straight string field calculated in Eq. (35) should be derivable from the formulas just derived as a special case. To do this let us change the variable of integration from α' to $s' = \alpha' R$ using $\cos\alpha' \to \pm 1$ and $R\sin\alpha' \to -r'\cos\theta'$, where θ' is an angle shown in FIG. 5. As a result

straight string

$$\frac{F_s^{(\infty)}(s_t,0,y)}{e} = \lim_{R\to\infty} \frac{\lambda_0}{4\pi\epsilon_0}\left(\frac{1-\beta^2\cos\overline{\alpha'}}{\overline{r'}+\beta R\sin\overline{\alpha'}} - \frac{1-\beta^2\cos\underline{\alpha'}}{\underline{r'}+\beta R\sin\underline{\alpha'}}\right) = \frac{\lambda_0}{4\pi\epsilon_0}\left(\frac{1/\gamma^2}{\overline{r'}(1-\beta\cos\overline{\theta'})} - \frac{1/\gamma^2}{\underline{r'}(1-\beta\cos\underline{\theta'})},\right) \quad (85)$$

This formula agrees with Eq.(35). For $y = 0$ the end points are given by Eq. (14).

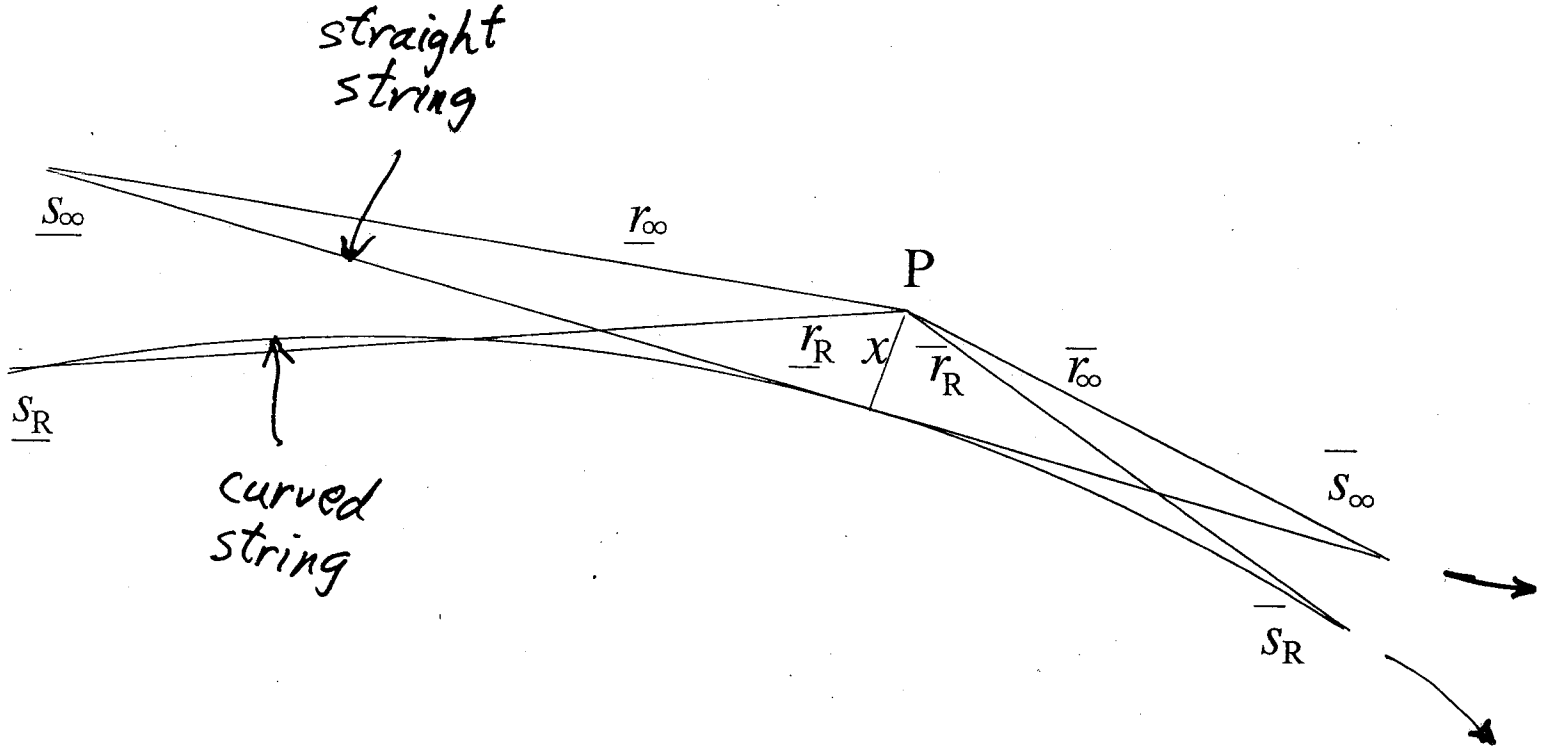
We are now in a position to perform the renormalization process described in the introduction. Symbolizing the curvature-present force by $\mathbf{F}^{(R)}$, the curvature-absent force by $\mathbf{F}^{(\infty)}$, and their difference by $\mathbf{G}$ one has

"unrenormalized" $\mathbf{F}^{(R)} = \mathbf{F}^{(\infty)} + \mathbf{G}$, where $\mathbf{G} = \mathbf{F}^{(R)} - \mathbf{F}^{(\infty)}$. "renormalized" i.e. curvature only (86)

The definitions of these terms is made more explicit by referring to FIG. 9. Force $\mathbf{F}^{(R)}(x,y,s)$ at position P is to be calculated based on the curved charge distribution running from $\underline{s}_R$ to $\overline{s}_R$ and force $\mathbf{F}(\infty)(x,y,s)$ is to be calculated based on the straight charge distribution running from $\underline{s}_\infty$ to $\overline{s}_\infty$. For transverse positions x, y very close to either end of the charge distribution $\mathbf{F}^{(R)}$ and $\mathbf{F}^{(\infty)}$ will exhibit the same logarithmic divergence. As a result the divergence will be absent from $\mathbf{G}$.

- useful as check of formulas
- not needed for string simulation (That's the point of strings)
- needed for point charge simulation

Pictorial Representation of Renormalization



VIII. COHERENT SYNCHROTRON RADIATION

The power radiated by N electrons in the form of synchrotron radiation has been calculated by Schwinger[12], both in the form of coherent and incoherent radiation:

$$P_{\text{coh.}}^{(N)} = \beta N^2 \frac{ce^2}{4\pi\epsilon_0 R^2}\left(\frac{\sqrt{3}R}{2L}\right)^{4/3},$$
$$P_{\text{incoh.}}^{(N)} = \beta N \frac{2}{3}\frac{ce^2}{4\pi\epsilon_0 R^2}\gamma^4, \qquad (94)$$

These contributions are equal for

$$\gamma_{\text{crit.}} = \left(\frac{3N}{2}\right)^{1/4}\left(\frac{\sqrt{3}R}{2L}\right)^{1/3}. \qquad (95)$$

The coherent power radiated should be equal to the rate at which the bunch does work on itself, which is given by

Calculable from formulas given →

$$P = \beta c \int_{-L}^{L} \frac{G_s(s_t,0,0)}{e}\lambda_0 ds_t, \qquad (96)$$

where $\lambda_0 = Ne/(2L)$.

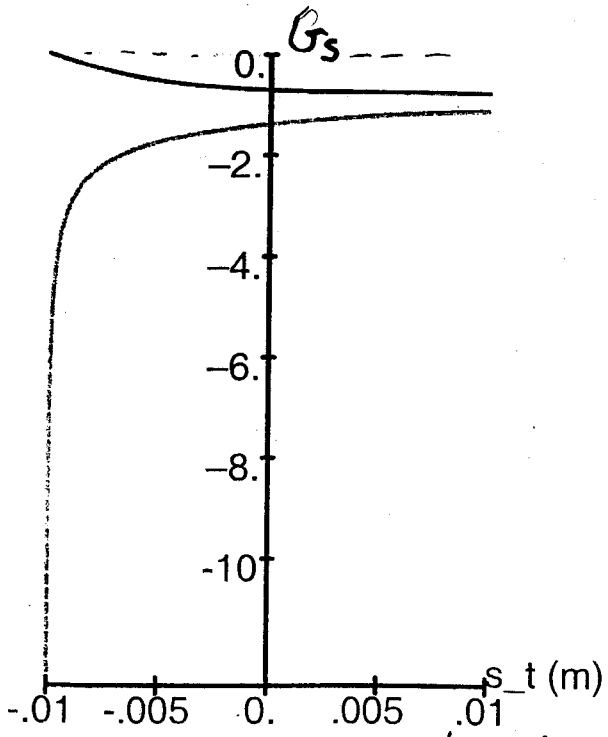
Saldin et al[2] give an approximate form, valid for $\gamma >> 1/u_s$ where u_s is a parameter defined by them that is comparable with our $(L/R)^{1/3}$; expressed in our notation, their longitudinal force is

High γ approx.

$$\frac{G_s^{(\gamma>>1/u_s)}}{e} = \frac{\lambda_0}{4\pi\epsilon_0}\frac{2}{3^{1/3}}\frac{1}{R^{2/3}}\frac{1}{|L+s_t|^{1/3}}. \qquad (97)$$

renormalization important

gamma=10



tail.

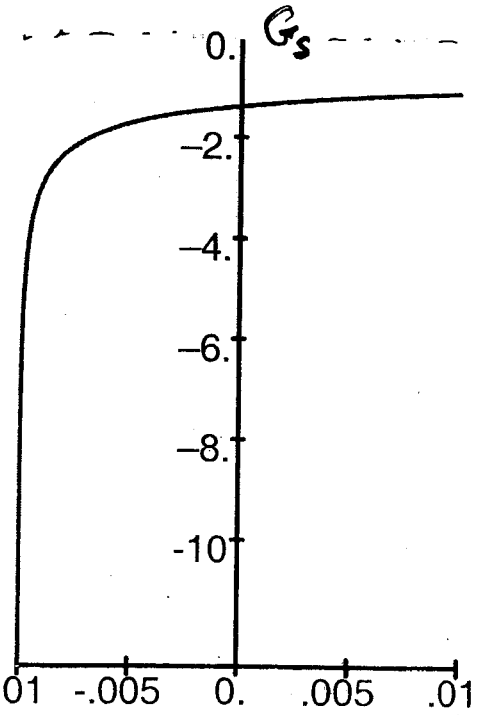
↑

strong deceleration

head

renormalization unimportant

gamma=1000



(86)

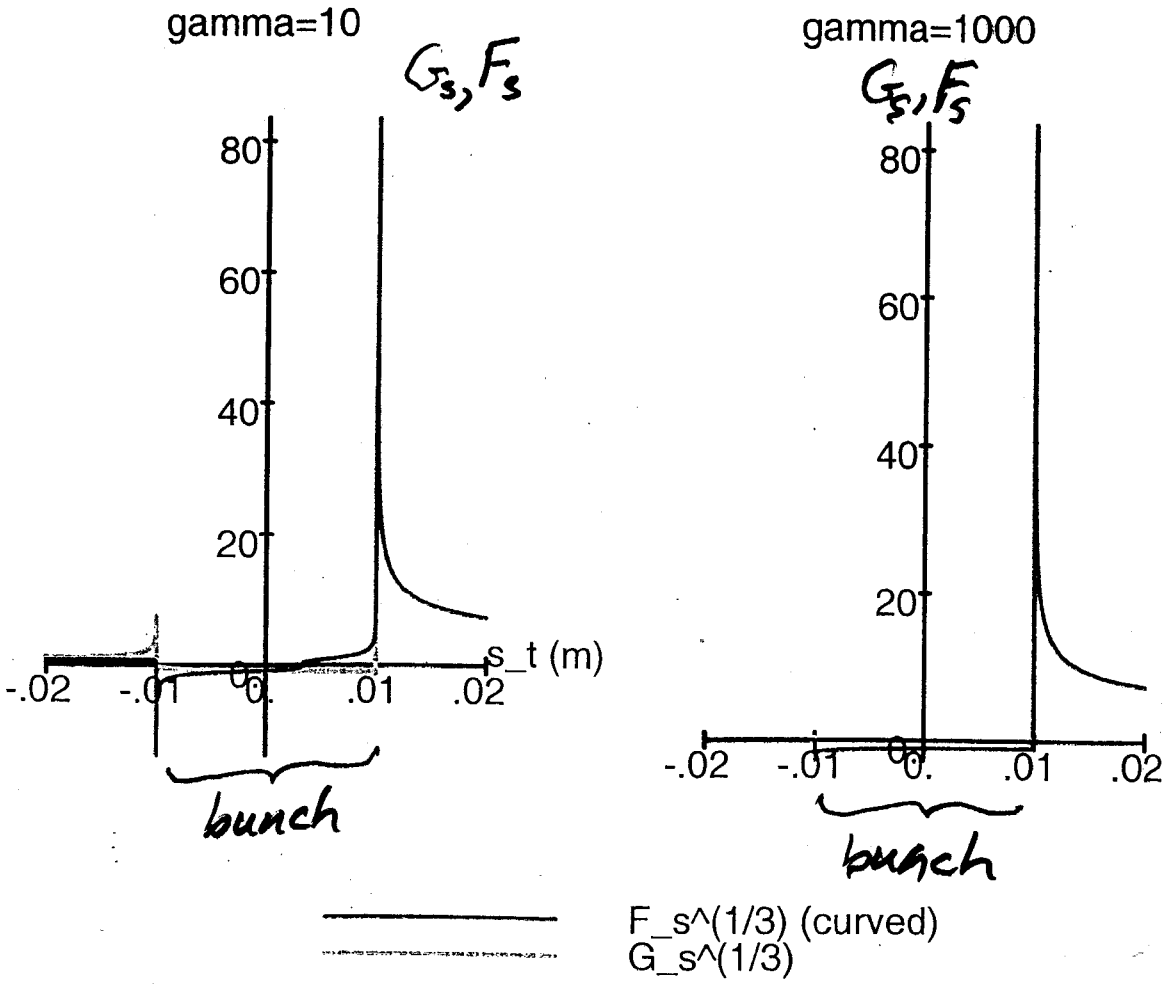
G_s (Eq.(~~88~~),exact)

G_s (Eq.(97),approx)

zero

- CSR is relatively less important at small γ than approx. formula suggest

Extend longitudinal range in front and behind



- strong acceleration in front of string.
- does not contribute to CSR.

Check against CSR

TABLE I: Coherent power, as calculated by self-force, P, and as by Schwinger from far fields, $P^{(N)}_{\text{coh.}}$. $L = 0.01\,\text{m}$, $R = 10\,\text{m}$, $\gamma = 1000$, $N = 10^{10}$.

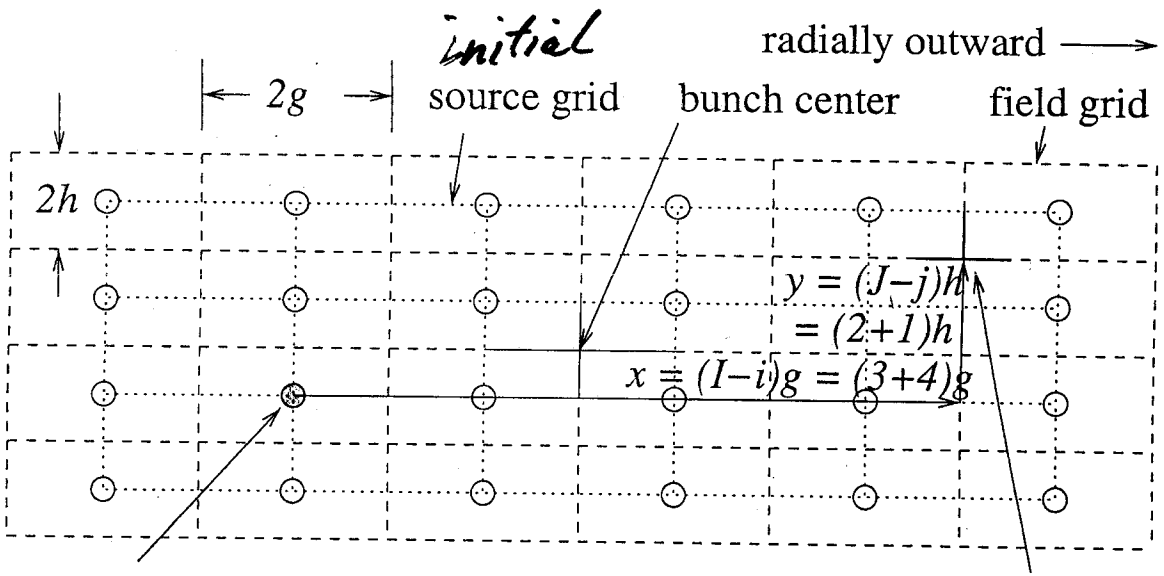
L m	R m	γ	P W		$P^{(N)}_{\text{coh.}}$ W	$\gamma_{\text{crit.}}$
0.01	10	10	207	✗	568	3336
		100	555		571	3336
		1000	563	⟷	571	3336
		10000	563	⟷	571	3336
0.001	10	1000	12179	⟷	12298	7187
0.01	100	1000	121.8	⟷	123.0	7187

• confirmation

There may also be emittance growth caused by space charge forces even in field free regions. That growth will have to be estimated using **F**, in which case the divergence has to be handled differently. The intention of treating point particles as strings is to obviate the need for renormalization. By starting all particles in a simulation with non-zero interparticle separation the divergence can only occur after some bunch evolution has occurred, and then the probability of "exact" spatial coincidence later on is small, especially for high precision computation. The occasional close encounter will be insignificant since the singularity is only logarithmic. Of course these comments have to be made quantitative by numerical investigation.

calculation

- Suggest gridding to avoid "early" coincidence in effective field approach



$i = -2M+1,\ldots,-1,1,\ldots,2M-1$
$j = -2N+1,\ldots,-1,1,\ldots,2N-1$

$I = -2M,\ldots,-2,0,2,\ldots,2M$
$J = 0,2,\ldots,2N$