Quadrupole-mode Maeasurements and their applications

- **The quadrupole-mode transfer function (QTF) is a powerful non-destructive tool to measure properties of dynamical systems. The QTF can be used to measure the betatron tunes and the rms beam emittances with a beam position monitor (BPM) systems. The QTF can also be used to compensate the optical mismatch during the beam injection process.**

- Introduction
- Quadrupole-mode Transfer Function, what is it?
- Applications
- Emittance and Tune Measurements
  (1) A numerical example with multi-particle-simulation.
  (2) Experimental test in longitudinal direction.
- Mismatch compensation
- Overcoming Intrinsic Spin Resonances
- Conclusion
- Ref: W. Guo and S.Y. Lee, Phys. Rev. E. 65, 066505 (2002); W. Guo, Ph.D. Thesis, Indiana University (2003).

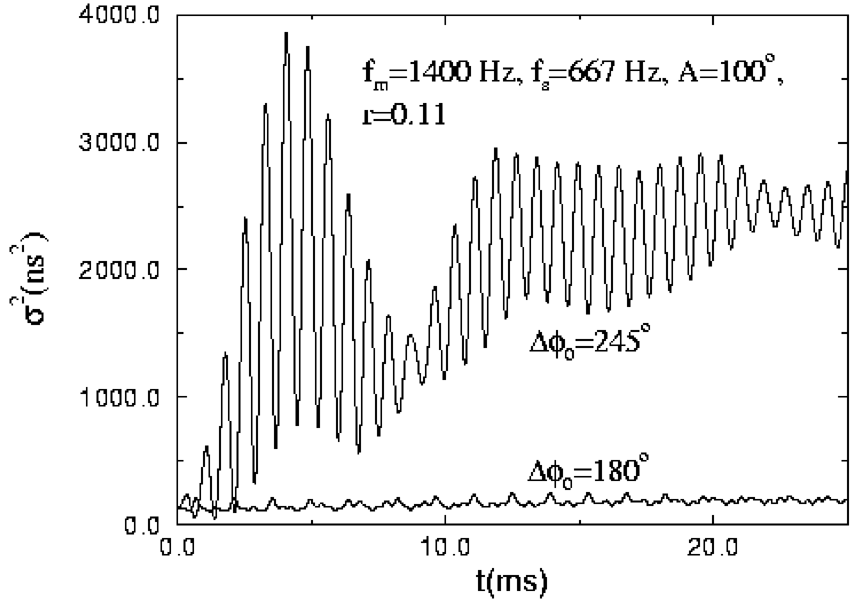
- **Applications of parametric resonances of dynamical systems in beams**

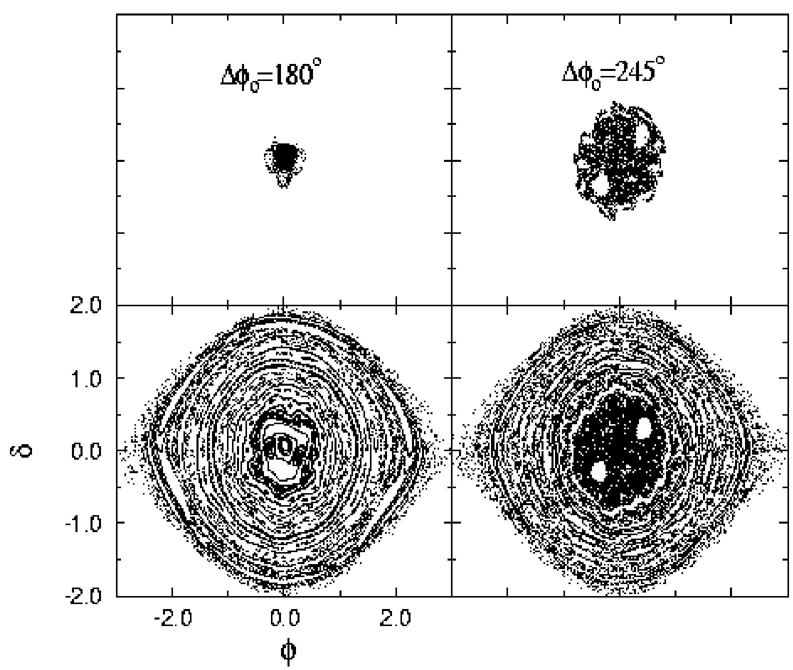
- **Characterizing chaos and properties of many dynamical systems: QI-dynamical system, double rf system, space charge dominated beams, etc.**
- **Parametric resonances in the longitudinal phase-space induced by rf cavity voltage and phase modulations can be employed to actively compensate the synchro-betatron coupling resonances, to create a bounded chaotic region in the longitudinal phase-space for a controlled bunch dilution.**
- **Voltage modulation at the second synchrotron sideband has also been applied to alleviate the coupled bunch instability driven by the cavity parasitic modes, and to manipulate bunch shape for bunch length compression.**
- **The idea of bunch manipulation has recently been extended to the transverse phase-space, where the coherent dipole mode excitation driven by a transverse rf dipole field has been successfully applied to overcome intrinsic spin resonances at the AGS. The rf dipole, excited adiabatically, induces coherent betatron oscillations, which can be used to measure the betatron tune without suffering emittance dilution.**
- **With advanced data analysis techniques, the dipole-mode transfer function can be used to reveal hidden dynamical variables in the Orbit Response Matrix (ORM) and the Model Independent Analysis (MIA) methods. Both techniques have been successfully implemented in improving the performance of high intensity accelerators.**

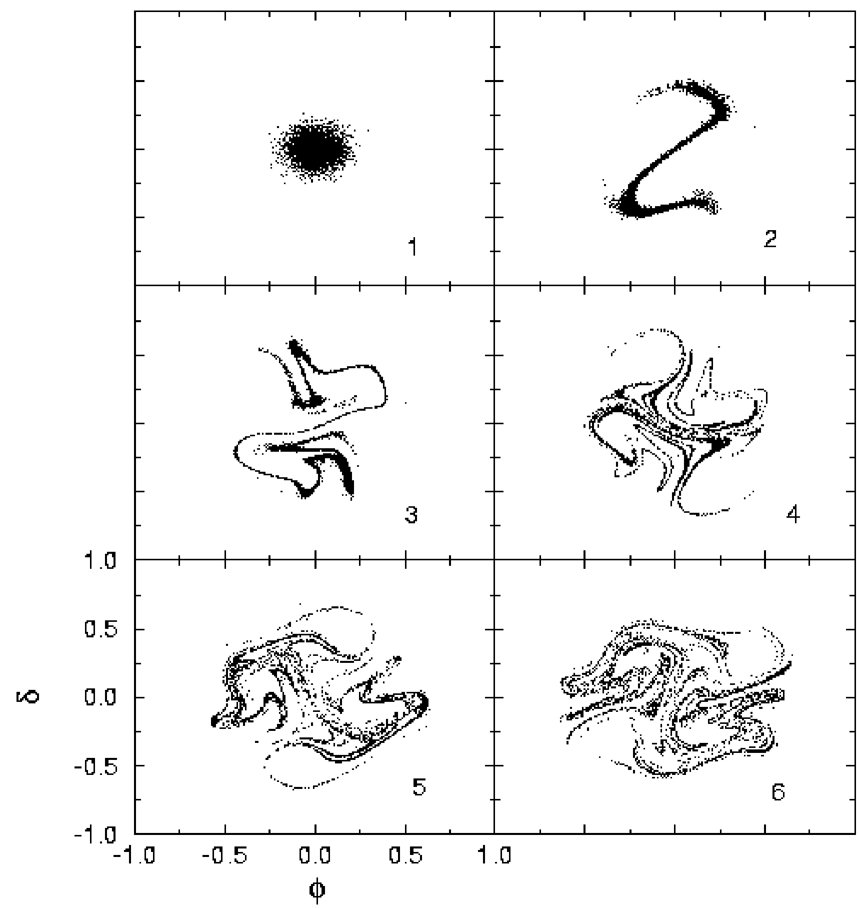
- **How about the quadrupole-mode transfer function?**

- **A fast field-changing quadrupole had been used to produce betatron tune jump for overcoming intrinsic spin resonances and for studying a strong betatron resonance.**
- **A tune modulation at a island tune can be used to study the effect of enhanced-diffusion process at a betatron resonance.**
- **Other Applications?**

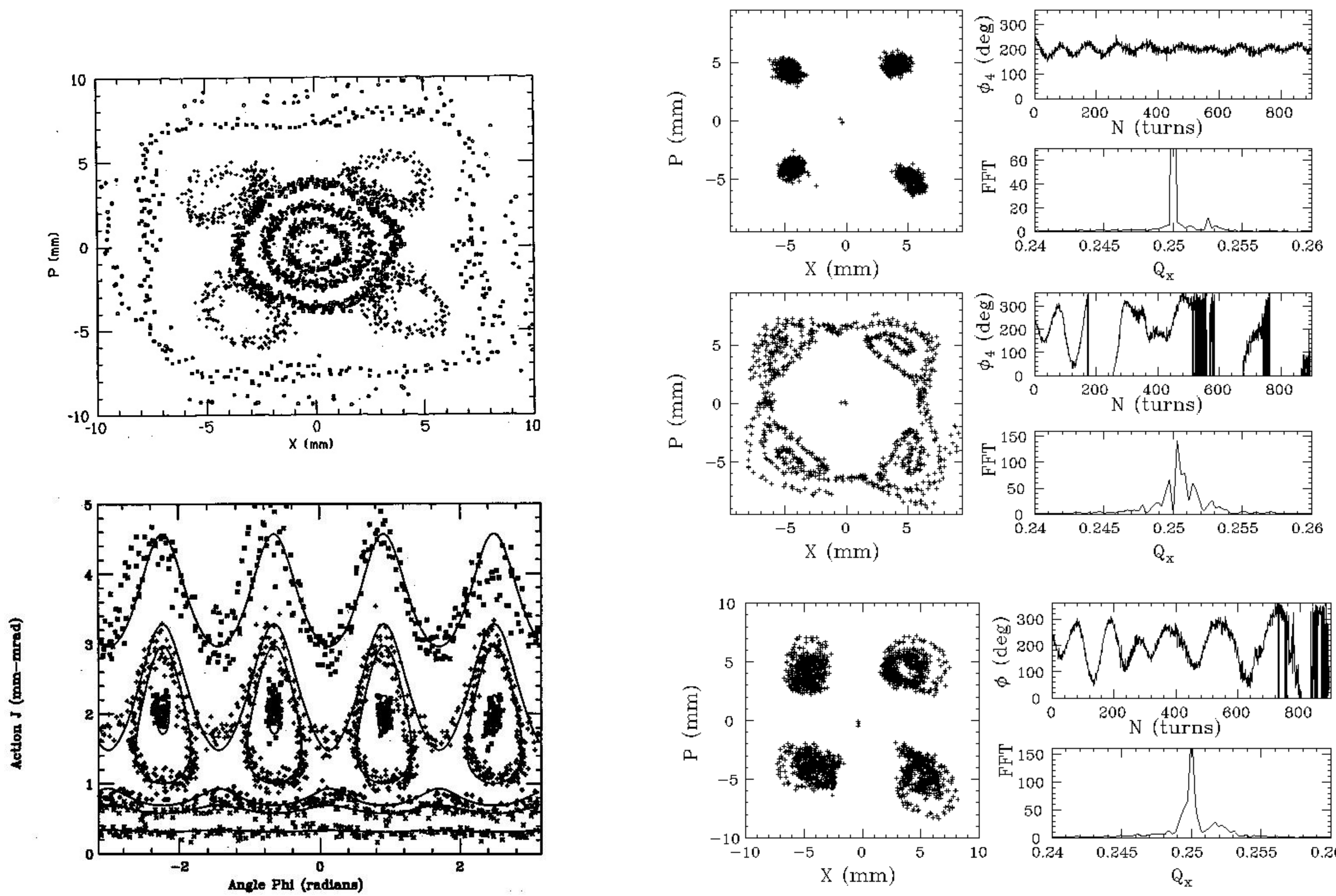
Phys. Rev. Lett. **80**, 2314 (1998)
Phys Rev. E **60**, 6051 (1999)



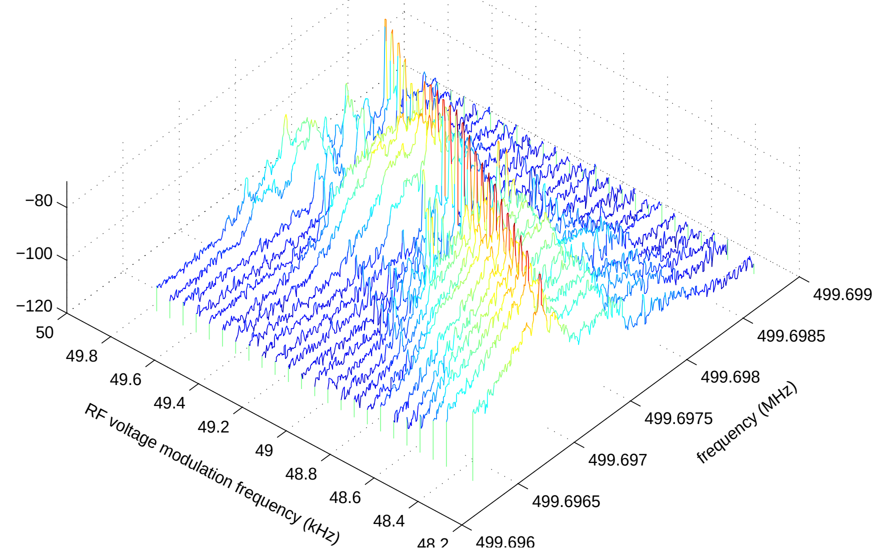


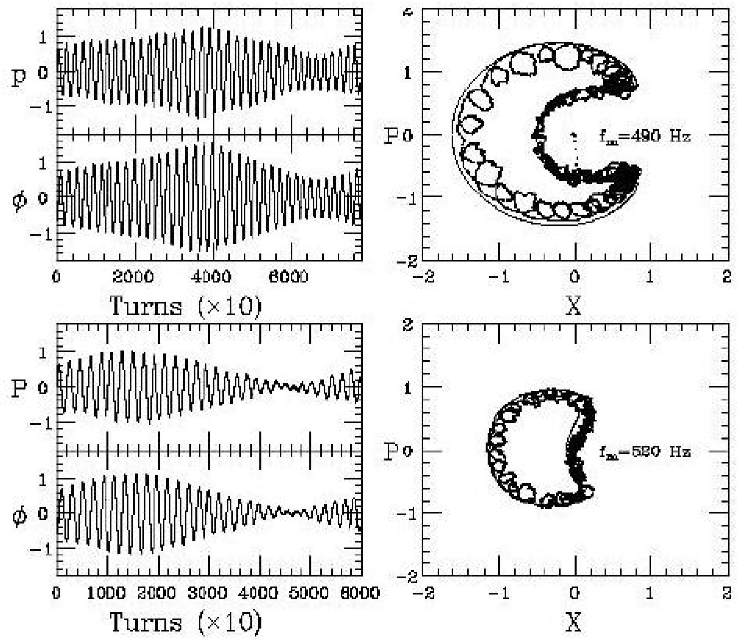


With electron cooled pencil beams, the beam can be used to probe the phase space distortion due to nonlinear magnetic fields. These experiments were carried out at the IUCF Cooler Ring, in collaboration with scientists at the SSC laboratory.

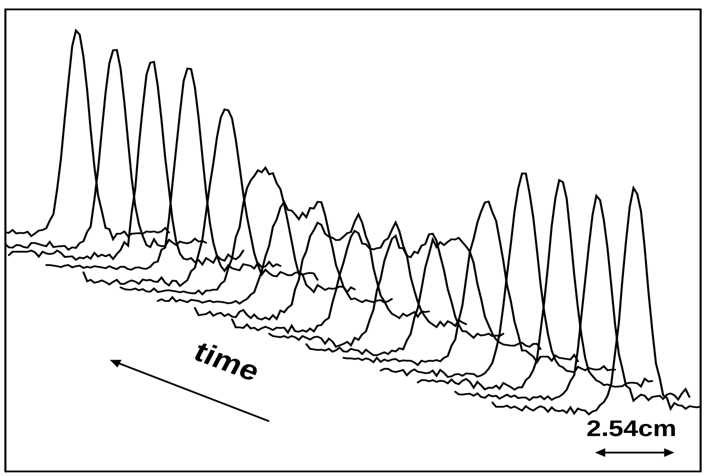


M.H. Wang et al (TLS)

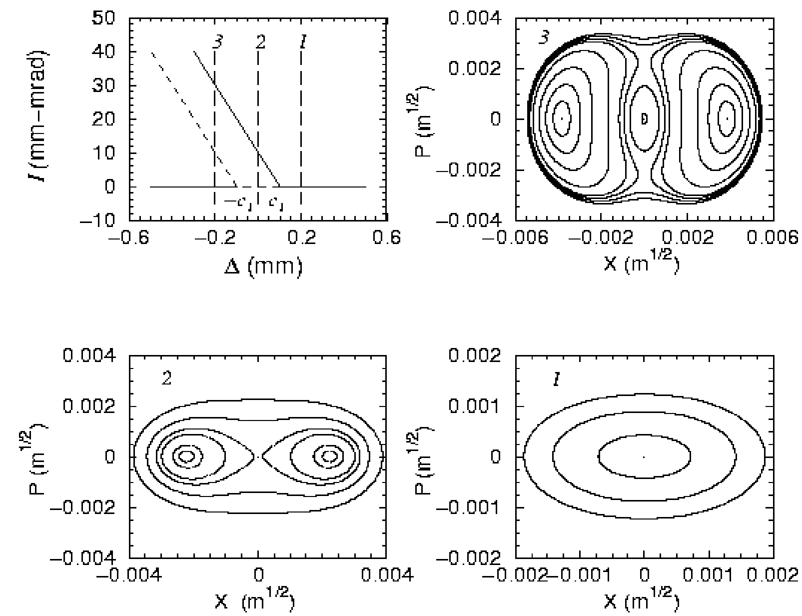




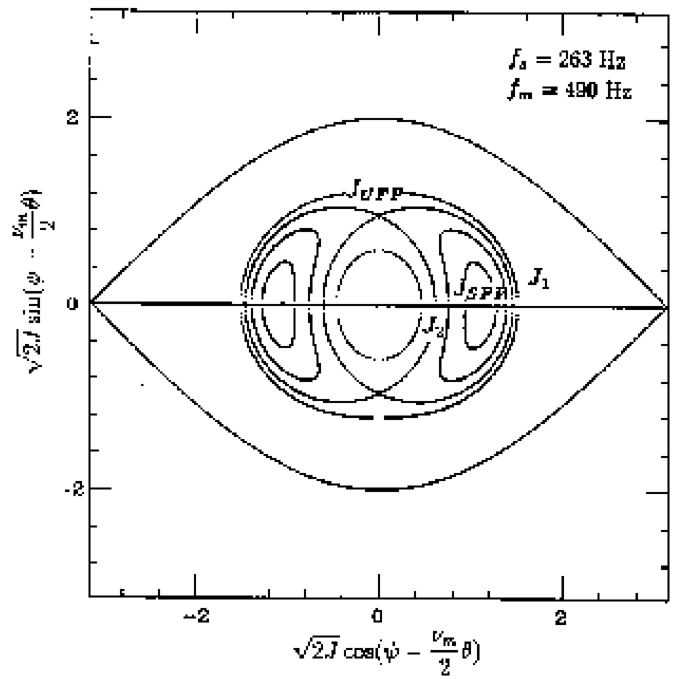
H. Huang et al CE22 (IUCF)

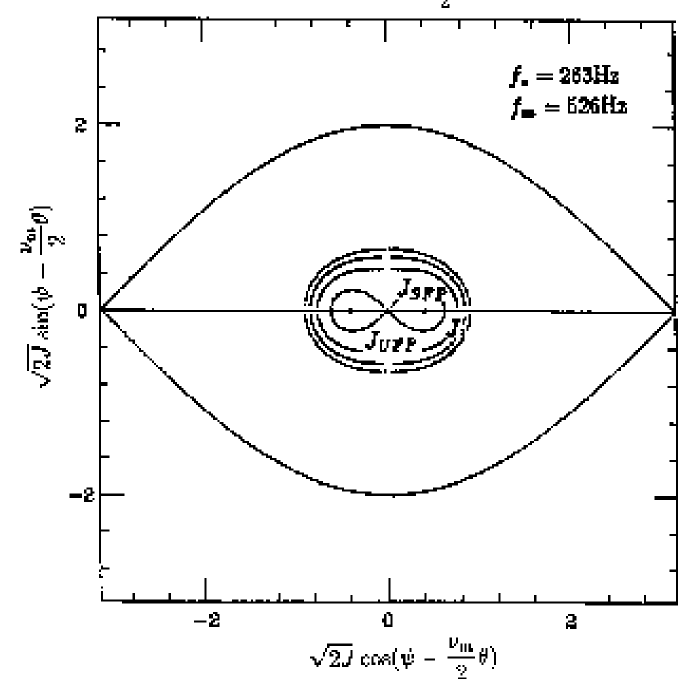


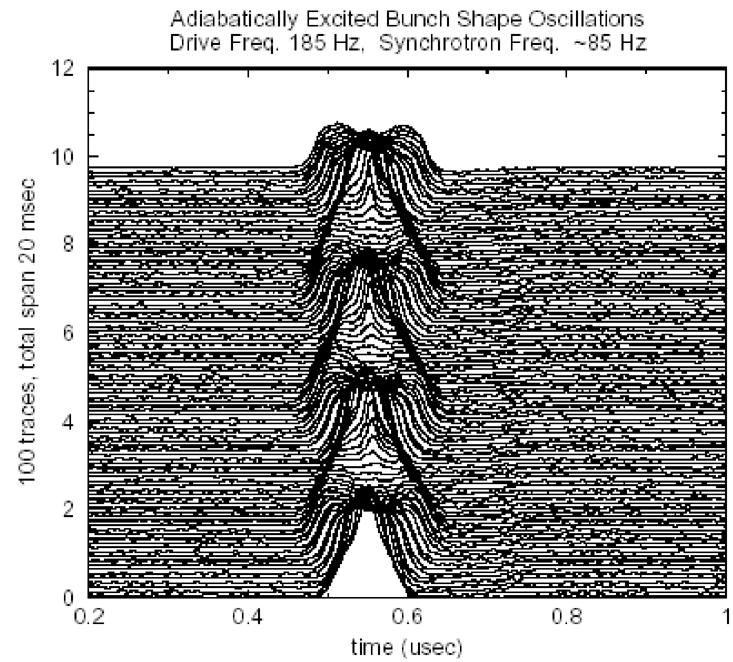
M. Bai et al (AGS)

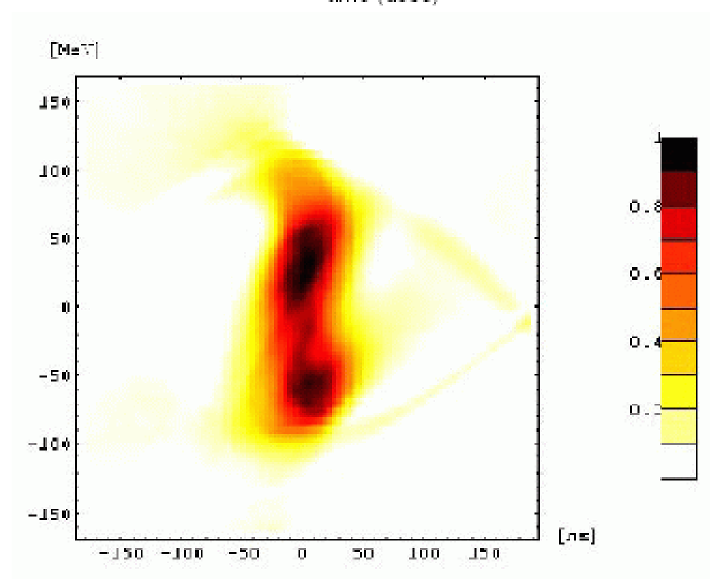


D. Li, et al, Guo et al (IUCF)









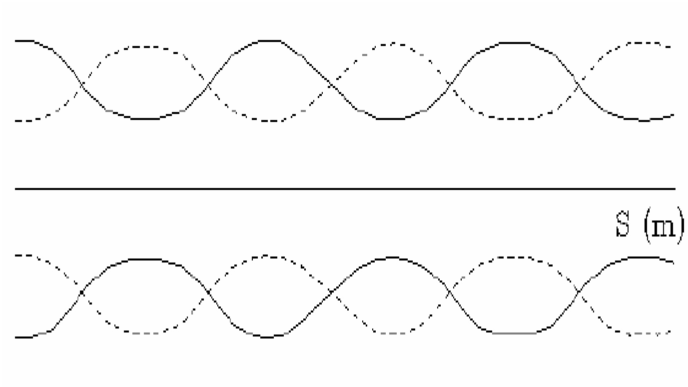
Bai et al, PAC99

Possible applications of quadrupole mode transfer function

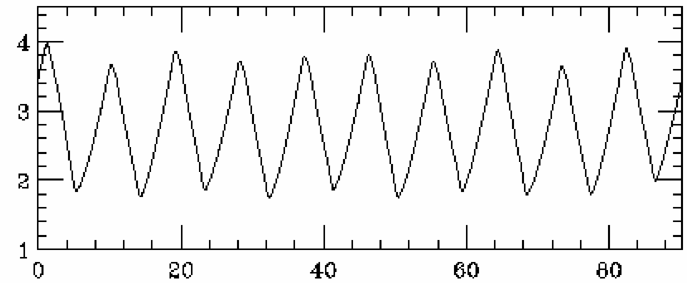
- Stopband compensation for space charge dominated beams: ORM for accelerator modeling is necessary! (Note that Kip Gardner has done a lot of experiments at the AGS and AGS Booster)
- Nondestructive measurement of rms emittance!
- Betatron tune measurement.
- Compensation for injection optical mismatch.
- Polarization preservation? Our result shows that the rf (AC) dipole provides better solution.

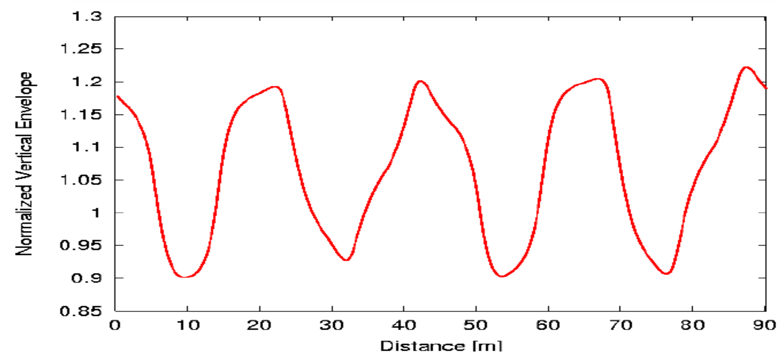
The envelope oscillations and half integer stopband compensation:

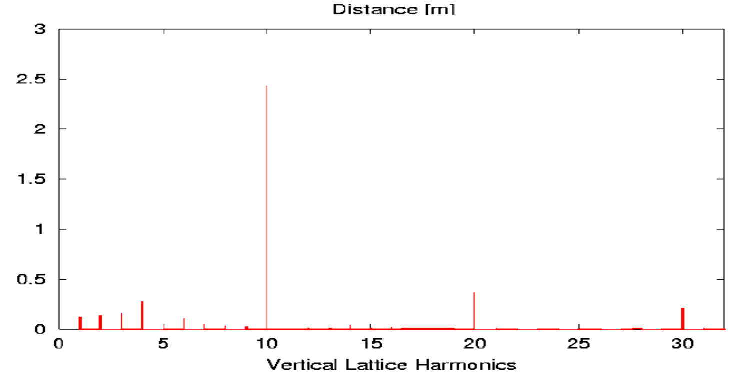
The envelope of particle beams in accelerators is determined by the (square root) betatron amplitude function and the beam emittance as shown schematically in the following graphs:



For space charge dominated beams, the envelope oscillations has another complication. The graph at right shows the additional envelope oscillations for the PSR, obtained from a PIC space charge simulation code [S. Cousineau, Ph.D. thesis, Indiana University, 2003].







Half-integer envelope resonance is particularly important to space charge dominated beams

- **If we compare the envelope oscillation of the regular betatron amplitude with that of a uniform focusing accelerator, the beam envelope modulation can drive particle-envelope interaction due to the space charge force. However, the oscillation frequency of the envelope differs substantially from twice the betatron frequency of particles. The effect is not very important!**
- **Now, we consider a "not so perfect" lattice with space charge. As the envelope tune is pushed downward near an integer, where the lattice perturbation exists. The envelope will be forced to oscillate at the forced oscillation frequency. This is exactly what we see from the PIC code calculation at n=4 for the PSR lattice in the vertical plane.**
- **This large envelope oscillation at n=4 will induce emittance growth by driving particle resonantly outward. See e.g. S. Cousineau, Ph.D. thesis, Indiana University (1993).**

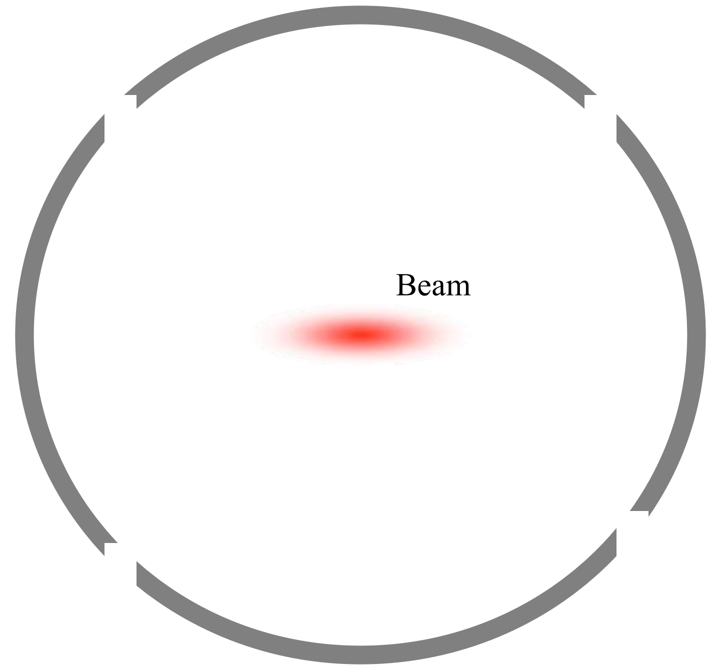
$$H = \frac{1}{4\pi} p^2 + \frac{1}{4\pi} k_y(\theta) y^2 - \frac{K}{4\pi R^2} y^2 \Theta(R - |y|) - \frac{K}{4\pi}\left(1 + 2\ln\frac{y}{R}\right)\Theta(|y| - R)$$

$$K = \frac{2 N r_0 L}{\beta^2 \gamma^3 \varepsilon}$$

A Quadrupole Pick-up:

$$\sigma(r,\phi,a,\Theta) = \frac{\lambda}{2\pi a}\,\frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(\Theta-\phi)}$$

$$= \frac{\lambda}{2\pi a}\left[1 + 2\sum_{i=1}^{\infty}\left(\frac{r}{a}\right)^k \cos k(\Theta-\phi)\right],$$



BPM

$$\sigma(a,\Theta) = \int \sigma(x,z,a,\Theta)\rho(x,x',z,z')\,dx\,dz\,dx'\,dz'$$

$$= \frac{\lambda}{2\pi a}\left\{1 + 2\frac{\langle x\rangle}{a}\cos\Theta + 2\frac{\langle z\rangle}{a}\sin\Theta\right.$$

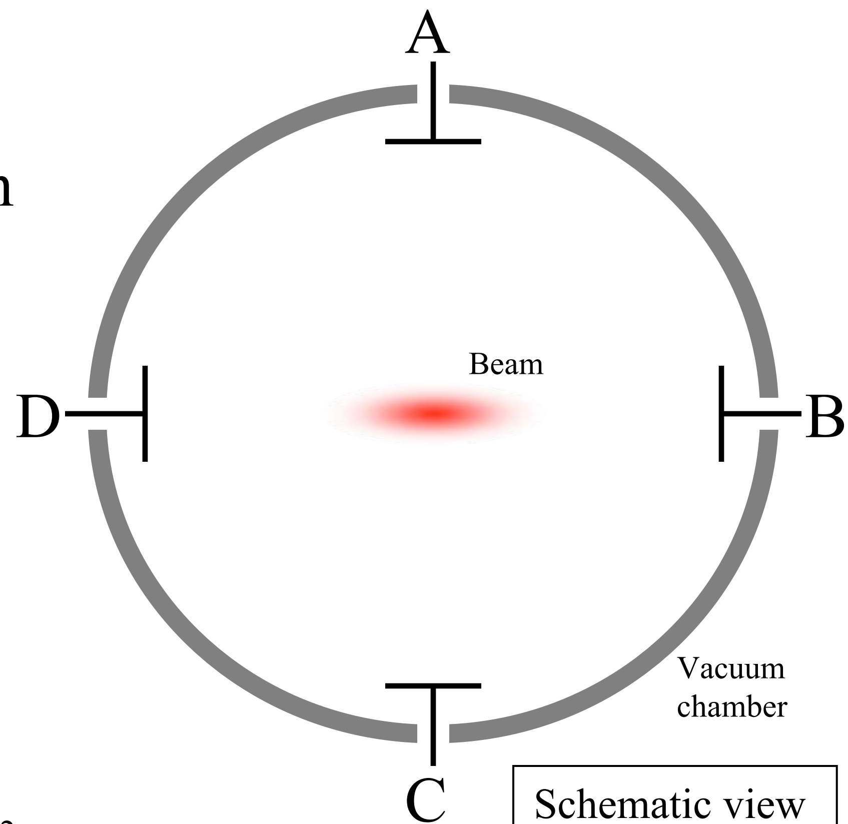
$$\left. + 2\left(\frac{\langle x^2\rangle - \langle z^2\rangle}{a^2}\right)\cos 2\Theta + 4\frac{\langle xz\rangle}{a^2}\sin 2\Theta + \cdots\right\}.$$

$$\frac{(\mathrm{B}+\mathrm{D}) - (\mathrm{A}+\mathrm{C})}{\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}} \propto \left\langle x^2\right\rangle - \left\langle z^2\right\rangle + \bar{x}^2 - \bar{z}^2$$

[See *Andreas JANSSON,* Ph.D. Thesis, (Stockholm, 2001); *Nucl. Instrum. Methods Phys. Res., A 479 (2002) 233*]

A Quadrupole Pick-up:

- A non-invasive instrument sensitive to beam size.
- A standard position pick-up can be wired up to be used as quadrupole pick-up.
- The signal originates from the quadratic term in the position response of the pick-up, which is usually very small.



Schematic view of pick-up seen along beam path

$$\frac{(B+D)-(A+C)}{A+B+C+D} \propto \sigma_x^2 - \sigma_z^2 + \bar{x}^2 - \bar{z}^2$$

σ = r.m.s. beam width

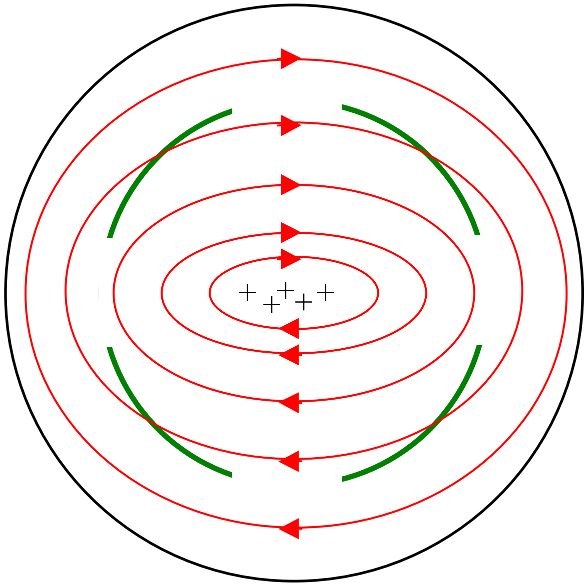
[See *Andreas JANSSON*, Ph.D. Thesis, (Stockholm, 2001); *Nucl. Instrum. Methods Phys. Res., A 479 (2002) 233*]

QUADRUPOLE PICK-UPS

- A quadrupole pick-up can be made out of an electrostatic position pick-up by adding and subtracting the four output signals in such a way that the sum signal and the vertical and horizontal position signal is suppressed. The remainder is the quadrupole signal. Extracting the quadrupole signal can be very hard since it is some orders of magnitude smaller than the dominating sum and difference (common-mode) signals.
- A recent working example to enhance the quadrupole mode pick up was developed for the CERN PS. The otherwise dominating common-mode signal is removed by coupling to the radial component of the magnetic field created by the beam. Since the B-field is divergence free, the sum of the outgoing radial flux equals incoming radial flux, and thus there can be no common-mode signal. This pick-up can be used even if the beam-size-to-aperture-ratio is very small. [See Andreas JANSSON, Ph.D. Thesis, (Stockholm, 2001); *Nucl. Instrum. Methods Phys. Res.,* A **479** (2002) 233]

PS Pick-up Design

- Unlike (most) previous quadrupole pick-ups, this one is purpose-built
- Measures magnetic flux in radial direction.
  - If beam round and centered, there is no radial component of the flux, i.e. no signal.
  - Only field deviations due to non-zero position or quadrupole moment are measured.
- Reciprocal of air-coil quadrupole magnet



Pick-up seen along beam path

See [Andreas JANSSON, Ph.D. Thesis, (Stockholm, 2001); *Nucl. Instrum. Methods Phys. Res.*, A **479** (2002) 233]

Quadrupole-mode transfer function measurements

$$q_2 = \frac{L+R-U-D}{\Sigma} = \frac{2}{a^2}\left(\langle x^2\rangle + \langle x\rangle^2 - \langle z^2\rangle - \langle z\rangle^2\right)$$

- Profile monitors – measure the deviation from the profile in a regular lattice for the space charge dominated beams.
- One needs also BPM to model betatron motion using dipole-mode transfer function methods. (Model Independent Analysis)
- What can one learn from the quadrupole mode measurements using a quadrupole pickup?
  (1) The deviation from the regular betatron oscillation gives the perturbation strength. (2) Detailed data analysis can be used to confirm accelerator modeling. (3) It provides information on machine and beam parameters. (4) It provides active compensation of optical mismatch, etc.
- The precision of measurements can be improved by making transfer function measurement!

Transfer Function

- The transfer function is defined as the ratio of response to a given known excitation. This transfer function can be measured from a network analyzer, or spectrum analyzer with input signal calibration.
- Transfer function method has been widely applied in measurements of longitudinal impedance, beam cooling properties, "model independent analysis", etc.
- However, the quadrupole-mode transfer function (qtf) has not been considered. To measure qtf, an rf quadrupole is used as a kicker, and the response can be measured by quadrupole pick-up. With a single harmonic excitation, the quadrupole mode function is

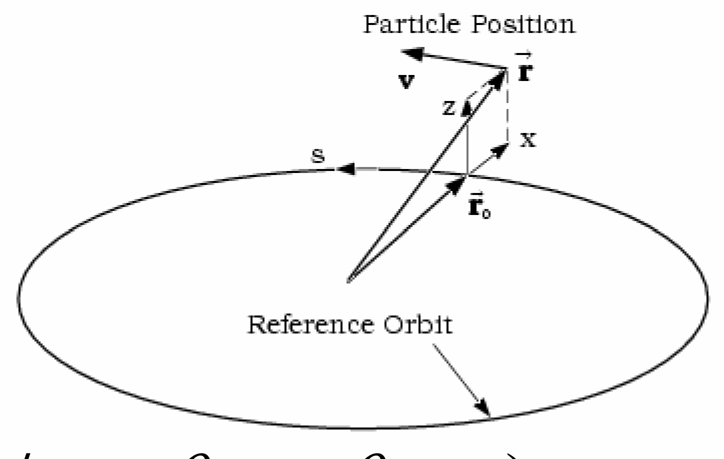
$$\langle x^2 \rangle = \tfrac{1}{2}(\langle X^2 \rangle + \langle P^2 \rangle) + \tfrac{1}{2}(\langle X^2 \rangle - \langle P^2 \rangle)\sin(\nu_m \theta)$$

$$q_2 = \tfrac{1}{a^2}(b_0 + b_1 \sin(\nu_m \theta))$$

The Hamiltonian of particle motion in accelerators is governed by

$$H = \tfrac{1}{2} y'^2 + \tfrac{1}{2} K_y(s) y^2 + \tfrac{1}{2} K_{rf}(s) y^2 \cos(\omega_m t + \vartheta_0)$$

Transforming into the action angle variables, we obtain the Hamiltonian as



$$H(J_y, \psi_y) = \nu_y J_y + C_1 J_y \cos(2\phi_y - n\theta - \nu_m \theta + \chi)$$

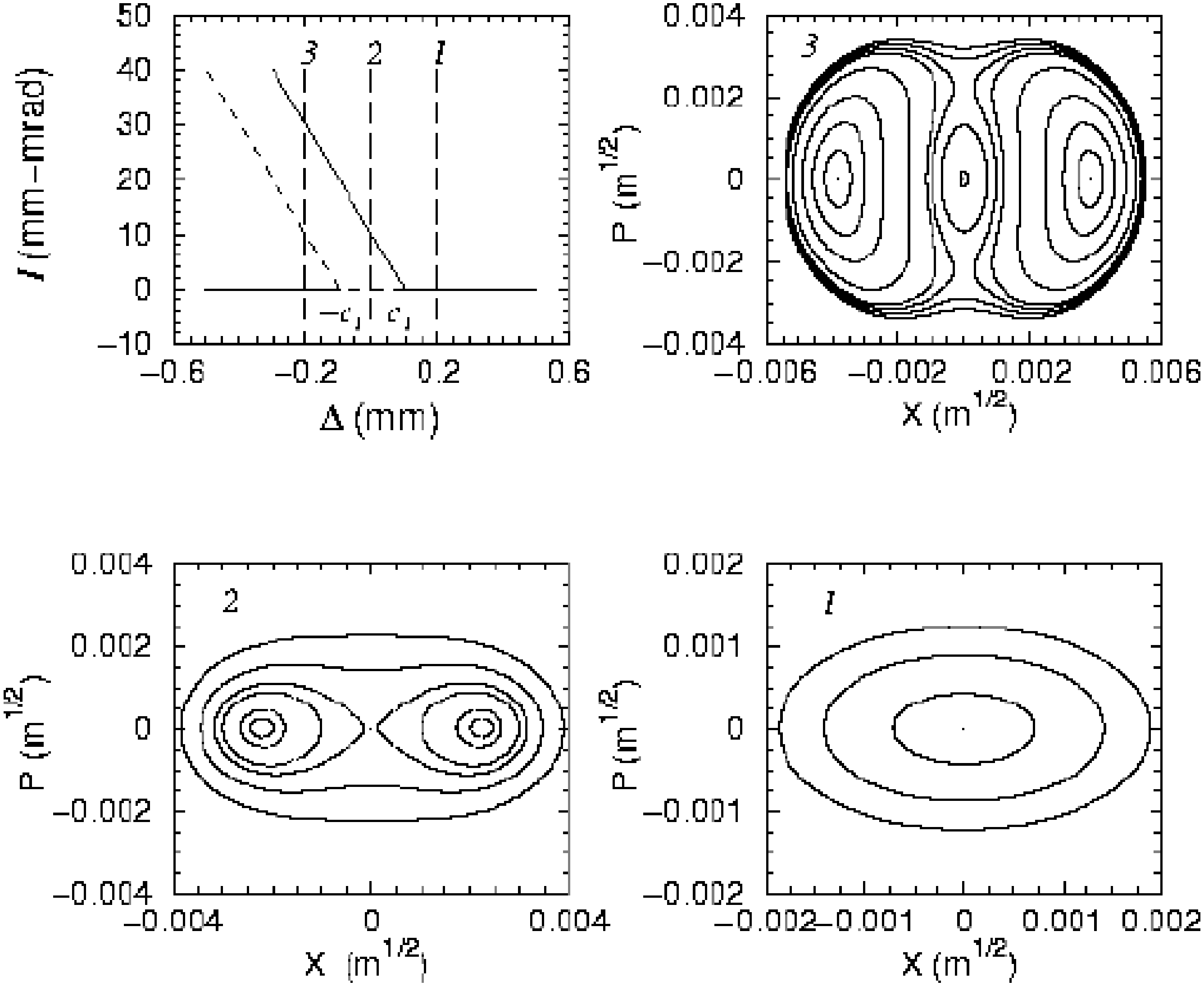
$$I = J_y, \psi = \phi_y - \tfrac{1}{2} n\theta - \tfrac{1}{2} \nu_m \theta + \tfrac{1}{2}\chi$$

$$H = \delta I + C_1 I \cos(2\psi) + [\tfrac{1}{2}\alpha I^2]$$

$$\delta = |\nu_y - \tfrac{1}{2} n| - \tfrac{1}{2}\nu_m$$

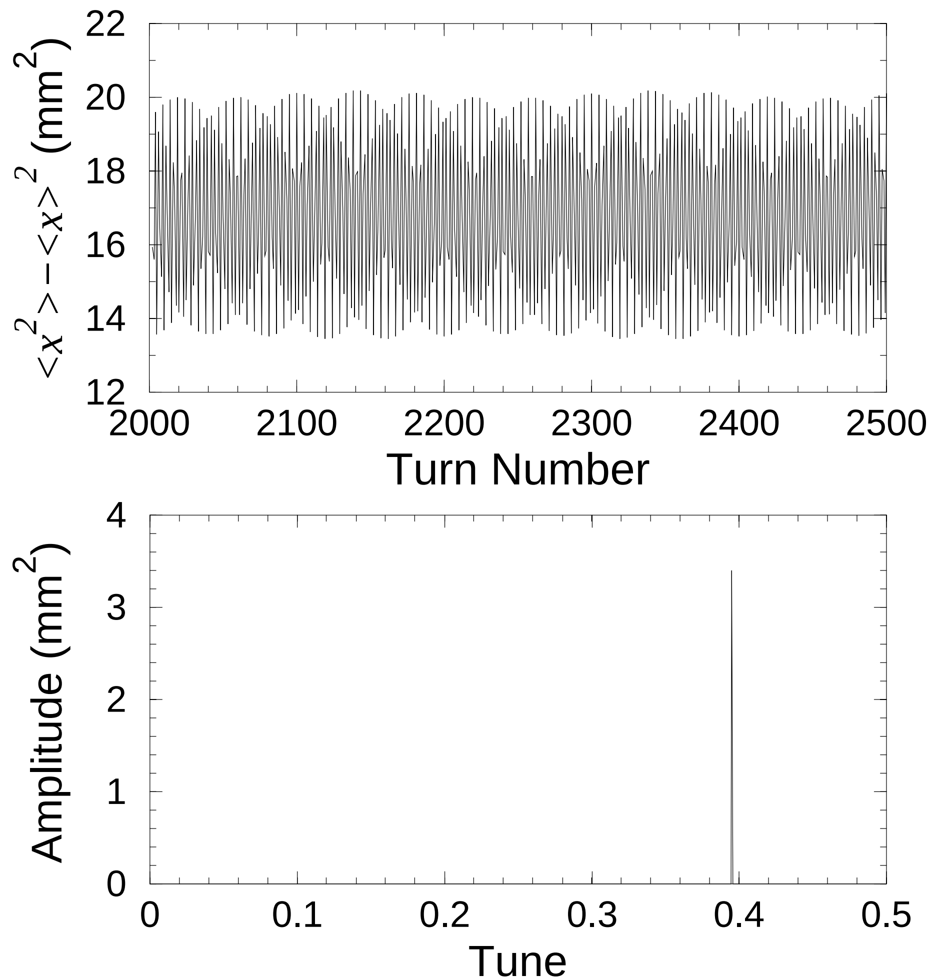
$$C_1 = \left| \oint \frac{\beta(s)}{8\pi} K_{rf}(s) e^{-jns/R \pm j(2\mu(s) - 2\nu s/R \pm \vartheta(s))} ds \right|$$

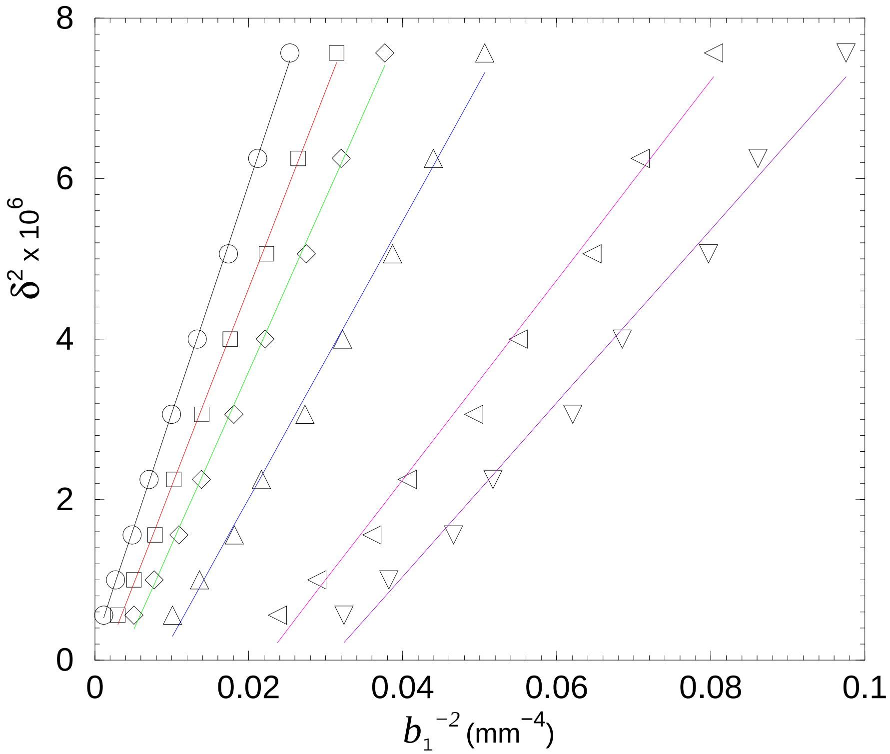
$\Delta = \delta / \alpha$



Numerical Example for rms emittance measurements

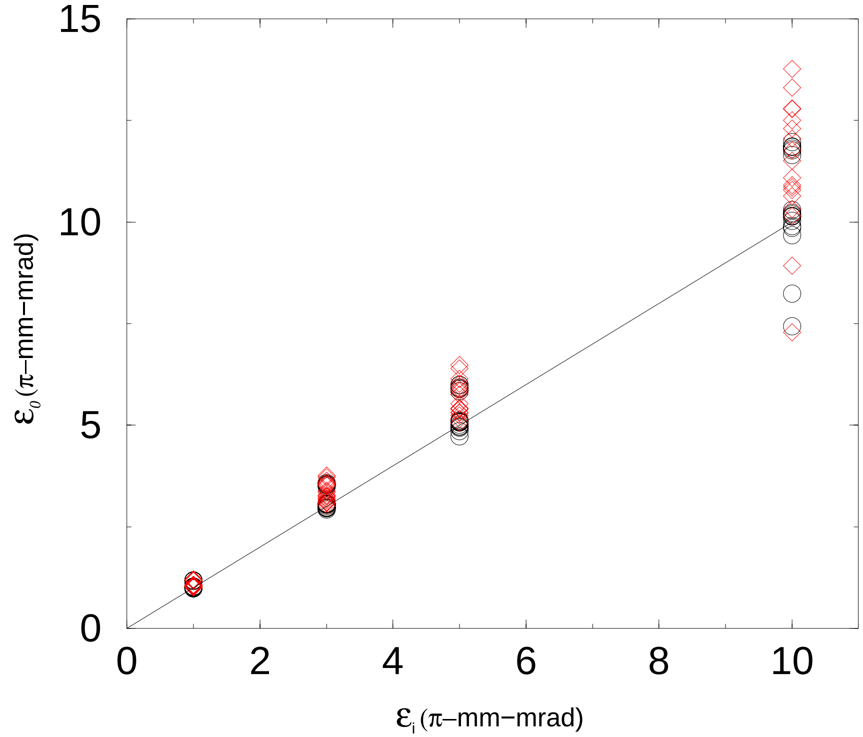
Since there is no hardware in accelerators for the QTF measurements, we carry out numerical multi-particle simulations to test various applications of the QTF. The plot at the right shows the result of multi-particle (10,000 particles) simulation. The FFT of the rms beam size variation gives a sharp power spectrum of the QTF. From the spectrum, we can derive the b_1 parameter as a function of the proximity parameter δ.





$$\delta^2 - C_1^{\,2} = (4C_1^{\,2}\beta_x^{\,2}\varepsilon^2)\frac{1}{b_1^{\,2}},$$

$$(\delta + F\,|\,\alpha_{xx}\,|\,\varepsilon)^2 - C_1^{\,2} = (4C_1^{\,2}\beta_x^{\,2}\varepsilon^2)\frac{1}{b_1^{\,2}}.$$



With an rf quadrupole, the effective Hamiltonian in the *resonance rotating frame* is $H(\psi,J_y)=\delta J_y+J_y C_1\cos 2\psi_y + \frac{1}{2}\alpha_{yy}J_y^2$. If the rf quadrupole is adiabatically turned on, the particle distribution is a function of the Hamiltonian. In particular, the Boltzmann distribution is

$$\rho(J_y,\psi_y)=\mathcal{N}\exp\left\{-\frac{\delta J_y+C_1J_y\cos 2\psi_y+\frac{1}{2}\alpha_{yy}J_y^2}{E_{\mathrm{T}}}\right\}. \tag{C8}$$

The normalization constant $\mathcal{N}$ and the thermal energy E_{T} are determined by the conditions: $\int\rho\,dJ_y d\psi_y=1$, and $\sqrt{\langle Y^2\rangle\langle P_y^2\rangle-\langle YP_y\rangle^2}=\epsilon_0$, where ϵ_0 is the rms emittance, $Y=\sqrt{2J_y}\cos\psi_y$, $P_y=-\sqrt{2J_y}\sin\psi_y$, and $\langle\cdots\rangle$ is the ensemble average over the beam distribution. Note that we are working in the parametric space where the beam bunch is slightly perturbed, i.e., the phase space has not entered the bifurcation region of the Mathieu instability. Thus we can use the rms emittance to characterize a beam property.

Using the normalization condition, we find

$$2\pi\mathcal{N}\sqrt{\frac{E_{\mathrm{T}}}{2\alpha_{yy}}}I_0\left(\xi\frac{\partial}{\partial u}\right)\left(\frac{\mathrm{erfcr}(u)}{u}\right)=1, \tag{C9}$$

where $u=\delta/\sqrt{2E_{\mathrm{T}}\alpha_{yy}}$, $\xi=C_1/\sqrt{2E_{\mathrm{T}}\alpha_{yy}}$, $\mathrm{erfcr}(u)$ is the reduced complementary error function of Eq. (C7), and $I_0(x)$ is the zeroth order modified Bessel function. Using the rms beam-emittance condition, we obtain

$$2\pi\mathcal{N}\frac{E_{\mathrm{T}}}{2\alpha_{yy}}\sqrt{\left[\left(\frac{\partial}{\partial u}+\frac{\partial}{\partial\xi}\right)I_0\left(\xi\frac{\partial}{\partial u}\right)\frac{\mathrm{erfcr}(u)}{u}\right]\left[\left(\frac{\partial}{\partial u}-\frac{\partial}{\partial\xi}\right)I_0\left(\xi\frac{\partial}{\partial u}\right)\frac{\mathrm{erfcr}(u)}{u}\right]}=\epsilon_0.$$

$$\begin{aligned} b_1 &= 2\beta_y(\langle Y^2\rangle-\langle P_y^2\rangle) \\ &= -4\pi\beta_y\mathcal{N}\frac{E_{\mathrm{T}}}{2\alpha_{yy}}\left[\frac{\partial}{\partial\xi}I_0\left(\xi\frac{\partial}{\partial u}\right)\frac{\mathrm{erfcr}(u)}{u}\right], \end{aligned} \tag{C11}$$

where variables u and ξ are defined in the previous paragraphs. Using the asymptotic expansion, we can obtain the coefficient b_1 of the quadrupole beam-transfer function.

First Experimental Test of Emittance Measurement Using the Quadrupole-Mode Transfer Function

M. H. Wang, Y. Sato, and S.Y. Lee, PAC2003

We carried out the first experimental test of the emittance (thermal energy) measurement using the quadrupole-mode transfer function. We show that this method can be applied to measure the intrinsic thermal energy dynamical systems nondestructively. Since the QTF does not depend on the beam distribution, this method can be used to measure the rms emittance of different dynamical system.

Excitation of the Longitudinal Quadrupole Mode

When the rf cavity voltage with angular frequency $h\omega_0$ is applied to charged particles in the accelerator, where h is the harmonic number and ω_0 is the angular revolution frequency of a synchronous particle, beam particles will execute stable synchrotron motion around the synchronous particle. Let ϕ and $\delta = \Delta p/p_0$ be the rf phase and the fractional off-momentum coordinates of a non-synchronous particle, and ϕ_s be the rf phase angle of a synchronous particle. The equations of motion are

$$\dot{\phi} = h\omega_0\eta\delta\,,$$

$$\dot{\delta} = \frac{\omega_0}{2\pi\beta^2 E} eV(\sin\phi - \sin\phi_s)\,,$$

where the overdot is the derivative with respect to time t, η is the phase slip factor, V is the rf cavity voltage, β is the Lorentz velocity factor, and E is the beam energy. The small amplitude synchrotron tune is $Q_s = \sqrt{h|\eta\cos\phi_s|eV/(2\pi\beta^2 E_0)}$.

When a sinusoidal wave with angular frequency ω_m and amplitude ratio b applied to the rf cavity voltage.The rf cavity voltage becomes

$$V = V_0(1 + b\sin\omega_m t)\,.$$

When the applied modulation frequency is near twice the synchrotron frequency, the synchrotron motion is coherently excited. In terms of the conjugate action-angle variables (J, φ), defined by

$$\varphi = \phi - \phi_s = -\sqrt{2J}\left|\cos\phi_s\right|^{-1/4}\sin\psi,$$

$$P = -\frac{h|\eta|}{\nu_s}\left(\frac{\Delta p}{p_0}\right) = \sqrt{2J}\left|\cos\phi_s\right|^{1/4}\cos\psi\,.$$

The effective Hamiltonian of excited quadrupole mode in the resonance rotating frame is

$$H = \Delta J - \frac{1}{2}\alpha J^2 + GJ\cos(2\psi),$$

Here the resonance proximity parameter Δ, nonlinear detuning parameter α, and the resonance strength G are

$$\Delta = (\nu_s\left|\cos\phi_s\right|^{1/2} - \frac{\nu_m}{2}),$$

$$\alpha = \frac{\nu_s}{8}(1 + \frac{5}{3}\tan^2\phi_s),$$

$$G = \frac{1}{4}b\nu_s\left|\cos\phi_s\right|^{1/2}(1 + \frac{1}{3}\tan^2\phi_s).$$

Beam Distribution

The equilibrium distribution of this rf modulation system obeys the Fokker-Planck equation is given

$$\rho(J,\psi) = Ne^{H/E_T} ,$$

where *H* is the Hamiltonian, *N* is the normalization factor and E_T is the thermal energy of the beam. The normalization constant *N* and the thermal energy E_T are determined by

$$\int \rho(J,\psi)\,dJd\psi = 1,$$

$$\varepsilon_{//} = \sqrt{\langle X^2\rangle\langle P^2\rangle - \langle XP\rangle^2}.$$

Here, $\varepsilon_{||}$ is the longitudinal rms emittance, <…> is the ensemble average over the beam distribution. The theoretical longitudinal beam emittance is the equilibrium of the quantum fluctuation and radiation damping and is given by

$$\varepsilon_{//} = \frac{h^2\eta^2}{\nu_s^2}\left|\cos\phi_s\right|^{-1/2}(\hat{\sigma}_E)^2 = \frac{h^2\eta^2}{\nu_s^2}\left|\cos\phi_s\right|^{-1/2} C_q \frac{\gamma^2}{J_E\rho},$$

where $C_q=3.84\times10^{-13}$ m, $J_E \approx 2$ is the damping partition number. The theoretical longitudinal rms emittance of the TLS is expected to be $\varepsilon_{||} \approx 8.243\times10^{-3}$.

Measurement of the Transfer Function

When a charged particle passes a wall-gap monitor, executes synchrotron motion with amplitude τ, the image current can be expressed as the

$$I_e(t) = e\sum_{\ell}\delta(t - \hat{\tau}\cos(\omega_s t + \psi) - \ell T_0).$$

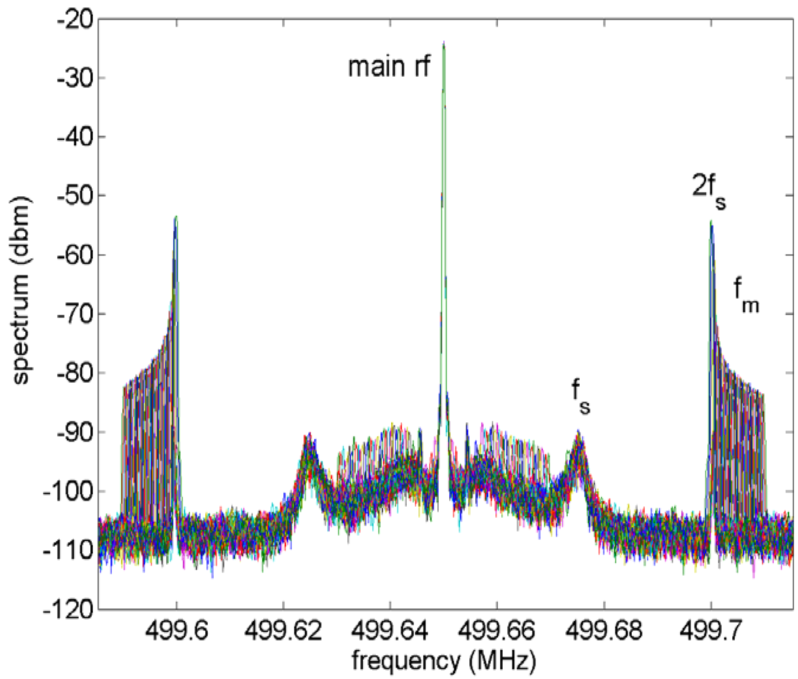
The coherent synchrotron modes of the bunch can be obtained by averaging the synchrotron mode over the bunch distribution

$$I(t) = N_B \int I_e \rho(\hat{\tau}, \psi)\hat{\tau}\, d\hat{\tau}\, d\psi = \sum_{n,m} A_{n,m} e^{i(n\omega_0 + m\omega_s)t}.$$

The amplitude $A_{n,m}$ is called the coherent beam mode of the *n*-th revolution harmonic and *m*-th synchrotron sideband. The coherent longitudinal quadrupole mode with the distribution and Hamiltonian shown in previous is

$$A_{n,2\ell} = (-1)^{\ell} \frac{2\pi e N N_B}{T_0} \int J_{2\ell}\left(n\omega_0 \frac{\sqrt{2J}\left|\cos\phi_s\right|^{-1/4}}{h\omega_0}\right) I_{\ell}\left(\frac{GJ}{E_{th}}\right) e^{\frac{\Delta J - \frac{1}{2}\alpha J^2}{E_{th}}}\, dJ.$$

Here J and I are Bessel and modified Bessel functions respectively.



The beam spectrum of rf voltage modulation. The modulation frequency is from 60 kHz to 50 kHz. The modulation amplitude ratio is 2.6%.

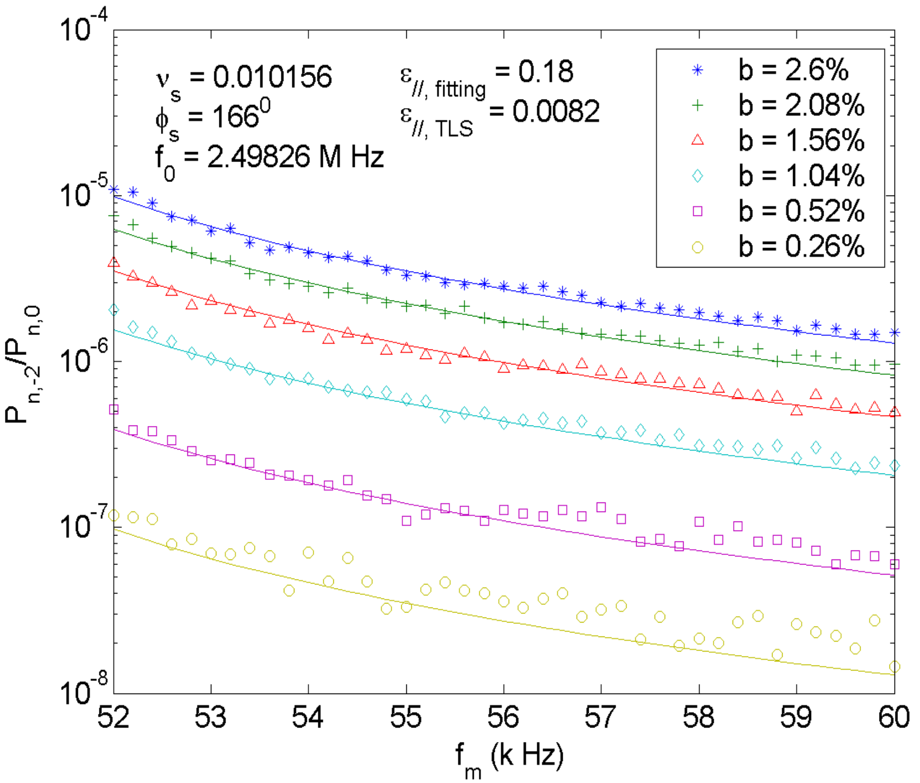
The beam spectrum obtained from a spectrum analyzer at center frequency 499.6649 MHz, span 130 KHz, modulation frequency from 60 kHz to 50 kHz with modulation amplitude ratio 2.6%.

The power spectrum measured from a spectrum analyzer is proportion to $|A_{n,m}|^2$. The quadrupole-mode transfer function (QTF) is defined by

$$q_2(n,\omega_m,b)=\frac{p_{n,\pm 2}}{P_{n,0}}=\left|\frac{A_{n,\pm 2}}{A_{n,0}}\right|^2 .$$

The QTF q_2 is shown as a function of modulation frequency f_m for different modulation amplitude ratio b from 2.6% to 0.26%. The symbols present the measured data and the solid line are the fitting result of rms longitudinal emittance of 0.18.

The solid lines in the figure show the fit with longitudinal emittance of $\varepsilon_{\parallel}$=0.18±0.01, the fit is much larger than the theoretical value of $\varepsilon_{\parallel,0}\approx$0.008.

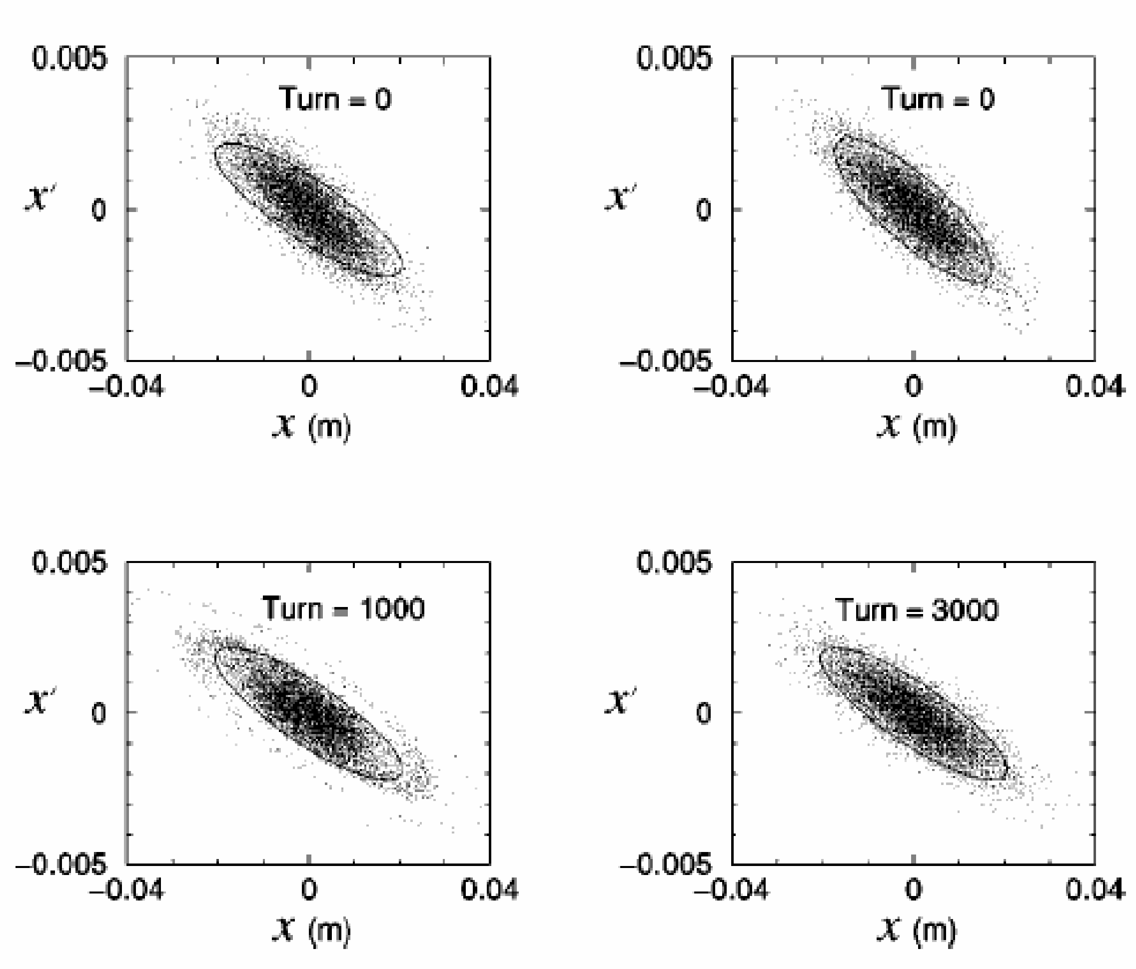


Summary of longitudinal emittance measurement

In conclusion, we have carried out experiments to test the applicability of using QTF to determine the longitudinal rms beam emittance. The derived rms beam emittance is much higher than the theoretical value. A new calculation including rigid dipole mode oscillations would be necessary to resolve the discrepancy between the derived emittance and the actual beam emittance. A single bunch experiment that can avoid the couple bunch mode excitation of the unwanted synchrotron sidebands would be very useful.

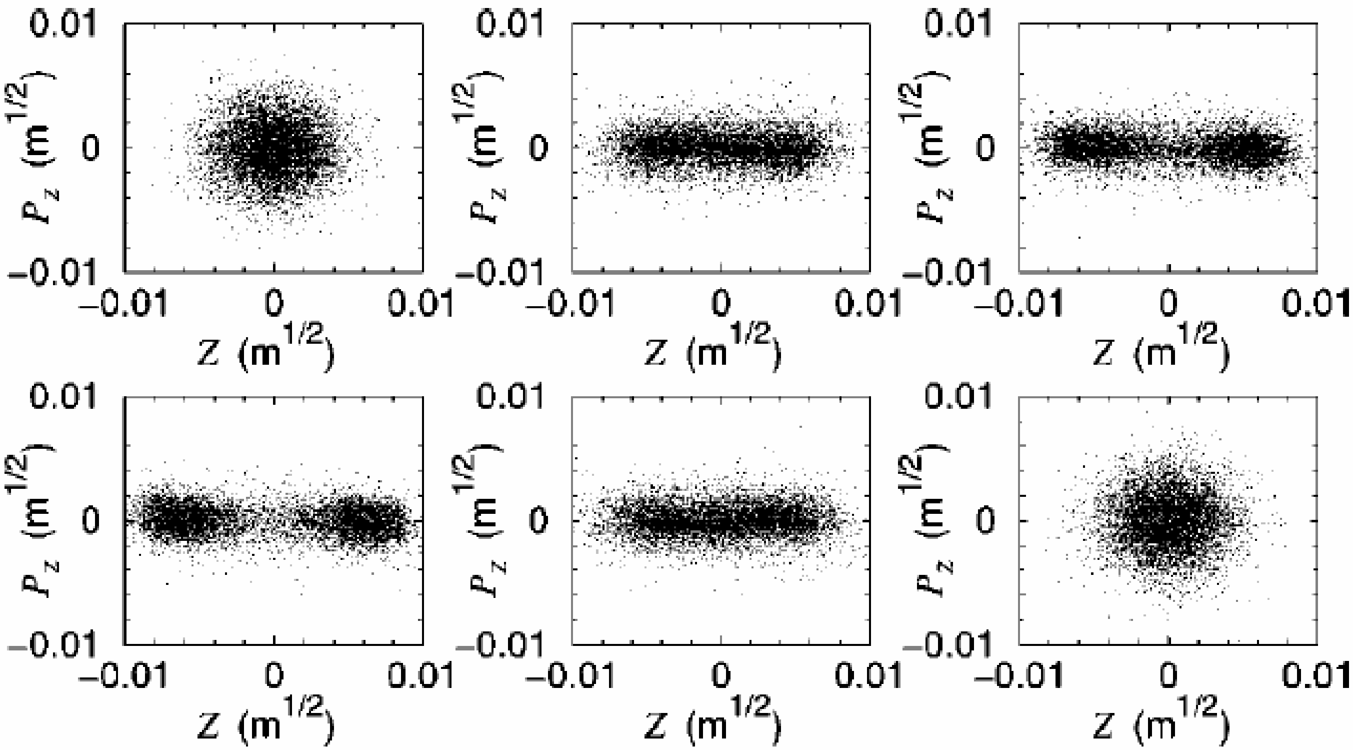
Application 2:

Optical Mismatch from the injection line into a storage ring: Without quadrupole transfer function, the emittance is diluted, and with quadrupole transfer function, the injection ellipse can be restored.



Application 3:

Numerical simulations of transverse phase space manipulation to test possible polarization preservation in passing through an intrinsic spin resonance. Our result indicates that the rf (AC) dipole modulation can provide better spin manipulation than the QTF method.



Conclusion

- Quadrupole-mode measurements and the half-integer stopband corrections are important for space charge dominated beams.
- Quadrupole-mode transfer function (QTF) can be used to obtain the rms emittance of the beam bunch without emittance dilution. Need experimental developments for this techniques. QTF can also be used to measure the betatron tune.
- QTF can also be used to compensate optical mismatch in the injection process (see the figure in next page).
- QTF is not as efficient as the rf (AC) dipole in polarized beam manipulation through intrinsic spin resonances.
- Hardware requirements: rf quadrupoles, quadrupole-mode monitors, etc.
- Other unsolved problems: (a) Can one derive dispersion relation for quadrupole mode transfer function? (b) Can one use the QTF to measure impedance that is important to the shape of the beam?