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Title: Stopband Correction of the AGS Booster Down Feed Effect Observed at the AGS Booster and Octupole Imperfection

Study Period:	April - July 1993
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Machine:	AGS Booster
Beam:	
Tools:	
Aim:	We had already reported about our studies and data. Here we report a progressing analysis about the normal octupole imperfection.

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I Introduction

We have already reported about the beam loss by the 4th order resonances. At low intensity we observed the considerable beam loss by a 4th resonance [1-4]. It can be happen in the future that we will upgrade the stop-band correction system to correct octupole errors. A special extra windings of the correction package can afford octupole corrections [5]. But we did not know the field strength of octupole imperfections because we did not have any octupole stop-band correction strings. If we knew the field strength of octupole imperfections, it would help to design the power of octupole correction system.

The momentum dependence of the resonance correction was maybe a new phenomena observed at the AGS Booster. It was a down feed from a higher order resonance to a lower with a dispersion displacement. It produced an undesired residual beam loss [6-8]. But on the other hand a measurement of this effect would be a good tool to estimate the strength of higher order imperfections. In this report we will estimate the strength of octupole imperfections of the AGS Booster.

II Down Feed to the Sextupole Resonance

We can estimate the strength of a octupole imperfection from the radial dependence (we will refer it "slope") of sextupole correction. The slope of 3Qx=14 correction was measured at B=1.7kG (B_p=2.36Tm), dB/dt=0 G/ms, $\xi x=0$ and $\xi y=0$ [9] as

$\delta N(\cos 14X) / \delta dRset = 69$ /cm
$\delta N(\sin 14X) / \delta dRset = -63$ /cm .

Here N(cos14X) and N(sin14X) are control digit of the correction used in the computer control program. They are

$N(\cos 14X) = 125(e/cP)\ \Sigma\ \beta x\sqrt{\beta x}\ S \cos 14\theta x$
$N(\sin 14X) = 125(e/cP)\ \Sigma\ \beta x\sqrt{\beta x}\ S \sin 14\theta x$

[10] here

$$2S = \int_s^{s+\Delta s} \partial^2 By/\partial x^2\ ds' \qquad T/m$$

$$(cP/e) = 3.335641 \qquad Tm/(GeV/c)$$

$$\theta x = \int_0^s (1/\beta x)ds' \Big/ \int_0^{2\pi R} (1/\beta x)ds' .$$

The s' is 0 at the beginning of A-super period. The dRset is a control digit of the radial steering by the RF used in the computer control program. It is [9]
dRset (cm) = 319 (dP/P) . (II-3)

The down feed of the octupole imperfection to the sextupole imperfection by the dispersion is

$N(\cos 14X) = 125\ (e/cP)\ \Sigma\ \beta x\sqrt{\beta x}\ 3\ O\ \underline{h}\ (dP/P) \cos 14\theta x$

here $\underline{h}$ is a dispersion function and

$$6\ O = \int_s^{s+\Delta s} \underline{\partial}^3 By/\underline{\partial} x^3\ ds'\ T/m^2 .$$

Then the slope is

$\delta N(\cos 14X)/\delta dRset = 125\ (e/cP)\ \Sigma\ \beta x\sqrt{\beta x}\ 30\ \underline{h} \cos 14\theta x\ /319$.

The expected octupole imperfection is

$\Sigma\ O \cos 14\theta x = 319/3/125\ (cP/e)\ [\delta N(\cos 14X)/\delta dRset]/ \langle \beta x\sqrt{\beta x}\ \underline{h}\rangle$.

We will use Twiss parameters calculated by A. Luccio and M. Blaskiewicz [11].

$\langle \beta x\sqrt{\beta x}\ \underline{h}\rangle \approx 47\ m^{5/2}$.

The integrated harmonic strength of 14th octupole imperfection is

$$|\Sigma O e^{14j\theta x}| = 319/3/125\ 3.3356\ \sqrt{(69^2+63^2)}\ /\ 47$$
$$= 5.6\ T/m^2 .$$

III Down Feed to the Quadrupole Resonances

The parabolic dependence of the quadrupole corrections (2Qx=9 and 2Qy=9) on dRset was very small. But it existed. We re-analyze the dRset dependence data [6,7] by fitting with function

$$N(***9*) = Co + Cr\ (dRset) + Crr\ (dRset)^2 . \qquad \text{(VII-1)}$$

The Crr were listed in Table I. Here N(***9*) is

$$N(\cos 9X) = (10^5/2\pi)(e/cP)\ \Sigma\ \beta x\ Q\ \cos 9\theta x$$
$$N(\sin 9X) = (10^5/2\pi)(e/cP)\ \Sigma\ \beta x\ Q\ \sin 9\theta x$$

$$N(\cos 9Y) = (10^5/2\pi)(e/cP)\ \Sigma\ \beta y\ Q\ \cos 9\theta y$$
$$N(\sin 9Y) = (10^5/2\pi)(e/cP)\ \Sigma\ \beta y\ Q\ \sin 9\theta y$$

[10] and

$$Q = \int_s^{s+\Delta s} \partial By/\partial x\ ds \qquad \text{Tesla}$$

$$\theta y = \int_0^s (1/\beta y)\,ds \Big/ \int_0^{2\pi R} (1/\beta y)\,ds .$$

The fitted values were comparable to errors or smaller than errors. But obviously the parabolic term existed. The sign of parabolic terms were always the same. It was not the non-linearity of dRset because the ratio of Crr to Cr was not constant. It was reasonable that the observed Crr on 2Qx=9 and 2Qy=9 had roughly the same amplitudes and the horizontal correction phase advanced to the vertical correction phase by a few tens degrees [12].

The down feed of the octupole imperfection to the quadrupole imperfection N(cos9X) by the dispersion is

$$N(\cos 9X) = (10^5/2\pi)(e/cP)\ \Sigma\ \beta x\ 3\ O\ [\underline{h}(dP/P)]^2\ \cos 9\theta x .$$

The expected octupole imperfections can be calculated as followings.

$$\Sigma\ O\ \cos 9\theta x = 319^2/3/(10^5/2\pi)\ (cP/e)\ Crr\ \langle \beta x \underline{h}^2 \rangle$$

$$\langle \beta x \underline{h}^2 \rangle \approx 28\ m^3$$
$$\langle \beta y \underline{h}^2 \rangle \approx 21\ m^3$$

$$|\Sigma O e^{9j\theta x}| = 319^2/3/(10^5/2\pi)\ 3.3356\ \sqrt{(13^2+10^2)}\ /\ 28$$
$$= 4.2\ T/m^2$$
$$|\Sigma O e^{9j\theta y}| = 319^2/3/(10^5/2\pi)\ 3.3356\ \sqrt{(10^2+15^2)}\ /\ 21$$
$$= 6.1\ T/m^2$$

The three estimated strength of the octupole imperfection were roughly the same. They were not necessarily the same because the weight functions were not the same. But they should be close because the error sources were the same. The conclusion is;

the integrated (or accumulated) harmonic normal octupole imperfection at B=1.7kG and dB/dt=0 was about 5 T/m² .

An estimation error depends on the character of the field error. When the error sources are many (= N) and random, the estimation error is a random error; $1/\sqrt{N}$. And when error source was only one, the error is the same as a variation of twiss parameters;

$(\underline{h}max-<\underline{h}>)/<\underline{h}>$ and $(\beta max-<\beta>)/<\beta>$.

Table I

Dependence on second order term of dRset of the half integer corrections; Crr (/cm²).

Date	N(cos9X)	N(sin9X)	N(cos9Y)	N(sin9Y)
April 21	-6 ± 7	2 ± 7		
April 23	-9 ±12	14 ±17		
April 24	-20 ±25	0 ±25		
April 27	-9 ±25	25 ±20		
April 28			-9 ±11	19 ±11
April 30	-40 ±30	20 ±20		
May 7	-20 ±20	10 ±25	-12 ±15	10 ±15
weighted mean	-13 ± 7	10 ± 7	-10 ± 9	15 ± 9
(Cr (/cm)	-110	-40	-76	-25)

IV Octupole Imperfection by the Random magnet Variation

The octupole harmonic imperfection produced by the random variation of the magnets can be estimated using the equation

$$|\Sigma\ O\ e^{jn\theta}| = \sqrt{<(\Sigma\ O\ e^{jn\theta})^2>} = \sqrt{N}\ \delta Orms$$

The estimated values from each imperfections are listed in Table II. We used the rms errors reported by E.Blesser and R.Thern [13]. The strength of quadrupoles and sextupoles were set to the typical values. The down feed effect from the decapole field of the dipole magnets were also calculated. Here the rms of the horizontal displacement was assumed to be 1mm.

The calculated strength of octupole imperfection was only 0.36 T/m^2, which was much smaller than the observed strength. There should be the other source of the imperfection. The eddy current correction windings, which had strong higher order multipoles, did not work during the measurement because dB/dt was 0. The systematically distributed sextupoles could have produced only 6n-th harmonic octupole components and no 9th neither 14th harmonic components. The quadrupole magnets have rather large skew octupole component. But it is skewed and is systematic. A remanent field is a possible source of the imperfection. But we have no proof. The measurement of the slopes at the various B and dB/dt will give us more information about it.

Table II

Octupole imperfections produced by magnet imperfections.

magnet	N	(δO)rms/Br	reference	$\lvert\Sigma\ O\ e^{jn\theta}\rvert$
variation of magnets				
B	36	$2\pi/36$ X 0.14	13	0.35
QF	24	0.270 X 0.01	11,13	0.03
QD	24	0.276 X 0.01	11,13	0.03
SexF	24	0.157 X 0.02	13	0.04
SexD	24	0.237 X 0.02	13	0.05
down feed from decapole				
B	36	$2\pi/36$ X9.8 X1.0E-3	13	0.10

V Down Feed from the Sextupole Imperfections

We will check if the slope of quadrupole imperfections could predict the strength of sextupole imperfections. The strength of corrections were fitted with functions

$$N(xxxx) = Co + Cb\ B + Cbt\ (dB/dt) \quad . \qquad \text{(II-1)}$$

Here N(xxxx) is an computer control unit of correction selected for a convenience. The xxxx in the bracket is replaced by a name of resonance and a harmonic numbers. Co, Cb and Cbt are fitting parameters. The unit of B, dB/dt, P are kG, G/ms=kG/s and GeV/c respectively.

The results of correction parameters were listed in Table III, which had reported in our study reports [6,9,14,15,16]. The strength of integrated correction field when N(xxxx)=1 is listed in Table IV. The strength of harmonic sextupole and skew sextupole imperfections were calculated from parameters listed in Table III and Table IV. The results were listed in Table V.

The strength of imperfections calculated from the observation of different resonances were roughly the same. The Cbt of normal sextupole field did not agreed with each other. That suggested that the error field located mainly at the low dispersion point.

$\underline{h} \approx (2.59+2.74)/(7.57+7.0) \langle \underline{h} \rangle$
≈ 0.62 m

And $\beta x/\beta y$ might be close to 1 because the strength of 3Qx=14 and Qx+2Qy=14, and the slope of 2Qx=9 and 2Qy=9 were close. The location which satisfies the above conditions is dipole magnet at 1 and 8 of each super period. The Co of skew sextupole field also had disagreement. The remanent error field might locate at high dispersion and high βy point. The random error could have produced that kind of disagreement. But the probability was very low.

The strength of normal and skew sextupole imperfections were roughly the same for Co and Cb. But the Cbt of normal sextupoles was larger than that of skew sextupoles. Which suggested the imperfection of eddy current correction system.

Table III

Stop band correction parameters.

resonance	string	Co	Cb (/kG)	Cbt (/(G/s))
normal sextupole field				
3Qx=14	N(cos14X)	48±70	-31±34	3.49±0.43
	N(sin14X)	-129±34	40±16	6.00±0.20
Qx+2Qy=14	N(cos14XY)	5±29	14±11	4.74±0.20
	N(sin14XY)	-103±24	17± 9	2.64±0.19
2Qx=9	δN(cos9X)/δdRset	75±40	-3±12	1.06±0.29
	δN(sin9X)/δdRset	52±40	13±12	0.45±0.29
2Qy=9	δN(cos9Y)/δdRset	49±25	21± 9	0.94±0.18
	δN(sin9Y)/δdRset	- 22± 9	1± 3	-0.44±0.06
skew sextupole field				
2Qx+Qy=14	SV4*20	720±120	-152±42	6.82±0.70
	SH4*20	604± 81	30±30	-0.30±0.64
Qx+Qy=9	δN(cos9XY)/δdRset	19.9±1.0	-0.4±0.6	0.024±0.03
	δN(sin9XY)/δdRset	9.8±1.0	1.6±0.6	0.044±0.03

Table IV

Strength of correction parameters.

resonance	string	strength(mT/m)	definition
normal sextupole			
3Qx=14	N(14X)	1.09	$\Sigma\delta S\ \beta x\sqrt{\beta x}\ e^{j14\theta}/\langle\beta x\sqrt{\beta x}\rangle$
Qx+2Qy=14	N(14XY)	1.29	$\Sigma\delta S\ \beta y\sqrt{\beta x}\ e^{j14\theta}/\langle\beta y\sqrt{\beta x}\rangle$
2Qx=9	$\delta N(9X)/\delta Rset$	2.25	$\Sigma\delta S\ \beta x\underline{h}\ e^{j9\theta}/\langle\beta xh\rangle$
2Qy=9	$\delta N(9Y)/\delta Rset$	2.64	$\Sigma\delta S\ \beta y\underline{h}\ e^{j9\theta}/\langle\beta yh\rangle$
skew sextupole			
2Qx+Qy=14	SV4*20	.160	$\Sigma\delta S'\beta x\sqrt{\beta y}\ e^{j14\theta}/\langle\beta x\sqrt{\beta y}\rangle$
	SH4*20	.163	$\Sigma\delta S'\beta x\sqrt{\beta y}\ e^{j14\theta}/\langle\beta x\sqrt{\beta y}\rangle$
Qx+Qy=9	$\delta N(9XY)/\delta Rset$	21.3	$\Sigma\delta S'\sqrt{\beta x}\sqrt{\beta y}\underline{h}\ e^{j9\theta}/\langle\sqrt{\beta x}\sqrt{\beta y}\underline{h}\rangle$

Table V

Strength of sextupole imperfections

imperfection sextupole	resonance	Co T/m	Cb T/m/kG	Cbt T/m/(G/s)
normal				
	3Qx=14	150	55	7.57
	Qx+2Qy=14	130	28	7.00
	2Qx=9 (slope)	205	30	2.59
	2Qy=9 (slope)	141	55	2.74
skew				
	2Qx+Qy=14	100	25	1.09
	Qx+Qy=9 (slope)	472	35	1.07

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