Linear and Non Linear Studies at RHIC Interaction Regions

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Abstract

Action and phase analysis on orbit measurements is presented as a technique to evaluate linear and non linear errors at RHIC IRs. This technique and the orbit bump method were used during the RHIC 2001 run to set the RHIC IR skew quadrupole correctors. There are also experimental evidence that the technique can be equally useful to evaluate gradient errors at the IRs. Precision and accuracy of the technique are evaluated.

Introduction

- Biggest optical error contributions have their origin at the IRs.
- RHIC has implemented a system of local correctors at each IR.
- What values should be used to set the correctors ?

Measurement of Local Errors and Correction

- Strengths can be calculated from the measured field errors of the magnets before installation.
- Orbit based Measurements
  - Minimization of badness function that depend on orbit measurements [a].
  - Orbit Bump Method [b].
  - Action and Phase Analysis (This Dissertation).

[a] R. Talman, A.B. D. workshop, Corpus Christi, 1991.

[b] V. Ptitsyn, PAC 2001, Chicago, 2001.

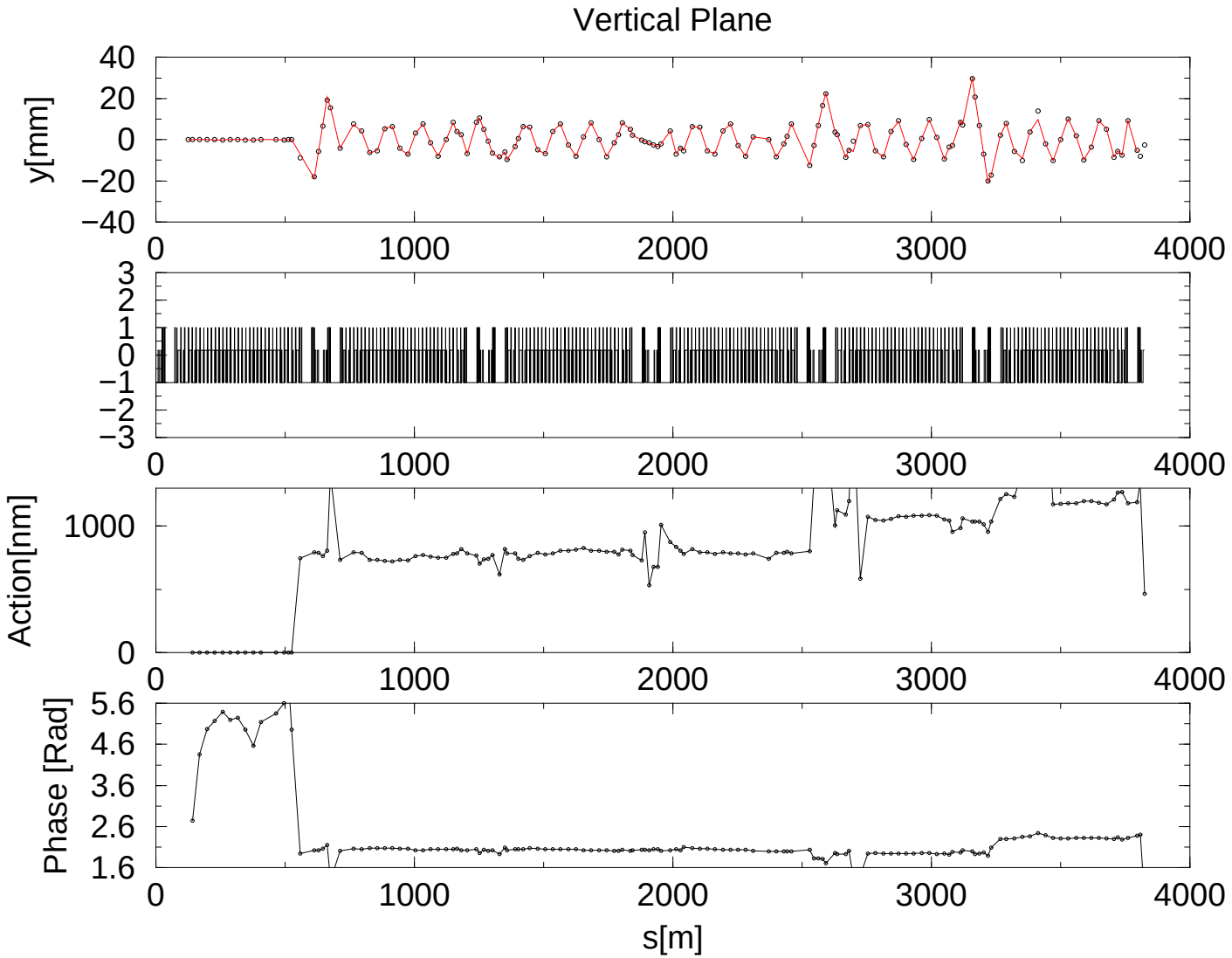
Action and Phase Analysis

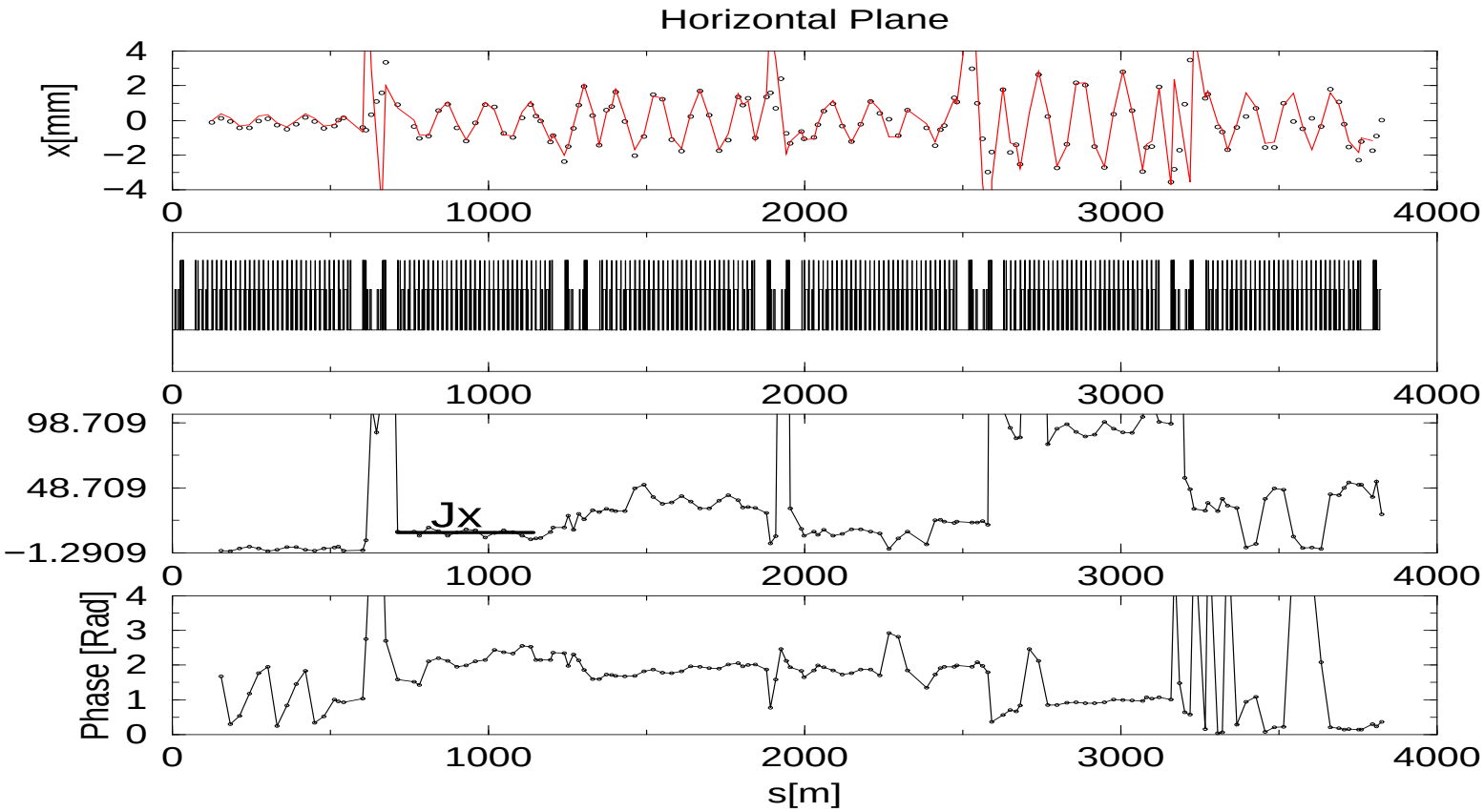
Two adjacent BPMs are used to find J and φ by inverting the equations:

$$\begin{aligned} x_1 &= \sqrt{2J\beta_x(s_1)}\sin(\psi_x(s_1)-\varphi) \\ x_2 &= \sqrt{2J\beta_x(s_2)}\sin(\psi_x(s_2)-\varphi) \end{aligned}$$

The procedure is repeated for all BPMs in the ring.

Orbit excited with bo7-tv7





$$A_1^{eq} = \frac{\sqrt{2J_x}}{y^{sc}\sqrt{\beta_x^{sc}}}$$

Skew Error Measurements in Blue Ring

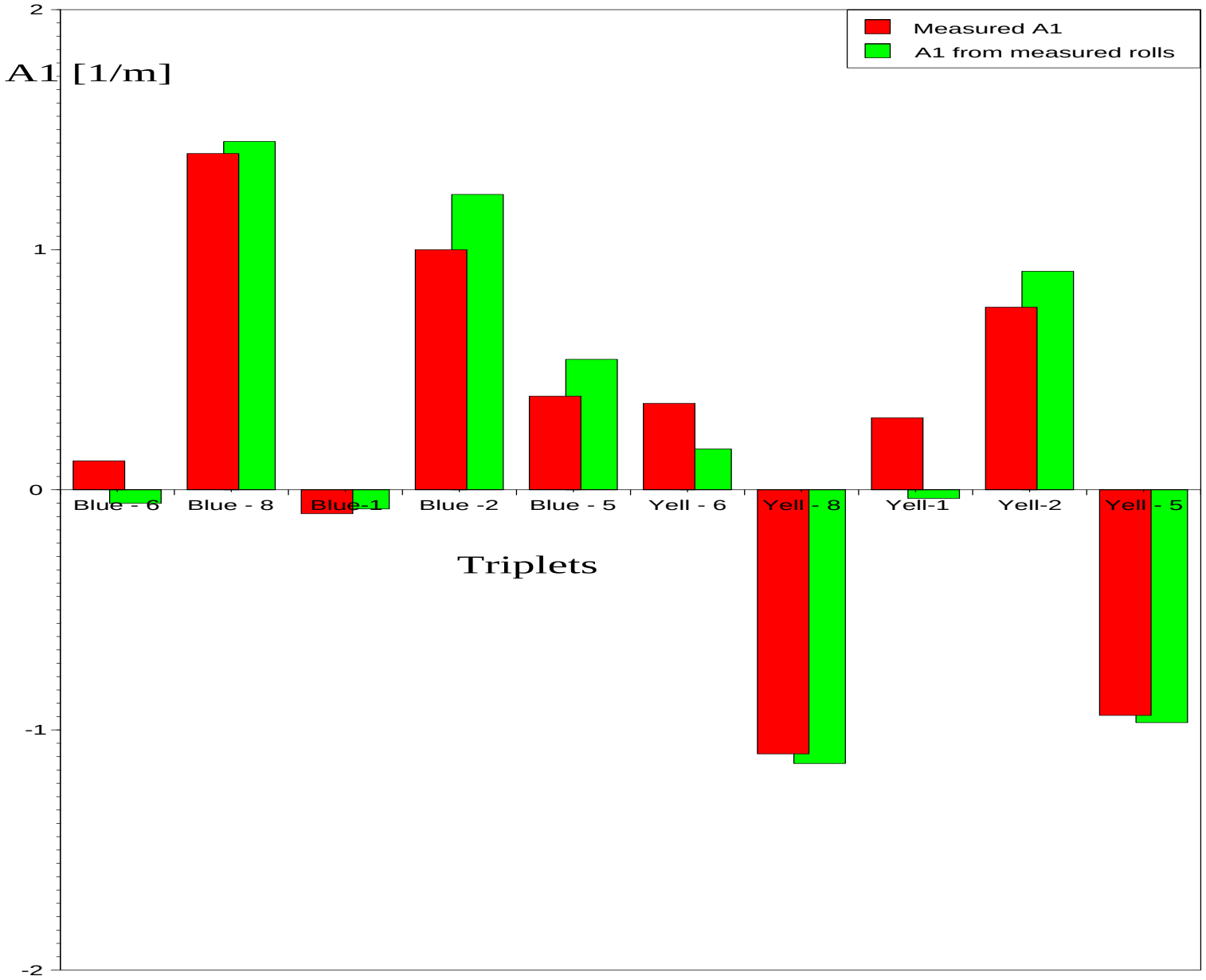
IR	Triplet	A_1^{eq} Orbit Bump (10^{-3} 1/m)	A_1^{eq} Action-Phase (10^{-3} 1/m)
IR8	7 8	-0.84 1.32	0.67 ± 0.07
IR10			1.00 ± 0.15
IR12			0.18 ± 0.03
IR2	1 2	-0.22 ± 0.16 1.23 ± 0.15	0.99 ± 0.14
IR4			0.63 ± 0.06
IR6	5 6	0.39 0.12	0.60 ± 0.15

Survey Measurements

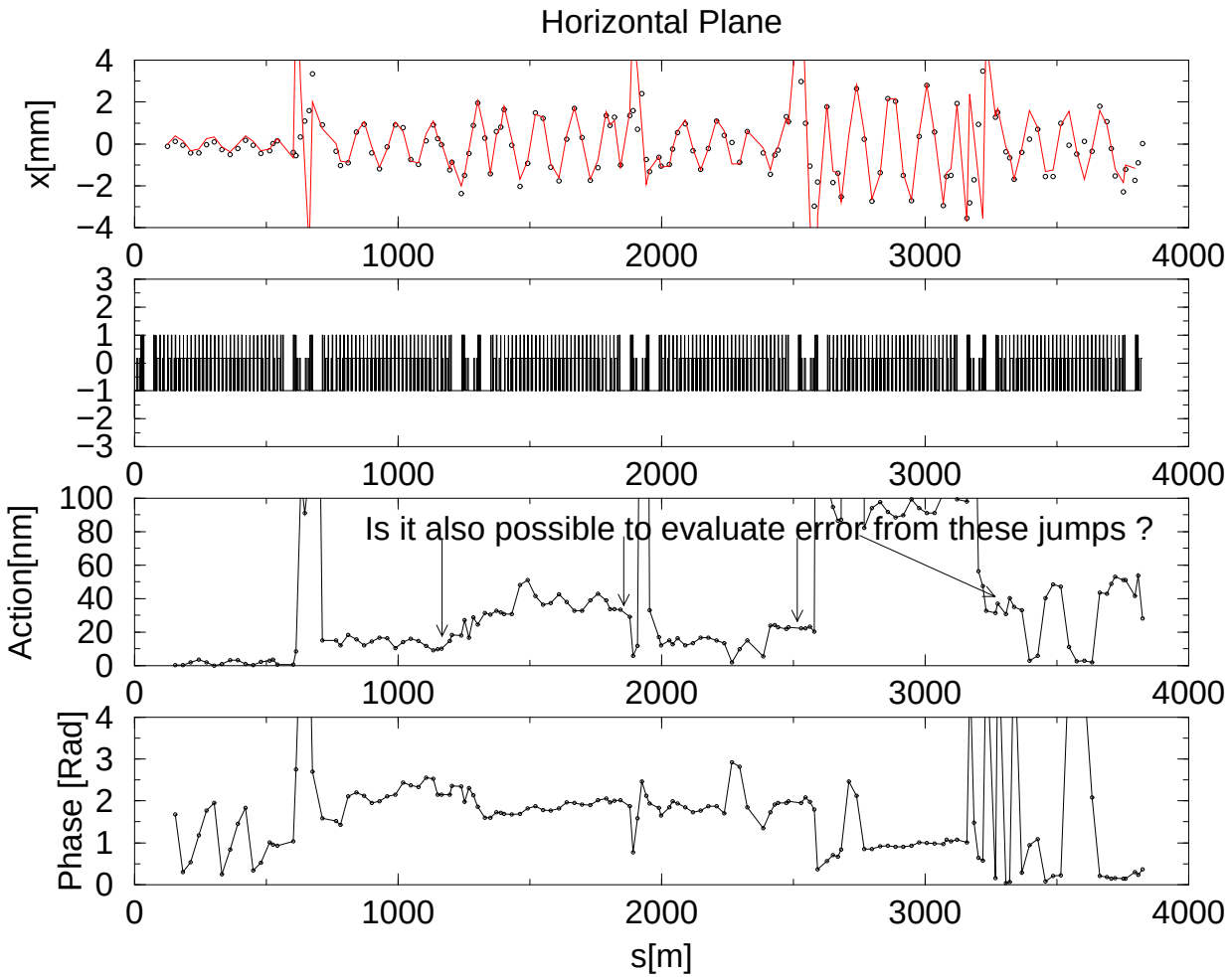
Skew quad errors can be calculated if the roll angles are known,

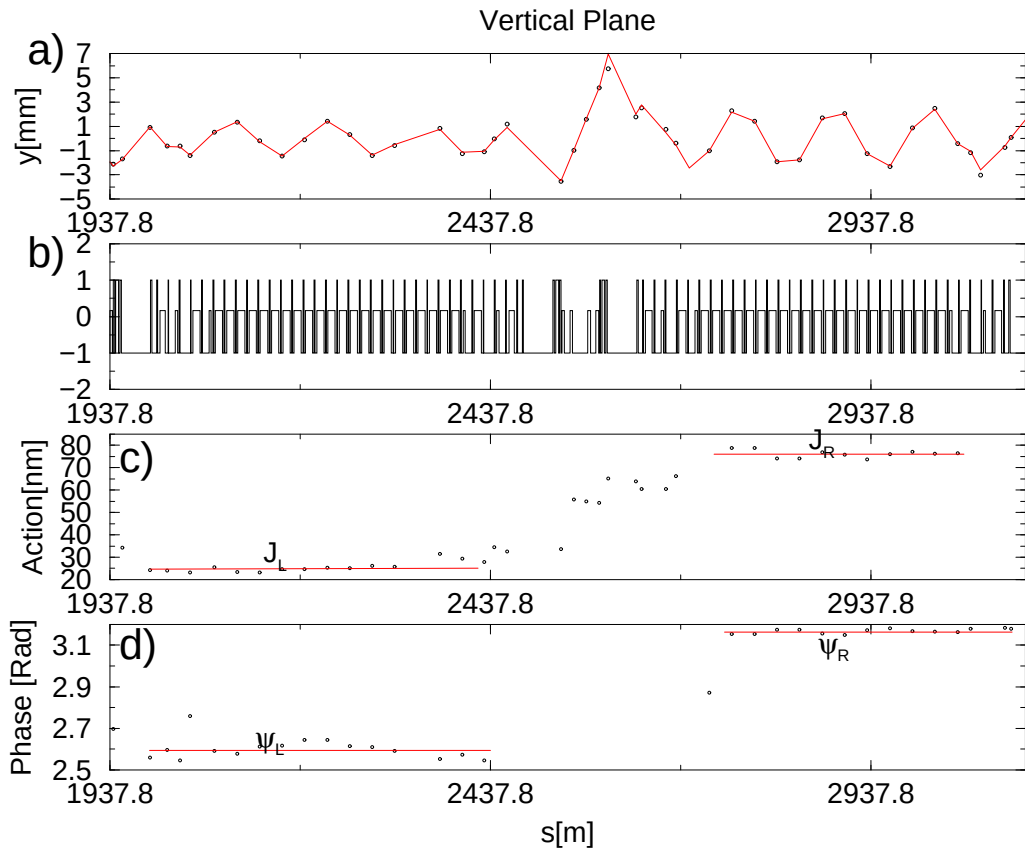
$$A_1^{eq} = \frac{\sum_{i=1}^{3}(-2\frac{\phi_i}{f_i})\sqrt{\beta_x^i\beta_y^i}}{\sqrt{\beta_x^{sc}\beta_y^{sc}}}$$

Roll angles of the quads were measured during the 2002 RHIC shutdown period.


Measured A1
A1 from measured rolls
A1 [1/m]
2
1
0
-1
-2
Blue - 6
Blue - 8
Blue-1
Blue -2
Blue - 5
Yell - 6
Yell - 8
Yell-1
Yell-2
Yell - 5
Triplets

General Method to Find Linear Errors from Orbit Measurements





$$\Delta x'^{eq} = \sqrt{\frac{2J_L + 2J_R - 4\sqrt{J_L J_R}\cos(\psi_L - \psi_R)}{\beta_I}}$$

Multipolar Expansion of $\Delta x'$ and $\Delta y'$

If one <u>single magnet</u> is responsible for the kick,

$$\begin{aligned}\Delta x' &= A_1 y - B_1 x + 2A_2 xy + B_2(-x^2 + y^2) + ... \\ \Delta y' &= A_1 x + B_1 y + 2B_2 xy + A_2(x^2 - y^2) + ...\end{aligned}$$

A_1 correspond to the skew error while B_1 correspond to gradient errors.

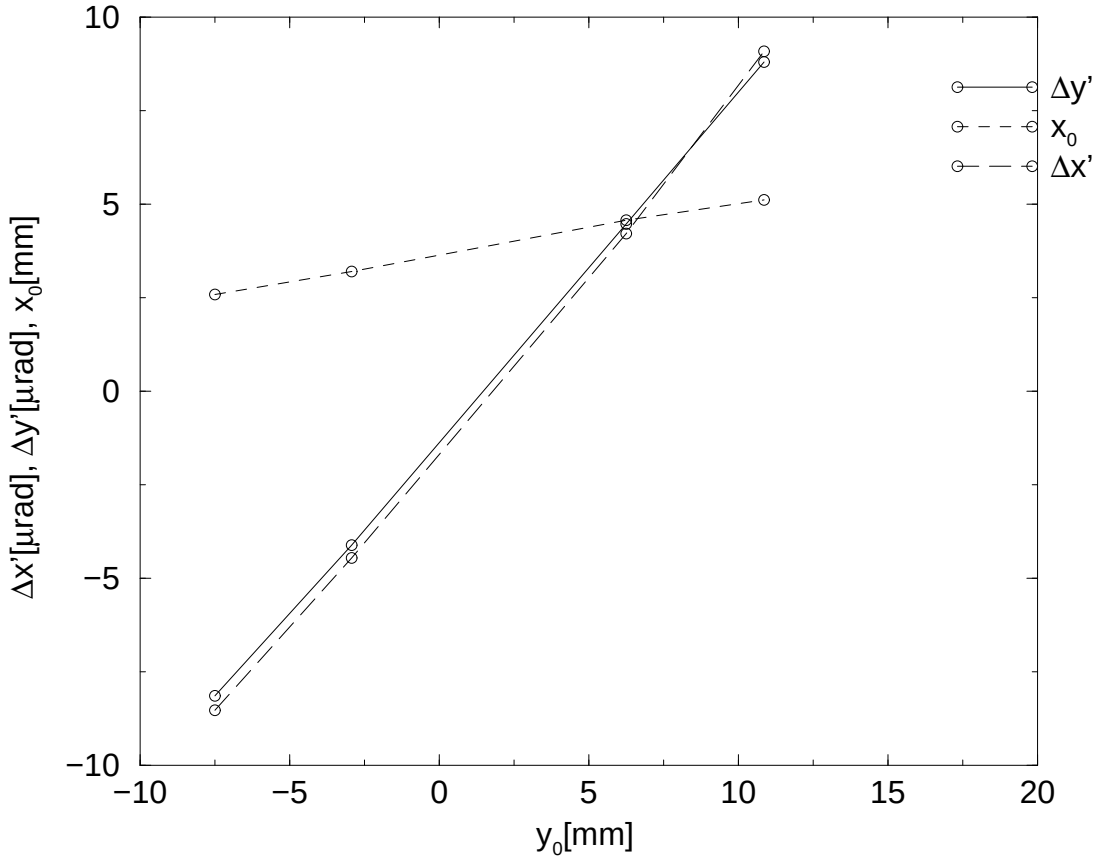
When more than one magnet is responsible for the kick, (for the special case of RHIC triplets)

$$
\begin{aligned}
\Delta x'^{eq} &= A_1^{eq} y_0 - B_1^x x_0 + 2{A_2}^x x_0 y_0 \\
&- {B_2}^x x_0^2 + {B_2}^y y_0^2 + ... \\
\Delta y'^{eq} &= A_1^{eq} x_0 + B_1^y y_0 + 2{B_2}^y x_0 y_0 \\
&+ {A_2}^x x_0^2 - {A_2}^y y_0^2 + ...
\end{aligned}
$$

where,

$$
B_1^x = \frac{B_1^a \beta_x^a + B_1^b \beta_x^b + B_1^c \beta_x^c}{\beta_x^{eq}}
$$

Measurement of Linear Errors during the RHIC 2001 run



Each point corresponds to a different dipole strenght.

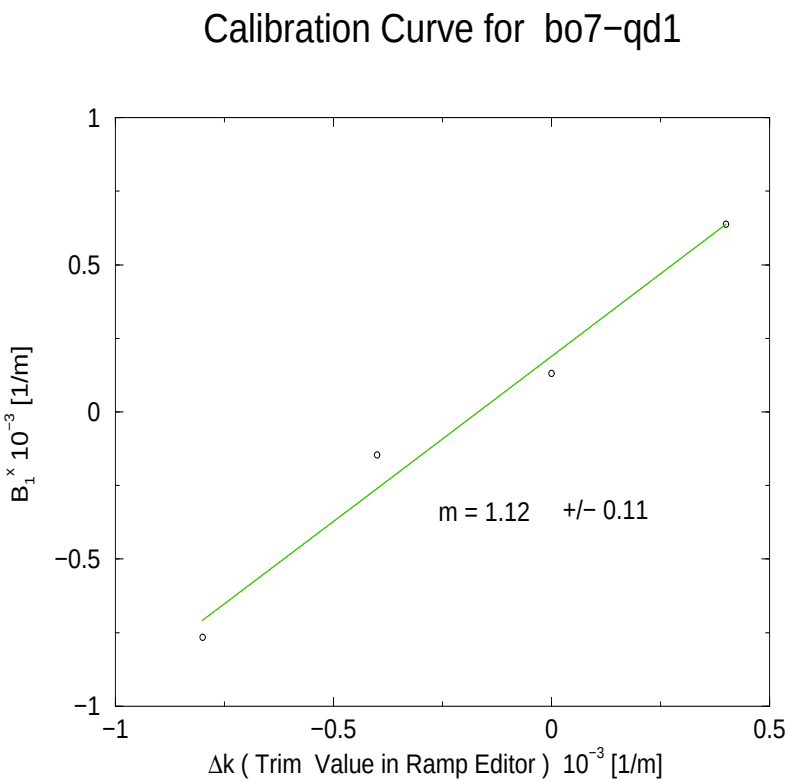
Results

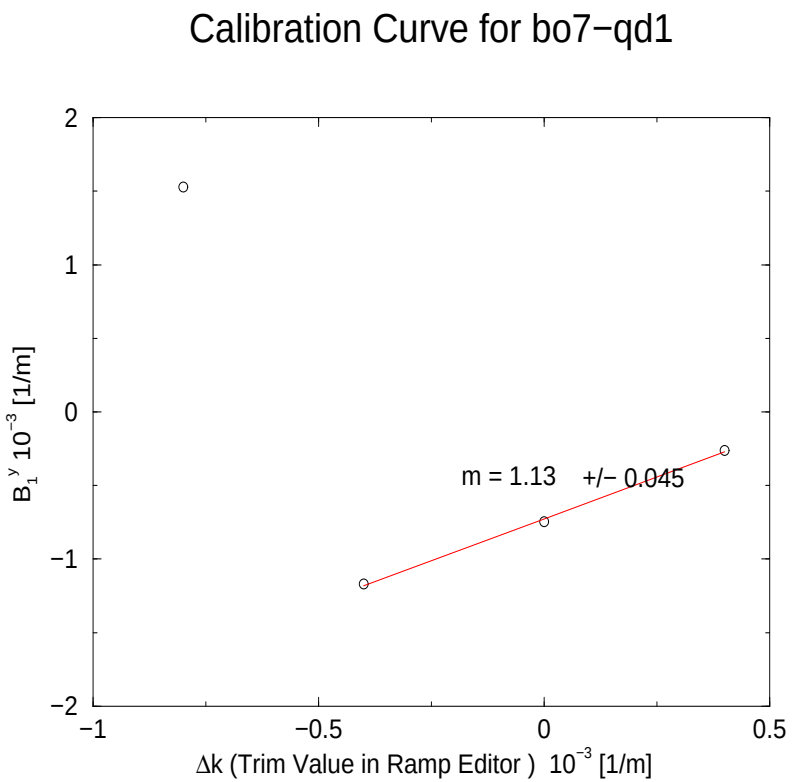
Triplet	A_1^{eq} (10^{-3} 1/m)	B_1^x (10^{-3} 1/m)	B_1^y (10^{-3} 1/m)
1	-0.866 ± 0.152	-0.655 ± 0.013	4.674 ± 1.805
2	-0.270 ± 0.001	-3.991 ± 0.000	0.961 ± 0.005
7	-0.067 ± 0.024	0.690 ± 0.040	0.200 ± 0.002
8	0.162 ± 0.031	-0.280 ± 0.030	-0.355 ± 0.003
9	-0.048 ± 0.007	-0.235 ± 0.009	-0.850 ± 0.010
10	0.206 ± 0.027	1.490 ± 0.019	0.715 ± 0.000

- A_1^{eq}s are an order of magnitude smaller that in the previous measurements as expected since correctors were on.
- Uncertainties as low as 0.02% can be achieved in the measurement of the total quadrupole strength compared to 0.04% achieved in other machines with the ORM [a].

[a] J. Safranek,NIM A,388,1997.

Calibration Curves for Quadrupole Gradients





$$B_1^x = \Delta k + {B_1}^x{}_{(IR)}$$
$$B_1^y = \Delta k + {B_1}^y{}_{(IR)}$$

Non Linear Experiments

- Sextupoles intentionally introduced in the SPS [a] and RHIC [b] lattices have been measured with the resonance terms method.
- Non linear corrections were possible with Orbit bump and tune measurements. [c]
- Action and phase could also be used for non linear analysis.

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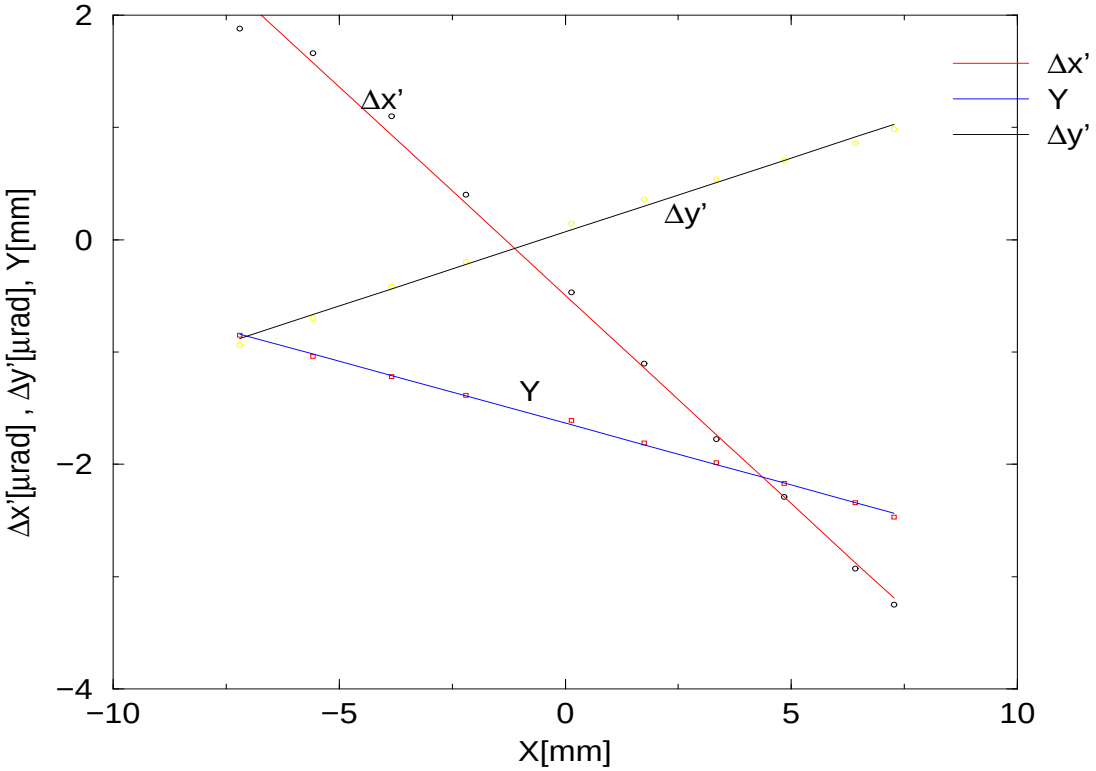
[a] R. Tomás et al, EPAC 2002

[b] W. Fischer et al, AP 81, BNL 2002

[c] F. Pilat et al, PAC2003.

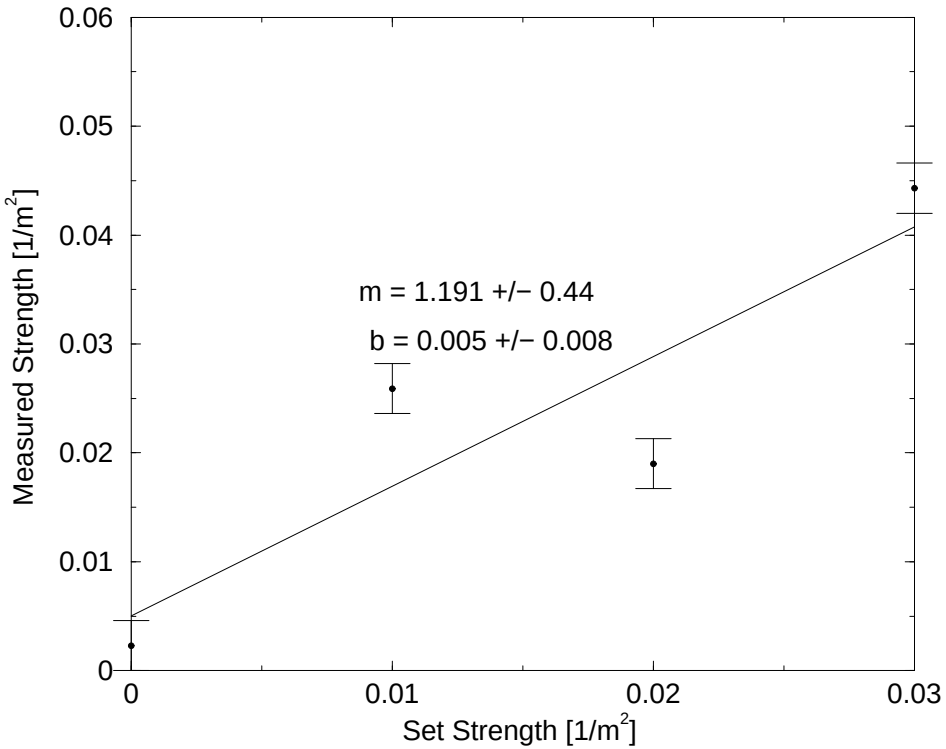


$$\Delta x'^{eq} = A_1^{eq} y_0 - B_1^x x_0 + 2A_2{}^x x_0 y_0 - B_2{}^x x_0^2 + B_2{}^y y_0^2 + \ldots$$

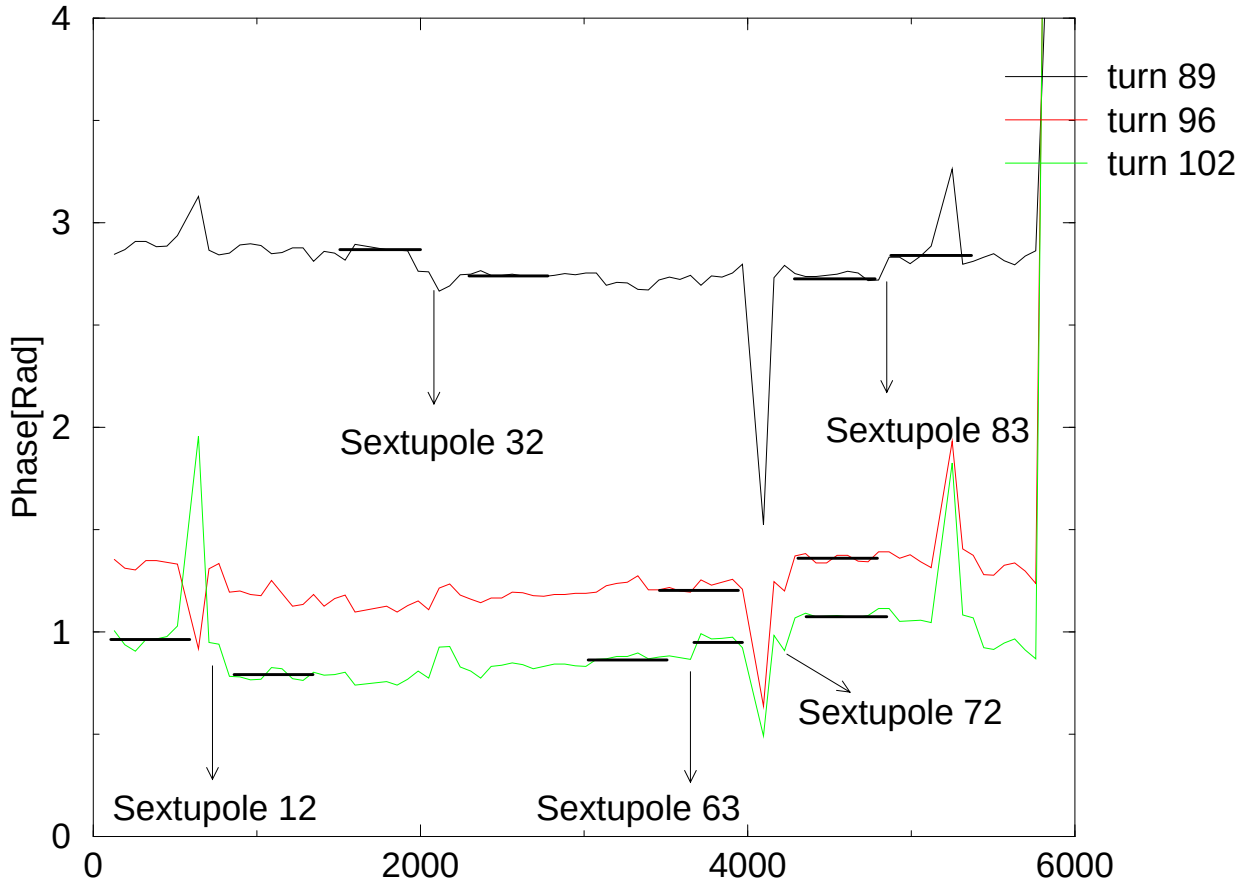
A_1^{eq}	0.122 ± 0.003
B_1^x	0.386 ± 0.001
B_1^y	-0.142 ± 0.002
A_2^{ya}	0.0121 ± 0.0003
A_2^x	-0.0012 ± 0.0011
B_2^y	0.0061 ± 0.0011
B_2^{xa}	0.0037 ± 0.0025

Calibration Curve for Sextupoles

Experiment to find non linear errors was repeated for 4 different sextupole strengths.



Analysis of Super Proton Synchroton Orbits

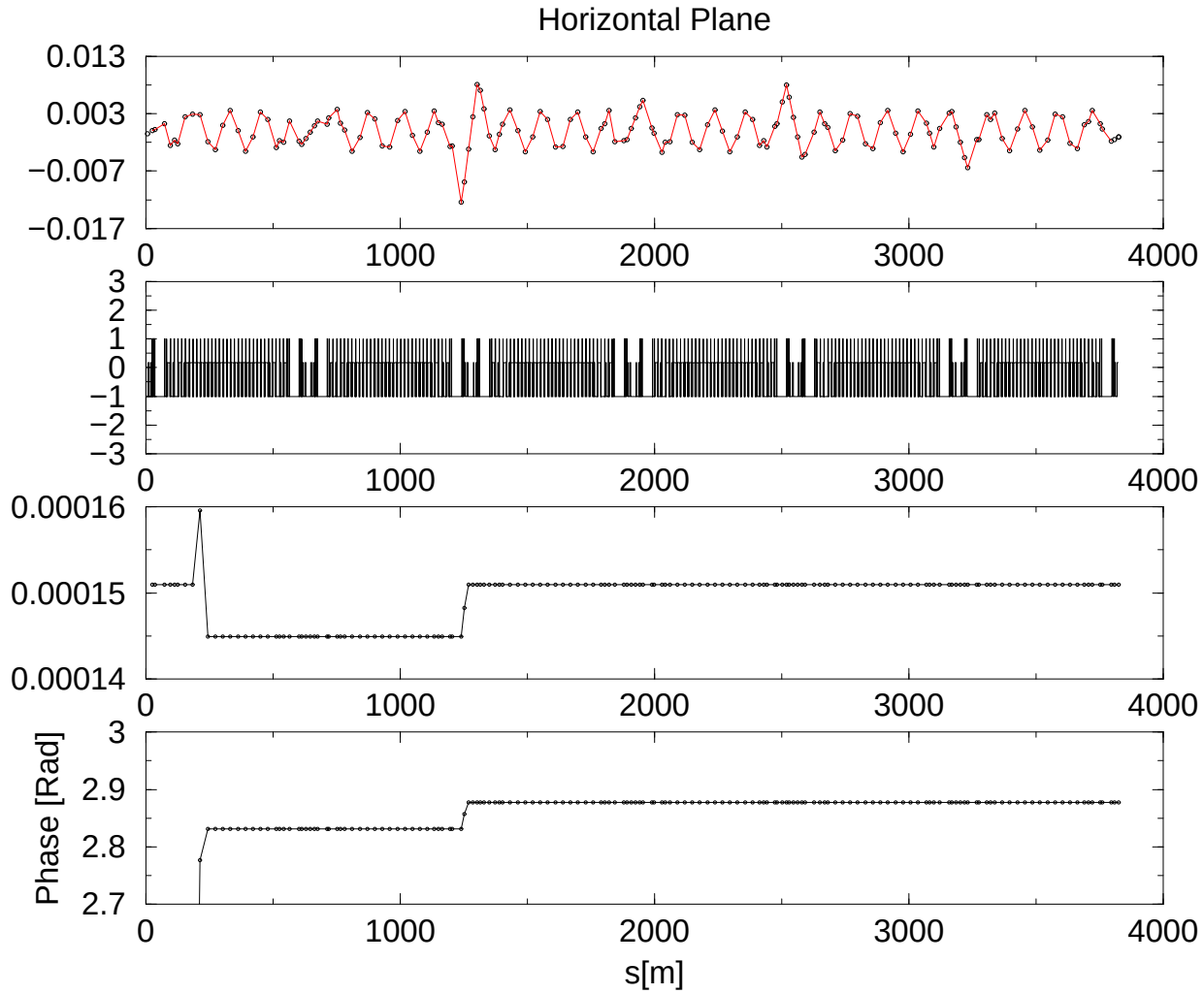


Summary

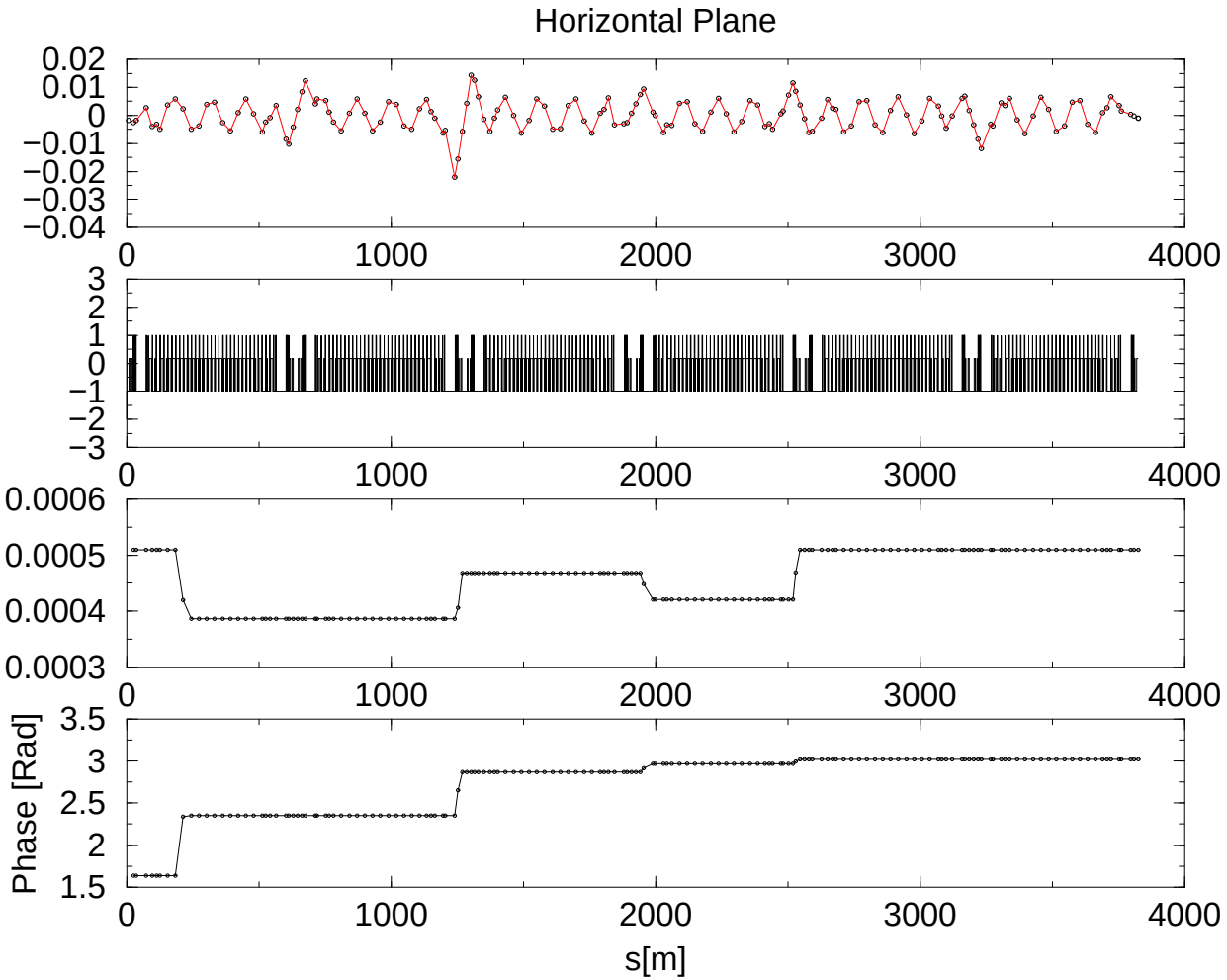
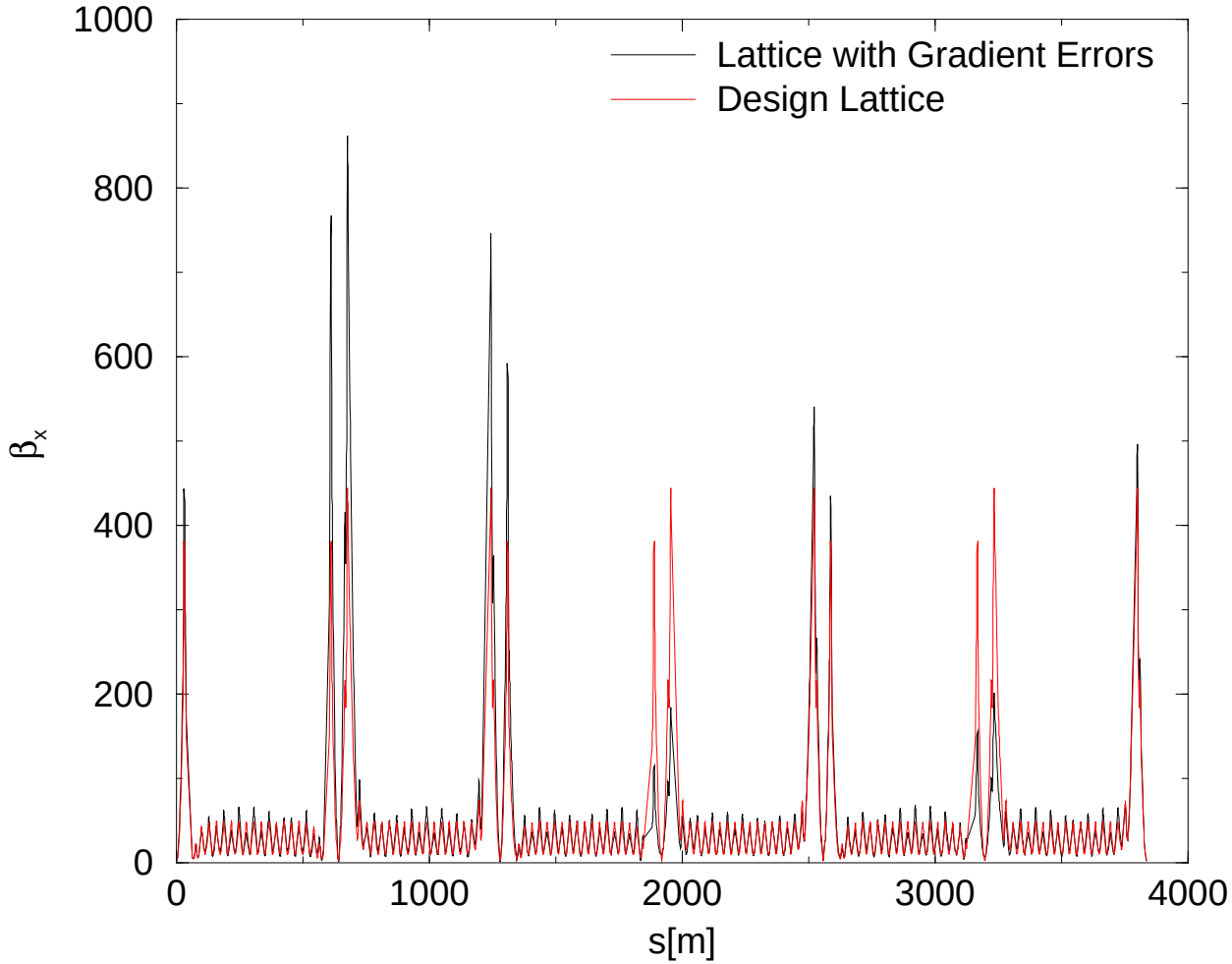
- Skew Quadrupole Errors has been measured with enough precision to do an effective local correction of linear coupling.
- The skew quadrupole error measurement has been validating with magnetic measurements and measurements done after the local correction.
- Measurement of the equivalent gradient strengths have been done with precisions as good as 0.02%.

- Measurement of known quadrupole gradient strengths allows to set the accuracy of the gradient measurements at about 0.1%.
- The possibility of detecting and measuring local sextupole errors with the action and phase method has been shown with RHIC and SPS orbits.

MAD Simulations



Gradient Errors in Several Places



Design Beta Functions vs Beta Functions with Errors