Programming the New Sextupole Strings in Booster

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1 The Sextupole Strings

Each superperiod of Booster contains eight sextupoles which are labled SVX1, SHX2, SVX3, SHX4, SVX5, SHX6, SVX7, and SHX8, where SH and SV denote, respectively, sextupoles located near horizontal and vertical beta maximums, and X refers to superperiod A, B, C, D, E, or F. We shall refer to the SH and SV sextupoles as horizontal and vertical sextupoles respectively. The sextupoles have a main winding consisting of 8 turns, a monitor winding consisting of one turn, and an auxiliary winding consisting of either one or two turns. The main windings are used for chromaticity adjustment and excitation of the $3Q_H = 13$ resonance for resonant extraction. The auxiliary windings are used for resonance correction.

The main windings are connected together to form four series strings called the Horizontal and Vertical strings and the C and F strings. The Horizontal and Vertical strings are used for chromaticity adjustment; the C and F strings are used for excitation of the $3Q_H = 13$ resonance. Within each string the sextupoles are all excited with the same polarity. The vertical string contains all 24 of the vertical sextupoles. The horizontal string used to contain all 24 of the horizontal sextupoles, but now four of these, SHC8, SHF8, SHB4 and SHE4, have been taken out of the string so that they can be powered independently for resonant extraction. With these four sextupoles removed, there are four "holes" in the horizontal string, each hole having a partner located three superperiods away. This arrangement of horizontal sextupoles is depicted schematically in **Figure 1**. Here the open red circles show the positions of the four holes; the filled circles show the positions of the remaining 20 horizontal sextupoles. The horizontal and vertical strings are powered by programmable monopolar power supplies that can deliver a maximum current of 300 A at a maximum of 90 V.

The main windings on sextupoles SHC8, SHF8, SHB4 and SHE4 are connected together to form the C and F strings. The C string contains the SHC8 and SHE4 sextupoles; the F string contains SHF8 and SHB4. The two strings are depicted by the blue lines in Figure 1. Each string is powered with its own ± 350 A programmable power supply. The $Q_H = 4 + 1/3$ resonance is excited by powering the two strings with opposite polarity.

The superperiod symmetry of the machine implies that the holes in the horizontal string will produce only even harmonics in azimuthal angle θ around the ring. Although this ensures that the $3Q_H = 13$ and $Q_H + 2Q_V = 13$ resonances (which are excited by harmonic 13) will not be excited, the $3Q_H = 14$ and $Q_H + 2Q_V = 14$ resonances (which are excited by harmonic 14) can be excited. To ensure that this does not happen, the currents in the four resonant extraction sextupoles will track the current in the horizontal string until extraction time. Note, however, that because the four holes are approximately equally spaced in betatron phase and in azimuth θ, harmonics 2, 6, 10, 14, 18, and so on, are suppressed to some extent. This means that excitation of the resonances $3Q_H = 14$ and $Q_H + 2Q_V = 14$ may not be a problem.

2 The Old Chrom Control Program

Let J_H be the current in the old string of 24 horizontal sextupoles and let J_V be the current in the string of 24 vertical sextupoles. If the user wants horizontal and vertical chromaticities ξ_H and ξ_V, the old Booster Chrom Control program calculates the required currents J_H and J_V. Conversely, if the user specifies J_H and J_V, the old program calculates ξ_H and ξ_V. The user interface for the old application is shown in Figure 2.

3 The New Chrom Control Program

The New Booster Chrom Control program deals with the four sextupole strings and their currents.

Let I_H be the current in the new string of 20 horizontal sextupoles.

Let I_V be the current in the string of 24 vertical sextupoles.

Let I_C be the current in the C string.

Let I_F be the current in the F string.

Then the chromaticities are given by

$$\xi_H = \xi_H^0 + \frac{S}{B\rho}(K_H I_H + K_V I_V + K_C I_C + K_F I_F) \tag{1}$$

and

$$\xi_V = \xi_V^0 + \frac{S}{B\rho}(L_H I_H + L_V I_V + L_C I_C + L_F I_F) \tag{2}$$

where ξ_H^0 and ξ_V^0 are the bare chromaticities, S is the integrated strength per unit current of each sextupole, $B\rho$ is the magnetic rigidity, and the constants K_H, K_V, K_C, K_F, L_H, L_V, L_C, and L_F are derived in Appendix I. Note that, as defined by (1) and (2), the bare chromaticities are what one would measure with zero current in the four sextupole strings. The values to be assigned to ξ_H^0 and ξ_V^0 are discussed in Appendix II.

It is convenient to parameterize I_C and I_F as

$$I_C = I_H + I_0 + I_R, \quad I_F = I_H + I_0 - I_R \tag{3}$$

where

$$I_R = (I_C - I_F)/2, \quad I_0 = -I_H + (I_C + I_F)/2. \tag{4}$$

The currents I_R and I_0 are responsible respectively for the excitation of the $3Q_H = 13$ resonance and the excitation of sextupole harmonics 0, 4, 8, 12, 16, and so on. (As noted in Section 1, harmonics 2, 6, 10, 14, 18, and so on, are suppressed by superperiod symmetry.) This suggests that we define the physics parameters

$$R = \frac{S}{B\rho} K_R I_R, \quad R_0 = \frac{S}{B\rho} K_0 I_0 \tag{5}$$

where the constants K_R and K_0 are chosen to give R and R_0 convenient units. Thus, the four physics parameters to be programmed by the user are ξ_H, ξ_V, R, and R_0.

If the four currents I_H, I_V, I_C, and I_F are specified, then ξ_H, ξ_V, R, and R_0 are calculated according to equations (1), (2), (4), and (5). Conversely, if ξ_H, ξ_V, R, and R_0 are specified, then the four currents can be calculated as follows. Using (3) in (1) and (2), we have

$$\Delta_H = (K_H + K_C + K_F)I_H + K_V I_V + \Gamma_H \tag{6}$$

and

$$\Delta_V = (L_H + L_C + L_F)I_H + L_V I_V + \Gamma_V \tag{7}$$

where

$$\Delta_H = \frac{B\rho}{S}(\xi_H - \xi_H^0), \quad \Delta_V = \frac{B\rho}{S}(\xi_V - \xi_V^0), \tag{8}$$

$$\Gamma_H = (K_C + K_F)I_0 + (K_C - K_F)I_R, \tag{9}$$

and

$$\Gamma_V = (L_C + L_F)I_0 + (L_C - L_F)I_R. \tag{10}$$

The currents

$$I_0 = \left(\frac{B\rho}{SK_0}\right) R_0, \quad I_R = \left(\frac{B\rho}{SK_R}\right) R \tag{11}$$

are obtained from equation (5). Using these in (9) and (10), one obtains Γ_H and Γ_V in terms of R and R_0. Equations (6) and (7) then may be solved to obtain I_H and I_V in terms of $\Delta_H - \Gamma_H$ and $\Delta_V - \Gamma_V$. One finds

$$I_H = L_V \left(\frac{\Delta_H - \Gamma_H}{D}\right) - K_V \left(\frac{\Delta_V - \Gamma_V}{D}\right) \tag{12}$$

and

$$I_V = -(L_H + L_C + L_F)\left(\frac{\Delta_H - \Gamma_H}{D}\right) + (K_H + K_C + K_F)\left(\frac{\Delta_V - \Gamma_V}{D}\right) \tag{13}$$

where

$$D = (K_H + K_C + K_F)L_V - (L_H + L_C + L_F)K_V. \tag{14}$$

Finally, using (11) and (12) in (3), one obtains currents I_C and I_F.

Thus, given ξ_H, ξ_V, R, and R_0, the new Booster Chrom Control program calculates currents I_H, I_V, I_C and I_F using equations (6–14) and equation (3). Conversely, if the currents are specified, the new program calculates ξ_H, ξ_V, R, and R_0 using equations (1), (2), (4), and (5). Figure 3 shows schematically the user interface for the new application.

Note that for the case in which $I_R = 0$ and $I_0 = 0$, we have

$$I_H = I_C = I_F \tag{15}$$

and we see that the 20 sextupoles in the horizontal string and the four sextupoles C8, E4, F8, and B4 all have the same current. This is the normal mode of operation when the $3Q_H = 13$ resonance is not being excited for resonant extraction. In this case one has

$$R = 0, \quad R_0 = 0, \tag{16}$$

$$\xi_H = \xi_H^0 + \frac{S}{B\rho}\left\{(K_H + K_C + K_F)I_H + K_V I_V\right\} \tag{17}$$

and

$$\xi_V = \xi_V^0 + \frac{S}{B\rho}\left\{(L_H + L_C + L_F)I_H + L_V I_V\right\}. \tag{18}$$

Here the chromaticities and currents I_H and I_V are calculated exactly as they are in the old Chrom Control program.

During resonant extraction I_R is nonzero. In this case one would generally like to have $I_0 = 0$, but in some situations this would require that either I_C or I_F be larger than the available current. Thus, during resonant extraction, one will generally have to adjust R_0 so that I_H, I_C, and I_F all stay within the bounds of available current. Figure 4 shows how the currents might be programmed for a typical BAF magnet cycle.

4 Appendix I

The horizontal and vertical chromaticities in Booster are given by [1, 2]

$$\xi_H = \xi_H^0 + \frac{1}{4\pi}\frac{S}{B\rho}\sum_j \beta_H^j D_j\, I_j \tag{19}$$

and

$$\xi_V = \xi_V^0 - \frac{1}{4\pi}\frac{S}{B\rho}\sum_j \beta_V^j D_j\, I_j \tag{20}$$

where ξ_H^0 and ξ_V^0 are the bare chromaticities, $B\rho$ is the magnetic rigidity, β_H^j and β_V^j are the horizontal and vertical beta functions, D_j is the periodic dispersion and I_j is the current in the jth sextupole. The sum is taken over all 48 sextupoles. The integrated strength per unit current of each sextupole is [3]

$$S = B''L = 2 \times 6.566 \times 10^{-3} \ \ (\mathrm{T/m})/\mathrm{A} \tag{21}$$

where B is the vertical magnetic field on the midplane of the sextupole, L is the magnetic length, and the primes denote differentiation with respect to the horizontal coordinate.

Let us now define

$$K_H = \frac{1}{4\pi}\sum_h \beta_H^h D_h, \quad K_V = \frac{1}{4\pi}\sum_v \beta_H^v D_v, \tag{22}$$

$$K_C = \frac{1}{4\pi}\sum_c \beta_H^c D_c, \quad K_F = \frac{1}{4\pi}\sum_f \beta_H^f D_f, \tag{23}$$

$$L_H = -\frac{1}{4\pi}\sum_h \beta_V^h D_h, \quad L_V = -\frac{1}{4\pi}\sum_v \beta_V^v D_v, \tag{24}$$

and

$$L_C = -\frac{1}{4\pi}\sum_c \beta_V^c D_c, \quad L_F = -\frac{1}{4\pi}\sum_f \beta_V^f D_f \tag{25}$$

where the sum over h is the sum over all horizontal sextupoles except SHC8, SHE4, SHF8, and SHB4; the sum over v is the sum over all vertical sextupoles; the sum over c is the sum over sextupoles SHC8 and SHE4; and the sum over f is the sum over sextupoles SHF8 and SHB4. Equations (19) and (20) then become

$$\xi_H = \xi_H^0 + \frac{S}{B\rho}(K_H I_H + K_V I_V + K_C I_C + K_F I_F) \tag{26}$$

and

$$\xi_V = \xi_V^0 + \frac{S}{B\rho}(L_H I_H + L_V I_V + L_C I_C + L_F I_F) \tag{27}$$

where the four currents I_H, I_V, I_C, and I_F are defined in Sections 1 and 3. The sums in (22–25) are readily evaluated using the MAD code. One finds

$$K_H = \frac{472.86}{4\pi}, \quad K_V = \frac{122.90}{4\pi}, \quad K_C = K_F = \frac{47.267}{4\pi} \tag{28}$$

$$L_H = -\frac{162.03}{4\pi}, \quad L_V = -\frac{355.38}{4\pi}, \quad L_C = L_F = -\frac{16.067}{4\pi} \tag{29}$$

and

$$K_H + K_C + K_F = \frac{567.39}{4\pi}, \quad L_H + L_C + L_F = -\frac{194.16}{4\pi} \tag{30}$$

where the units are meters squared. Here the horizontal and vertical tunes were taken to be $Q_H = 4.82$ and $Q_V = 4.83$. Note that it follows from superperiod symmetry that

$$K_C = K_F, \quad L_C = L_F. \tag{31}$$

Since the constants defined in (22–25) depend on the beta functions and dispersion, they will depend on the machine tunes. This dependence can be put into the Chrom Control program by means of a lookup table which gives the values of the constants for various tunes.

5 Appendix II

As defined by (1) and (2), the bare chromaticities, ξ_H^0 and ξ_V^0, are what one would measure with zero current in the four sextupole strings. There are several contributions to ξ_H^0 and ξ_V^0 which may be expressed by writing

$$\xi_H^0 = \xi_H^N + \frac{1}{B\rho}\left(A_H + \dot{B}\,B_H\right) + C_H \tag{32}$$

and

$$\xi_V^0 = \xi_V^N + \frac{1}{B\rho}\left(A_V + \dot{B}\,B_V\right) + C_V. \tag{33}$$

Here ξ_H^N and ξ_V^N are the so called natural chromaticities [1, 2] due to the intrinsic quadrupole structure of the lattice (they do not depend on sextupole fields). These will depend on the machine tunes and can be provided by a lookup table. For $Q_H = 4.82$ and $Q_V = 4.83$, the MAD code gives

$$\xi_H^N = -4.941, \quad \xi_V^N = -5.258 \tag{34}$$

The A_H and A_V terms are due to remanant sextupole fields, and the B_H and B_V terms are due to sextupole fields that scale with $\dot{B} = dB/dt$. The values of A_H, A_V, B_H, and B_V must be determined from measurements. Their dependence on the machine tunes and can be provided by a lookup table. The C_H and C_V terms are due to sextupole fields that scale with $B\rho$. For given tunes, C_H and C_V are assumed to be constant until the Booster dipoles begin to saturate. Their dependence on the tunes and on saturation in the dipoles can be provided by a lookup table. For $Q_H = 4.82$ and $Q_V = 4.83$, the MAD code gives (with no dipole saturation)

$$C_H = -2.342, \quad C_V = +2.194 \tag{35}$$

Thus, for $Q_H = 4.82$ and $Q_V = 4.83$ we can write

$$\xi_H^0 = -7.283 + \frac{1}{B\rho}\left(A_H + \dot{B}\,B_H\right) \tag{36}$$

and

$$\xi_V^0 = -3.064 + \frac{1}{B\rho}\left(A_V + \dot{B}\,B_V\right). \tag{37}$$

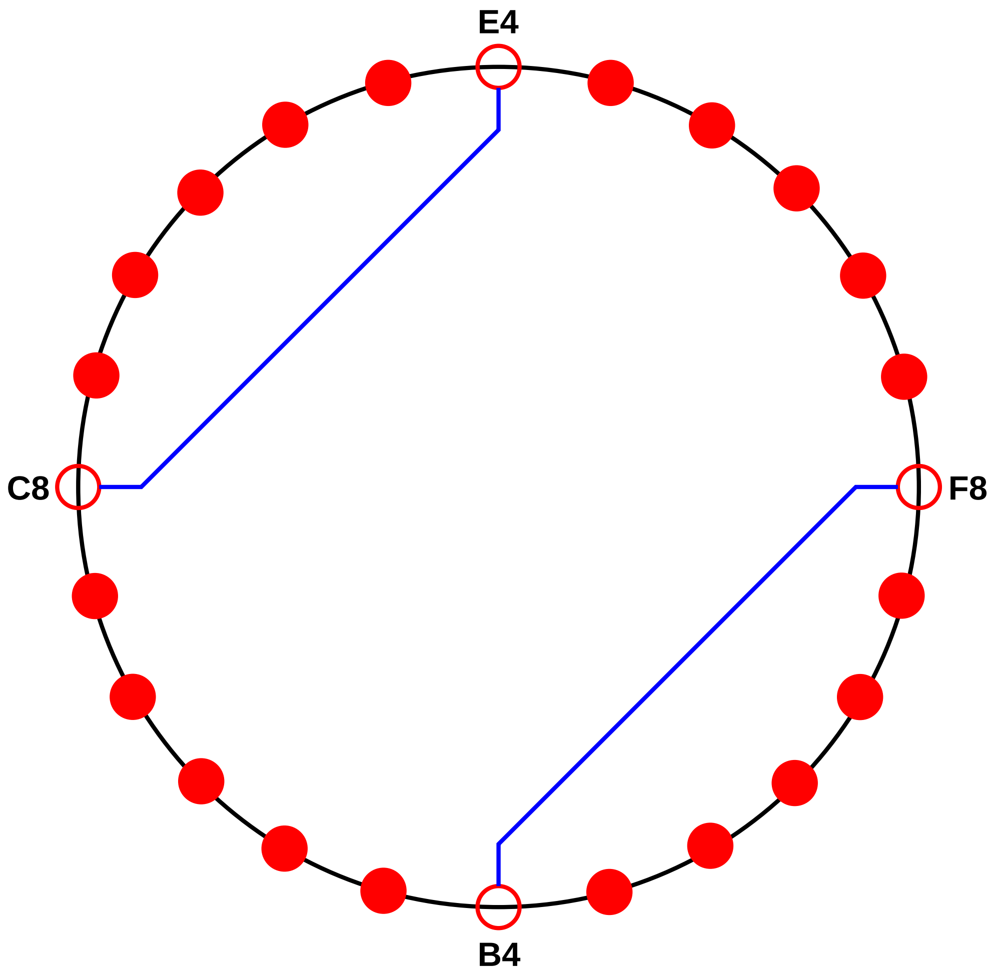

Figure 1: Horizontal Sextupoles in the Booster Ring

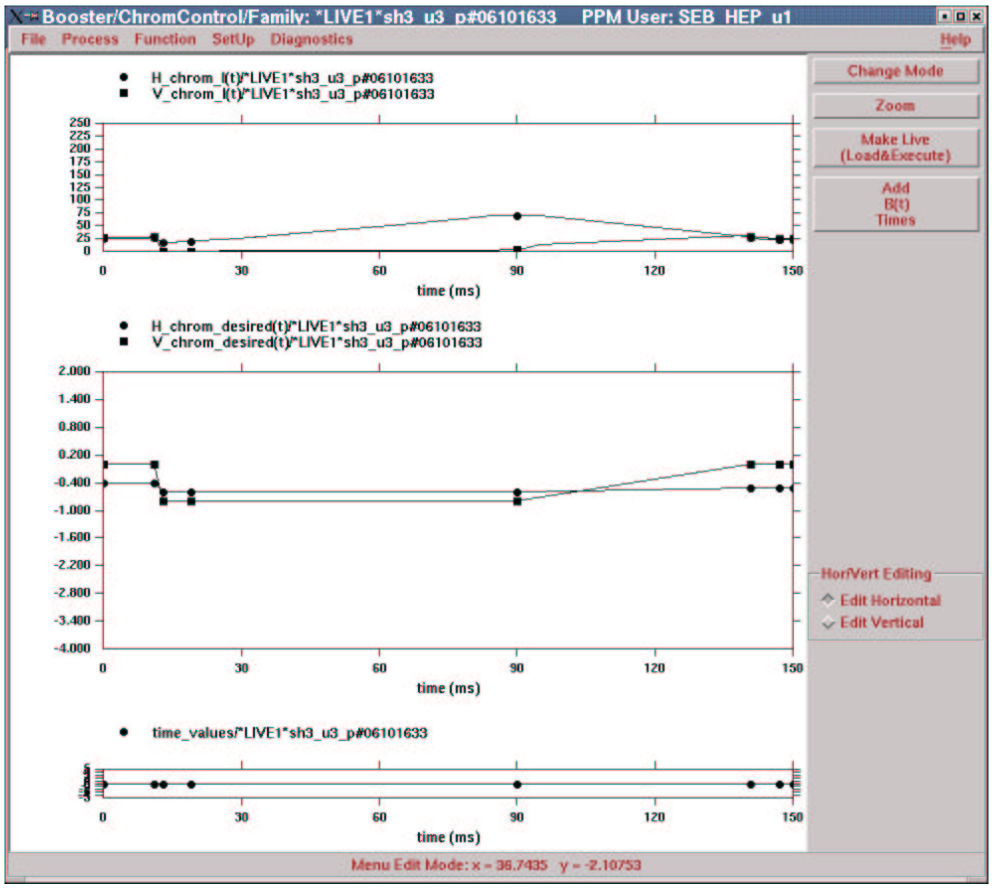

Figure 2: User Interface for Old Booster Chromaticity Control Program

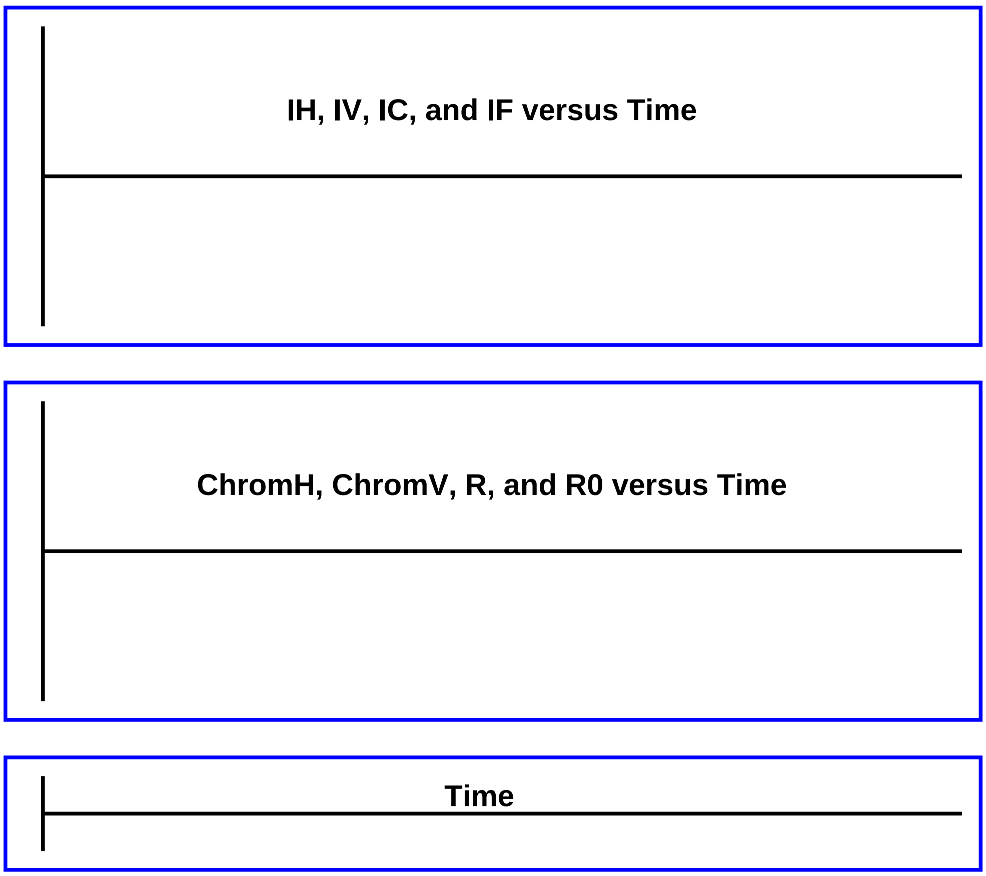

Figure 3: New User Interface

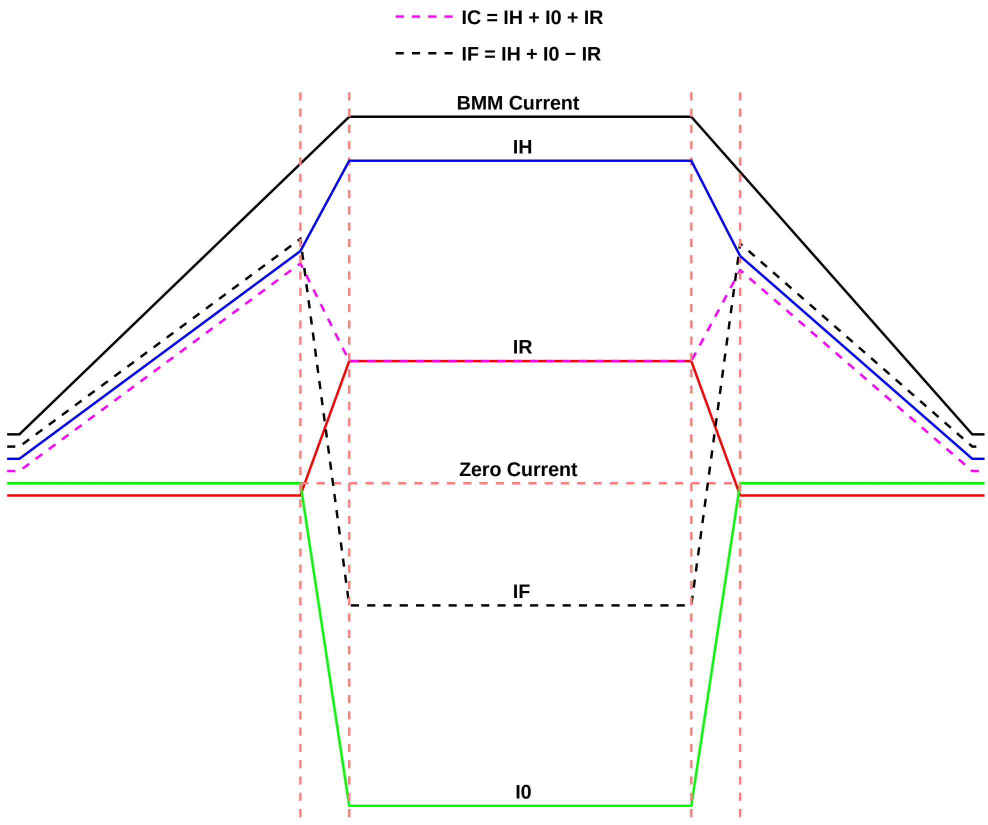

Figure 4: Booster Sextupole Currents for a Typical BAF Magnetic Cycle.

References

[1] D.A. Edwards and M.J. Syphers, "An Introduction to the Physics of High Energy Accelerators", John Wiley & Sons, Inc., New York, 1993, p. 98

[2] A.W. Chao and M. Tigner (editors), "Handbook of Accelerator Physics and Engineering", World Scientific, Singapore, 1999, p. 50

[3] E. Bleser, "Booster Sextupole Production Measurements", Booster Technical Note 182, March 13, 1991